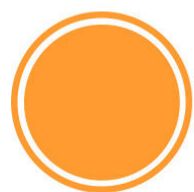


This is a compiled collection of **entire Irodov solution** videos (within JEE scope) available on my youtube channel  [Arihant Senior](#)

Release date:
April-2023

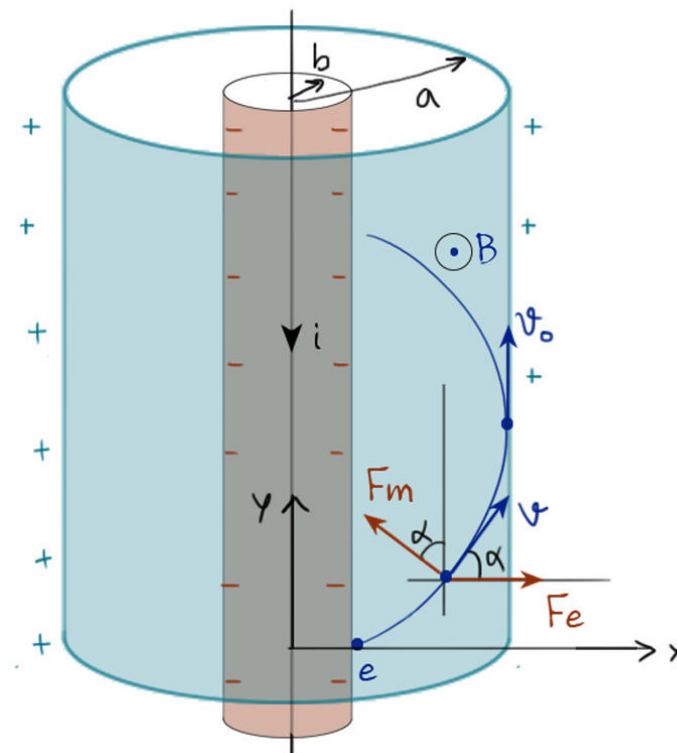
Currently, I am solving another book **Pathfinder** on  [Arihant Senior](#).
Do subscribe to get your daily dose of advanced physics!



Revision Book for **IRODOV XII**

Ankit Singhvi
Dual Degree, IIT Madras

Electrostatics
Magnetism
Optics
Modern Physics



I keep updating the videos and the book. So please download **updated version** in August 2023 from  [Arihant Senior](#)

3.1	1	3.25	16	31	46	3.109	61	3.137	76	3.163	91	3.192	106	121			
3.2	1	3.26	16	3.52	31	46	3.110	61	3.138	76	3.164	91	3.193	106	3.229	121	
3.3	2	3.27	17	3.53	32	3.82	47	3.111	62	3.139	77	3.165	92	3.194	107	3.230	122
	2	3.28	17	3.54	32	3.83	47	3.112	62	3.140	77	3.166	92	3.195	107	3.231	122
3.4	3	3.29	18	3.55	33	3.84	48	3.113	63	3.141	78	3.167	93	3.196	108	123	
3.5	3	3.30	18	3.56	33	3.85	48	63	3.142	78	3.168	93	3.197	108	123		
3.6	4	3.31	19	3.57	34	3.86	49	3.114	64	79	3.169	94	3.198	109	3.232	124	
3.7	4	3.32	19	3.58	34	3.87	49	3.115	64	79	3.170	94	3.199	109	3.233	124	
3.8	5	3.33	20	3.59	35	3.88	50	3.116	65	80	3.171	95	3.200	110	3.234	125	
3.9	5	3.34	20	3.60	35	3.89	50	3.117	65	3.143	80	3.172	95	3.201	110	3.235	125
							3.118,										
3.10	6	3.35	21	3.61	36	51	3.119	66	3.144	81	3.173	96	3.202	111	3.236	126	
3.11	6	3.36	21	3.62	36	3.90	51	3.120	66	3.145	81	3.174	96	3.203	111	3.237	126
3.12	7	3.37	22	3.63	37	3.92	52	3.121	67	3.146	82	3.175	97	3.204	112	127	
	7	3.38	22	3.64	37	3.93	52	3.122	67	3.147	82	3.176	97	3.205	112	3.238	127
3.13,																	
3.14	8	3.39	23	3.65	38	3.94	53	3.123	68	3.148	83	3.177	98	3.206	113	128	
3.15	8	3.40	23	3.66	38	3.95	53	3.124	68	3.149	83	3.178	98	3.207	113	3.239	128
3.16	9	3.41	24	3.67	39	3.96	54	3.125	69	84	3.179	99	3.208	114	3.240	129	
3.17	9	3.42	24	3.68	39	3.97	54	3.126	69	3.150	84	3.180	99	3.209	114	3.241	129
	10	3.43	25	3.69	40	3.98	55	3.127	70	3.151	85	3.181	100	3.210	115	3.242	130
	10	3.44	25	3.70	40	3.99	55	3.128	70	3.152	85	3.182	100	3.211	115	3.243	130
	11	3.45	26	3.75	41	3.100	56	3.129	71	3.153	86	3.183	101	3.219	116	3.244	131
	11	3.46	26	3.76	41	3.101	56	3.130	71	3.154	86	3.184	101	3.220	116	3.245	131
3.18	12		27	3.77	42	3.102	57	72	3.155	87	3.185	102	3.221	117	3.246	132	
3.19	12		27	42	3.103	57	3.131	72	3.156	87	3.186	102	3.222	117	3.247	132	
3.20	13		28	3.78	43	3.104	58	3.132	73	3.157	88	3.187	103	3.223	118	3.248	133
3.21	13		28	3.79	43	58	3.133	73	88	3.188	103	3.224	118	3.249	133		
3.22	14		29	3.80	44	3.105	59	3.134	74	3.158	89	3.189	104	3.225	119	134	
3.23	14	3.47	29	44	3.106	59	3.135	74	3.159	89	104	3.226	119	3.250	134		
		3.48,															
		3.49,															
3.24	15	3.50	30	3.81	45	3.107	60	75	3.160	90	3.190	105	3.227	120			
	15	3.51	30	45	3.108	60	3.136	75	3.161	90	3.191	105	3.228	120			

3.251	135	3.295	150	3.316	165	3.385	180	5.20	195		210	6.37	225	6.216	240		255	6.274	270
3.252	135	3.296	150	3.317	165	3.386	180	5.21	195	5.74	210	6.38	225	6.217	240	6.243	255	6.275	270
3.253	136	3.297	151	3.318	166	3.387	181		196	5.75	211	6.39	226	6.218	241	6.244	256	6.276	271
3.254	136	3.298	151	3.319	166	3.388	181	5.22	196	5.76	211	6.40	226	6.219	241	6.245	256	6.277	271
3.255	137	3.299	152	3.320	167	3.389	182	5.23	197		212	6.41	227	6.220	242	6.246	257	6.278	272
	137	3.300	152	3.321	167	3.390	182	5.24	197	5.77	212	6.42	227	6.221	242	6.249	257	6.279	272
3.256	138		153	3.322	168	3.391	183		198	5.79	213	6.43	228	6.222	243	6.250	258	6.280	273
3.257	138	3.301	153	3.323	168	3.392	183	5.26	198	5.80	213		228	6.223	243	6.251	258	6.289	273
3.258	139		154	3.324	169		184	5.27	199	6.1	214	6.46	229	6.224	244	6.252	259		
3.259	139	3.302	154	3.325	169		184		199	6.2	214	6.47	229	6.225	244	6.253	259		
3.260	140	3.303	155	3.326	170		185	5.28	200	6.3	215	6.48	230	6.226	245	6.254	260		
	140	3.304	155	3.327	170		185	5.31	200	6.20	215		230	6.227	245	6.255	260		
	141	3.305	156	3.328	171		186	5.32	201		216	6.49	231	6.228	246	6.256	261		
3.261	141	3.306	156	3.329	171		186	5.34	201	6.21	216	6.50	231	6.229	246	6.257	261		
3.262	142	3.307	157	3.330	172		187		202	6.22	217	6.51	232	6.230	247	6.258	262		
3.263	142	3.308	157	3.331	172		187	5.35	202		217	6.52	232	6.231	247	6.259	262		
3.264	143	3.309	158	3.332	173	3.393	188		203	6.23	218	6.53	233	6.232	248	6.260	263		
								5.36,											
3.265	143	3.310	158	3.333	173	3.394	188	5.37	203	6.24	218	6.133	233	6.233	248	6.261	263		
3.266	144		159	3.334	174		189	5.40	204	6.25	219	6.134	234	6.234	249	6.262	264		
3.269	144		159	3.335	174	3.396	189	5.41	204	6.26	219	6.135	234	6.235	249	6.263	264		
	145		160	3.336	175	3.397	190		205	6.27	220	6.136	235	6.236	250	6.264	265		
3.270	145		160	3.343	175	3.398	190		205	6.28	220	6.137	235	6.237	250	6.265	265		
3.271	146		161	3.344	176	5.13	191	5.42	206		221	6.138	236		251	6.266	266		
3.288	146		161	3.372	176	5.14	191	5.69	206	6.30	221	6.139	236	6.238	251	6.267	266		
3.289	147	3.311	162	3.373	177	5.15	192	5.70	207	6.31	222	6.140	237	6.239	252	6.268	267		
3.290	147	3.312	162	3.374	177	5.16	192	5.71	207	6.32	222	6.141	237		252	6.269	267		
3.291	148		163	3.378	178	5.17	193		208	6.33	223	6.142	238	6.240	253	6.270	268		
3.292	148	3.313	163	3.382	178	5.18	193		208	6.34	223	6.214	238	6.241	253	6.271	268		
3.293	149	3.314	164	3.383	179	5.19	194	5.72	209	6.35	224		239		254	6.272	269		
3.294	149	3.315	164	3.384	179		194	5.73	209	6.36	224	6.215	239	6.242	254	6.273	269		

3.3. Two small equally charged spheres, each of mass m , are suspended from the same point by silk threads of length l . The distance between the spheres $x \ll l$. Find the rate dq/dt with which the charge leaks off each sphere if their approach velocity varies as $v = a/\sqrt{x}$, where a is a constant.

$$v = \frac{a}{\sqrt{x}} \Rightarrow a_c = v \frac{dv}{dx} = -\frac{a}{\sqrt{x}} - a \left(-\frac{1}{2} \right) \frac{1}{x^{3/2}}$$

$$\Rightarrow a_c = -\frac{a^2}{2x^2}$$

Also,

$$|ma_c| = T \sin \theta - \frac{kq^2}{x^2} \quad T = mg \cos \theta + \frac{kq^2}{x^2} \sin \theta$$

$$\Rightarrow \frac{ma^2}{2x^2} = mg \cos \theta \sin \theta + \frac{kq^2}{x^2} \sin^2 \theta - \frac{kq^2}{x^2}$$

$$\frac{ma^2}{2x^2} = mg \sin \theta \cos \theta - \frac{kq^2}{x^2} \cos^2 \theta$$

$$\frac{ma^2}{2x^2} = mg \left(\frac{x}{2l} \right) (1) - \frac{kq^2}{x^2} (1)$$

Rearranging:

$$\frac{kq^2}{m} = \frac{gx^3}{2l} - \frac{a^2}{2}$$

Neglecting a^2 compared to $gx^3/2l$ (reason explained in next slide)

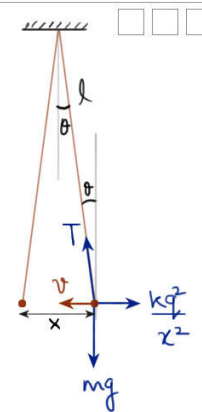
$$q = \sqrt{\frac{mg}{2kl}} x^{3/2}$$

differentiate wrt t

$$\Rightarrow \frac{dq}{dt} = \frac{3}{2} \sqrt{\frac{mg}{2kl}} x^{1/2} \cdot \frac{dx}{dt}$$

$$\Rightarrow \frac{dq}{dt} = \frac{3a}{2} \sqrt{\frac{mg}{2kl}}$$

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Why did we neglect a^2

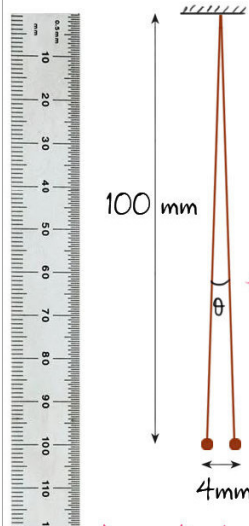
$$\frac{kq^2}{m} = \frac{gx^3}{2l} - \frac{a^2}{2}$$



In a gold leaf electroscope experiment, the charged leaves when left alone, come back closer together in hours. As charge leakage is very slow. Its not humanly possible to notice it move (like an hour hand of a clock).

$$v = \frac{a}{\sqrt{x}}$$

However, in our case, let's assume large velocity so we get larger value of a . Let the speed of the balls be 1mm/sec. Which is noticeable to eye. From shown positions, they stick together in less than 2 seconds.



A practical setup

$$\frac{kq^2}{m} = \frac{gx^3}{2l} - \frac{a^2}{2}$$

$$\frac{10 (4 \times 10^{-3})^3}{2 \times 100 \times 10^{-3}} = 3.2 \times 10^{-6}$$

$$\frac{v^2 x}{2} = \frac{(10^{-3})^2 \cdot (4 \times 10^{-3})}{2} = 2 \times 10^{-9}$$

So even with us considering a large a , term 2 is still 1600 times smaller than 1.

In reality, it will be even lesser than this. So its safe to neglect the second term.

Note: If instead of charge leaking in air, it was leaked via conduction to ground or something, this whole method wont work as a would be quite high.

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3.4. Two positive charges q_1 and q_2 are located at the points with radius vectors $\vec{r}_1$ and $\vec{r}_2$. Find a negative charge q_3 and a radius vector $\vec{r}_3$ of the point at which it has to be placed for the force acting on each of the three charges to be equal to zero.



Equilibrium condition for q_3 :

$$q_1 q_3 \frac{(\vec{r}_3 - \vec{r}_1)}{|\vec{r}_3 - \vec{r}_1|^3} + q_3 q_2 \frac{(\vec{r}_3 - \vec{r}_2)}{|\vec{r}_3 - \vec{r}_2|^3} = 0 \quad (1)$$

Also unit vectors along the line:

$$\frac{\vec{r}_3 - \vec{r}_1}{|\vec{r}_3 - \vec{r}_1|} = -\frac{(\vec{r}_2 - \vec{r}_3)}{|\vec{r}_3 - \vec{r}_2|} \quad (2) = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|} \quad (3)$$

From 1 and 2,

$$\frac{q_1}{|\vec{r}_3 - \vec{r}_1|^2} = \frac{q_2}{|\vec{r}_3 - \vec{r}_2|^2} \Rightarrow \sqrt{q_1} |\vec{r}_3 - \vec{r}_2| = \sqrt{q_2} |\vec{r}_3 - \vec{r}_1|$$

$$\Rightarrow \sqrt{q_1} (\vec{r}_3 - \vec{r}_2) = \sqrt{q_2} (\vec{r}_1 - \vec{r}_3) \Rightarrow \vec{r}_3 = \frac{\sqrt{q_2} \vec{r}_1 + \sqrt{q_1} \vec{r}_2}{\sqrt{q_1} + \sqrt{q_2}} \quad (4)$$

Equilibrium condition for q_1 :

$$q_1 q_3 \frac{(\vec{r}_1 - \vec{r}_3)}{|\vec{r}_1 - \vec{r}_3|^3} + q_1 q_2 \frac{(\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3} = 0$$

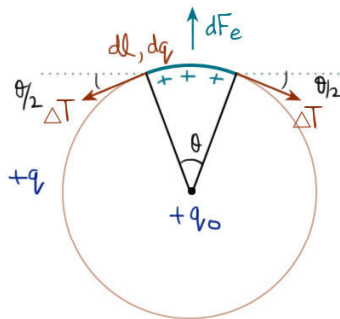
From 3 $\rightarrow q_3 = \frac{-q_2 |\vec{r}_1 - \vec{r}_3|^2}{|\vec{r}_2 - \vec{r}_1|^2}$

Putting $\vec{r}_3$ from 4

$$q_3 = \frac{-q_1 q_2}{(\sqrt{q_1} + \sqrt{q_2})^2} //$$

Youtube / Arihant Senior

3.5. A thin wire ring of radius r has an electric charge q . What will be the increment of the force stretching the wire if a point charge q_0 is placed at the ring's centre?



Let ΔT be increment in tension once q_0 is placed at center

And dF_e be the force on small charge dq due to q_0

As the dl element is still at rest, extra forces are balanced.

$$\therefore dF_e = 2\Delta T \sin \theta/2$$

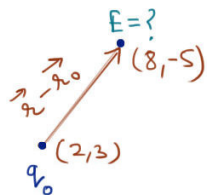
$$\frac{k q_0 dq}{r^2} = \Delta T \theta$$

$$\Rightarrow \frac{k q_0 \left(\frac{\theta}{2\pi} q \right)}{r^2} = \Delta T \theta //$$

$$\Rightarrow \Delta T = \frac{k q_0 q}{2\pi r^2} //$$

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3.6. A positive point charge $50 \mu\text{C}$ is located in the plane xy at the point with radius vector $\vec{r}_0 = 2\hat{i} + 3\hat{j}$, where $\hat{i}$ and $\hat{j}$ are the unit vectors of the x and y axes. Find the vector of the electric field strength $\vec{E}$ and its magnitude at the point with radius vector $\vec{r} = 8\hat{i} - 5\hat{j}$. Here r_0 and r are expressed in metres.



$$\vec{E} = \frac{kq(\vec{r} - \vec{r}_0)}{|\vec{r} - \vec{r}_0|^3} = \frac{9 \times 10^9 \cdot 50 \times 10^{-6}}{10^3} \frac{(6\hat{i} - 8\hat{j})}{5} = 4500 \frac{(3\hat{i} - 4\hat{j})}{5} //$$

$$|\vec{E}| = 4500 //$$

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3.7. Point charges q and $-q$ are located at the vertices of a square with diagonals $2l$ as shown in Fig. 3.1. Find the magnitude of the electric field strength at a point located symmetrically with respect to the vertices of the square at a distance x from its centre.

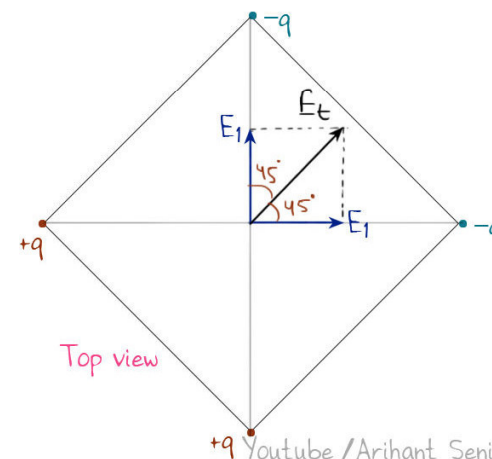
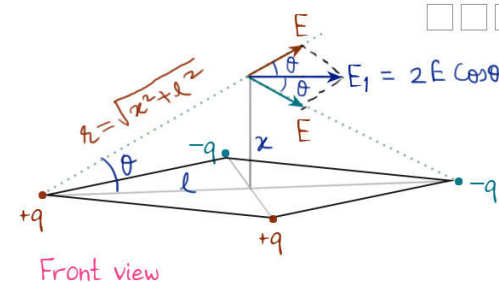
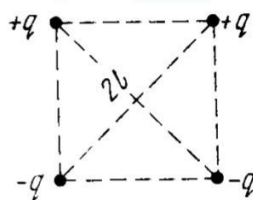
Let E be field due to one charge q and E_1 be the field due to one pair of opposite charges. Then,

$$E_1 = 2E \cos \theta$$

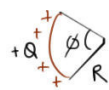
$$= 2 \frac{kq \cdot l}{r^2 \cdot r} = \frac{kq l}{(x^2 + l^2)^{3/2}}$$

$$\text{Net field } E_t = 2E_1 \cos 45^\circ$$

$$= 2 \left[\frac{2kq l}{(x^2 + l^2)^{3/2}} \right] \frac{1}{\sqrt{2}} = \frac{2\sqrt{2} kq l}{(x^2 + l^2)^{3/2}} //$$

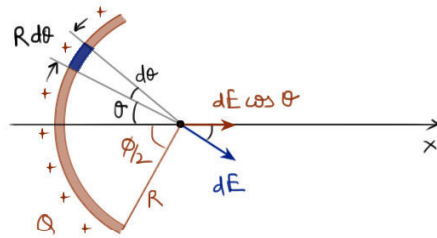


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E due to uniformly charged circular arc

Charge on element: $dq = \frac{d\theta}{\phi} \cdot Q$



E_x due to element: $dE_x = dE \cos \theta$

$$= \frac{k dq}{R^2} \cos \theta$$

$$= \frac{k}{R^2} \left(\frac{d\theta}{\phi} \right) Q \cdot \cos \theta$$

$$E = \int dE_x = \frac{kQ}{\phi R^2} \int_{-\phi/2}^{+\phi/2} \cos \theta \, d\theta = \frac{kQ}{R^2} \frac{\sin \phi/2}{\phi/2} //$$

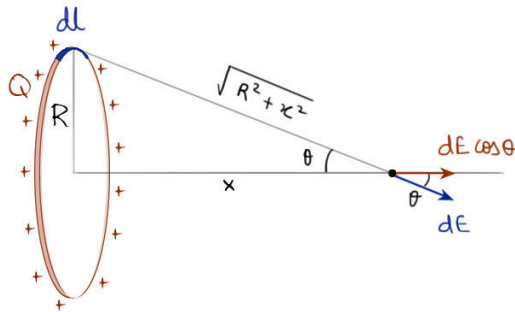
Due to symmetry, field along the normal will be zero. Only E_x remains.

$E = \frac{2kQ}{\pi R^2}$

$E = 2\sqrt{2} \frac{kQ}{\pi R^2}$

Youtube / Arihant Senior

E due to a uniformly charged ring at its axial point



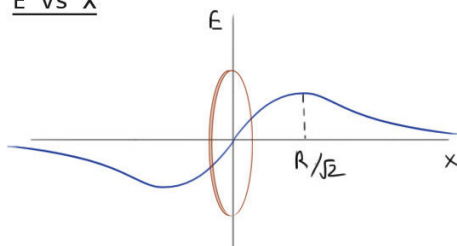
$$dE_x = dE \cos \theta = \frac{k dq}{(R^2 + x^2)} \cdot \frac{x}{\sqrt{R^2 + x^2}}$$

$$E = \int dE_x = \frac{kx}{(R^2 + x^2)^{3/2}} \int dq$$

$$\Rightarrow E_{axial} = \frac{kQx}{(x^2 + R^2)^{3/2}}$$

Due to symmetry, field along the normal will be zero. Only E_x remains.

E vs X



Position where E is maximum

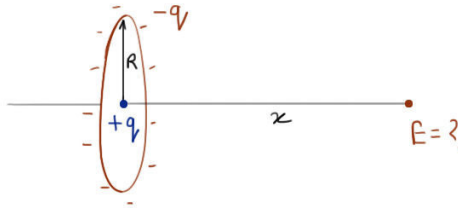
$$\frac{dE}{dx} = 0 \Rightarrow kQ \left[\frac{(x^2 + R^2)^{-3/2} (1) - x \cdot \frac{3}{2} (x^2 + R^2)^{-5/2} \cdot 2x}{(x^2 + R^2)^3} \right] = 0$$

$$\Rightarrow [x^2 + R^2 - 3x^2] = 0$$

$$\Rightarrow R^2 = 2x^2 \Rightarrow E_{max} \text{ at } x = \frac{R}{\sqrt{2}}$$

Youtube / Arihant Senior

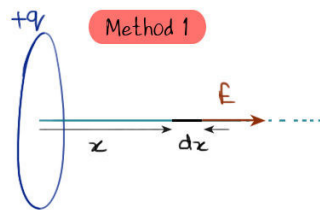
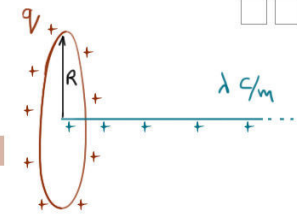
3.10. A point charge q is located at the centre of a thin ring of radius R with uniformly distributed charge $-q$. Find the magnitude of the electric field strength vector at the point lying on the axis of the ring at a distance x from its centre, if $x \gg R$.



$$\begin{aligned}
 E &= -\frac{kqx}{(x^2+R^2)^{3/2}} + \frac{kq}{x^2} \\
 &= \frac{kq}{x^2} \left[1 - \frac{x^3}{(x^2+R^2)^{3/2}} \right] \\
 &= \frac{kq}{x^2} \left[1 - \left(1 + \frac{R^2}{x^2} \right)^{-3/2} \right] \\
 &\approx \frac{kq}{x^2} \left[1 - \left(1 - \frac{3R^2}{2x^2} \right) \right] = \frac{3}{2} \frac{kqR^2}{x^4} //
 \end{aligned}$$

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3.11. A system consists of a thin charged wire ring of radius R and a very long uniformly charged thread oriented along the axis of the ring, with one of its ends coinciding with the centre of the ring. The total charge of the ring is equal to q . The charge of the thread (per unit length) is equal to λ . Find the interaction force between the ring and the thread.

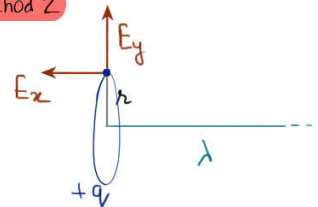


We know that E due to ring at distance x is: $E = \frac{kqx}{(x^2+R^2)^{3/2}}$

∴ Force dF on small element dx of wire is:

$$dF = dqE = (\lambda dx) \frac{kqx}{(x^2+R^2)^{3/2}} \Rightarrow F = kq\lambda \int_0^\infty \frac{x dx}{(x^2+R^2)^{3/2}} \xrightarrow{(x \rightarrow R \tan \theta)} F = \frac{kq\lambda}{R} //$$

Method 2



We know that E due to semi-infinite wire is:

$$E_x = \frac{k\lambda}{R}, \quad E_y = \frac{k\lambda}{R}$$

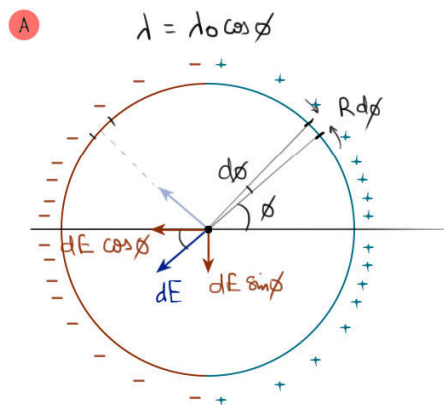
Force on ring due to y components will cancel out.

$$F = qE_x = \frac{kq\lambda}{R} //$$

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3.12. A thin nonconducting ring of radius R has a linear charge density $\lambda = \lambda_0 \cos \phi$, where λ_0 is a constant, ϕ is the azimuthal angle. Find the magnitude of the electric field strength

- (a) at the centre of the ring;
 (b) on the axis of the ring as a function of the distance x from its centre. Investigate the obtained function at $x \gg R$.



Charge on element: $dq = \lambda R d\phi = \lambda_0 R \cos \phi d\phi$

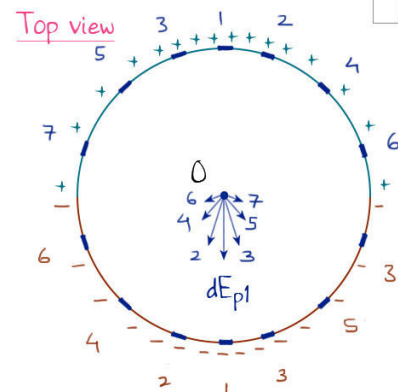
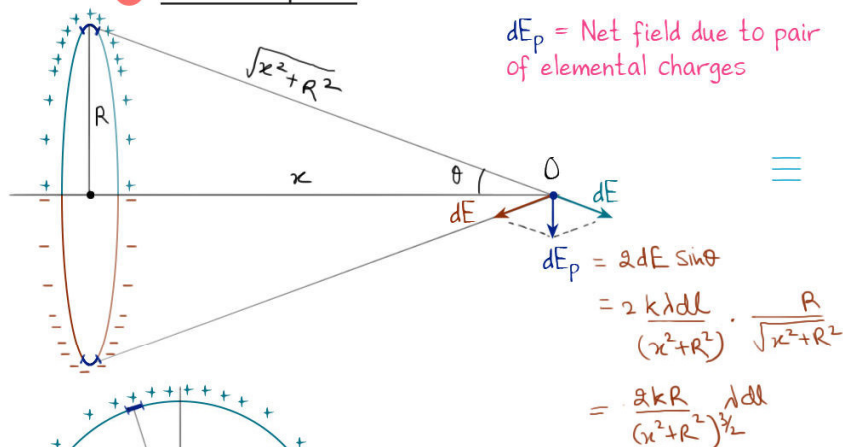
Due to symmetry vertical components will cancel out.
 Only E_x remains.

E_x due to element: $dE_x = dE \cos \phi$
 $= \frac{k (\lambda_0 R \cos \phi d\phi)}{R^2} \cos \phi$

$E_x = \int dE_x = \frac{k \lambda_0}{R} \int_0^{2\pi} \cos^2 \phi d\phi$

$E_x = \frac{k \pi \lambda_0}{R}$ // Youtube / Arihant Senior

B At axial point

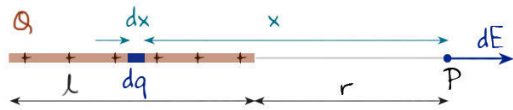


Due to symmetry, only Y component remains.

$\therefore dE_{net} = dE_p \cos \phi$
 $= \frac{2kR}{(x^2 + R^2)^{3/2}} \lambda dl \cdot \cos \phi$
 $= \frac{2kR}{(x^2 + R^2)^{3/2}} \lambda_0 \cos \phi R d\phi \cos \phi = \frac{2k \lambda_0 R^2}{(x^2 + R^2)^{3/2}} \int_0^{\pi} \cos^2 \phi d\phi = \frac{k \pi \lambda_0 R}{(x^2 + R^2)^{3/2}}$

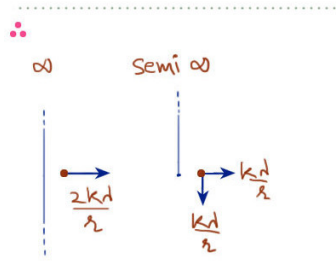
Electric field due to uniformly charged rod

A At an axial point

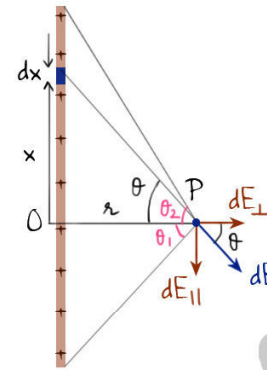


$$E = \int dE = \int \frac{k dq}{x^2} = \int \frac{k \lambda dx}{x^2} = \frac{k \lambda}{x^2} \int_{x_1}^{x_2} dx$$

$$= \frac{k \lambda}{x^2} \left(-\frac{1}{x} \right)_{x_1}^{x_2} \Rightarrow E_{\text{axial}} = \frac{k \lambda}{x^2} \left(\frac{1}{x_1} - \frac{1}{x_2} \right)$$



B At a general point



$$dE_{\parallel} = dE \sin \theta$$

$$= \frac{k \lambda dx}{(x^2 + R^2)^{3/2}} \cdot \frac{x}{\sqrt{x^2 + R^2}}$$

$x \rightarrow R \tan \theta$
integrating

$$E_{\parallel} = \frac{k \lambda}{R} \int_{-\theta_1}^{\theta_2} \sin \theta d\theta$$

$$E_{\parallel} = \frac{k \lambda}{R} [\cos \theta_2 - \cos \theta_1]$$

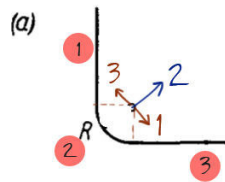
$$dE_{\perp} = dE \cos \theta = \frac{k \lambda dx}{(x^2 + R^2)^{3/2}} \cdot \frac{R}{\sqrt{x^2 + R^2}}$$

$x \rightarrow R \tan \theta$
integrating

$$E_{\perp} = \frac{k \lambda}{R} \int_{-\theta_1}^{\theta_2} \cos \theta d\theta \Rightarrow E_{\perp} = \frac{k \lambda}{R} [\sin \theta_2 + \sin \theta_1]$$

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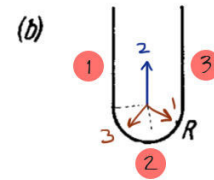
3.15. A thread carrying a uniform charge λ per unit length has the configurations shown in Fig. 3.2 a and b. Assuming a curvature radius R to be considerably less than the length of the thread, find the magnitude of the electric field strength at the point O.



E_1 and E_3 will cancel each other.

$$E_2 = \frac{k \lambda}{R^2} \frac{\sin \phi/2}{\phi/2} = \frac{k \lambda}{R^2} \left(\frac{2\pi R}{\pi} \right) \cdot \frac{\sin(\pi/2)}{\pi/2}$$

$$= \frac{\sqrt{2} k \lambda}{R}$$



$$E_1 + E_3 = 2E_{\parallel} = 2 \left(\frac{k \lambda}{R} \right)$$

For semi-infinite rod

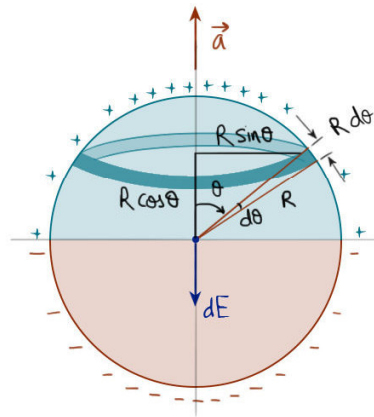
$$E_2 = \frac{k \lambda}{R^2} \frac{\sin \phi/2}{\phi/2} = \frac{k \lambda \pi R}{R^2} \cdot \frac{\sin \pi/2}{\pi/2}$$

$$= \frac{2 k \lambda}{R}$$

$$\Rightarrow E_1 + E_3 + E_2 = 0$$

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3.16. A sphere of radius r carries a surface charge of density $\sigma = \vec{a} \cdot \vec{r}$ where $\vec{a}$ is a constant vector, and $\vec{r}$ is the radius vector of a point of the sphere relative to its centre. Find the electric field strength vector at the centre of the sphere.



$$\sigma = \vec{a} \cdot \vec{r} = aR \cos \theta$$

$$dq = \sigma dA = (aR \cos \theta) (2\pi R \sin \theta \cdot R d\theta)$$

Using, $E = \frac{kQ}{(x^2 + R^2)^{3/2}}$

Field due to elemental ring dE at center:

$$dE = \frac{k dq (R \cos \theta)}{R^3} = \frac{k (aR \cos \theta) (2\pi R \sin \theta \cdot R d\theta) \cdot R \cos \theta}{R^3}$$

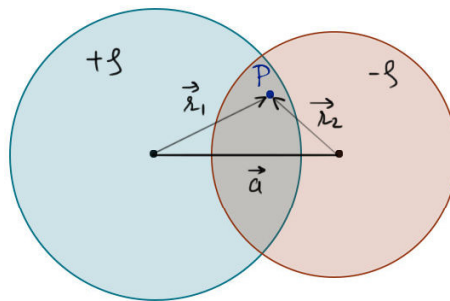
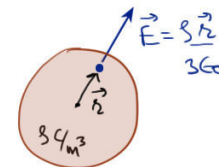
$$E = \int dE = 2\pi k a R \int_0^\pi \cos^2 \theta \sin \theta d\theta$$

$$= \frac{aR}{3\epsilon_0} \downarrow = -\frac{R}{3\epsilon_0} \vec{a}$$

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Field inside cavity of two overlapping spheres.

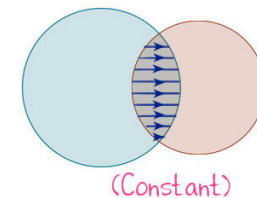
We know that inside a uniformly charged sphere, electric field is: $\vec{E} = \frac{\rho}{3\epsilon_0} \vec{r}$



∴ When two uniformly charged positive and negative spheres of same charge density are overlapped. Field in the cavity is given by superposition principle. That is:

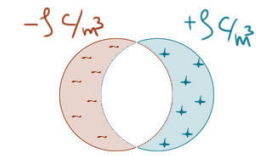
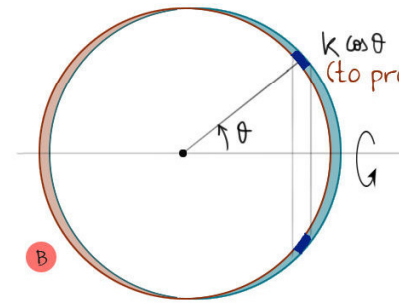
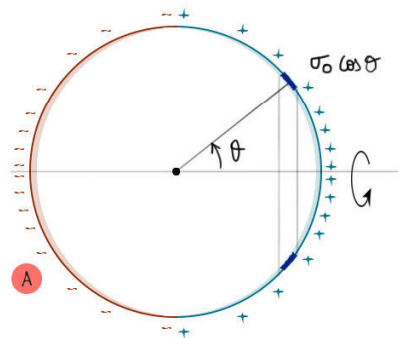
$$E_P = \vec{E}_1 + \vec{E}_2 = \frac{\rho}{3\epsilon_0} \vec{r}_1 + \frac{(-\rho)}{3\epsilon_0} \vec{r}_2$$

$$= \frac{\rho}{3\epsilon_0} (\vec{r}_1 - \vec{r}_2) = \frac{\rho}{3\epsilon_0} \vec{a}$$



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3.17. Suppose the surface charge density over a sphere of radius R depends on a polar angle θ as $\sigma = \sigma_0 \cos \theta$, where σ_0 is a positive constant. Show that such a charge distribution can be represented as a result of a small relative shift of two uniformly charged balls of radius R whose charges are equal in magnitude and opposite in sign. Resorting to this representation, find the electric field strength vector inside the given sphere.

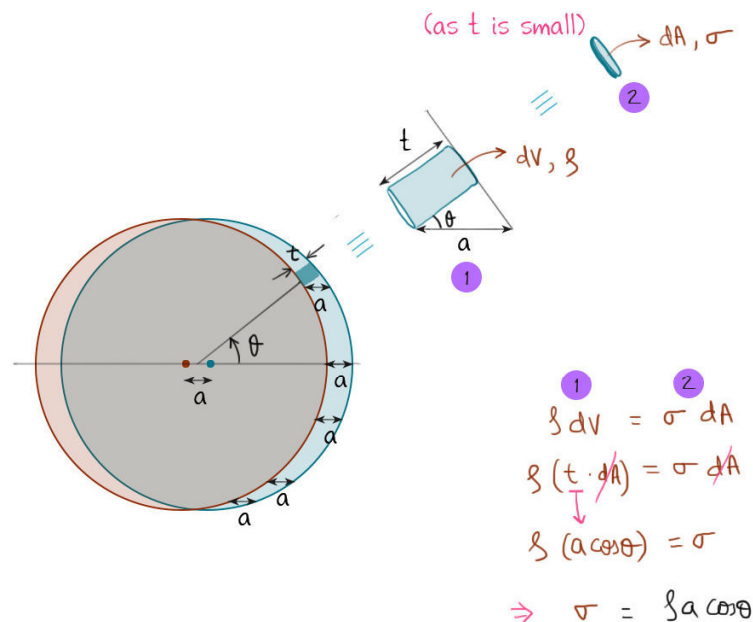


Overlapping almost completely

We need to,

show that surface charge distribution of form: $\sigma = \sigma_0 \cos \theta$ on a shell (A) is similar to that of a almost overlapping pair of similar, uniform and oppositely charged solid spheres (B)

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Calculating field inside shell using our derived result:

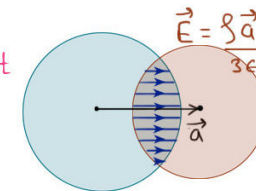
$\sigma_{\text{shell}} = \sigma_{\text{overlapping spheres}}$

$$\sigma_0 \cos \theta = \oint a \cos \theta$$

$$\Rightarrow \oint a = \sigma_0$$

$$\Rightarrow \oint = \frac{\sigma_0}{a}$$

Using a known result for field inside two overlapping shell:



∴ Field inside the given shell is:

$$\vec{E} = \frac{\oint \vec{a}}{3\epsilon_0} = \left(\frac{\sigma_0}{a} \right) \frac{\vec{a}}{3\epsilon_0} = \frac{\sigma_0}{3\epsilon_0}$$

Hence proved that charge distribution of overlapping spheres is similar to that of shell with surface charge density: $\sigma = \sigma_0 \cos \theta$

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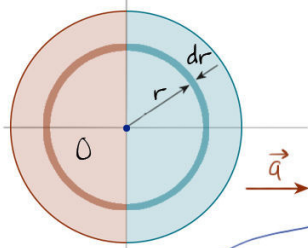
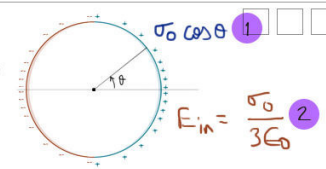


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3.18. Find the electric field strength vector at the centre of a ball of radius R with volume charge density $\rho = \vec{a} \cdot \vec{r}$ where $\vec{a}$ is a constant vector, and $\vec{r}$ is a radius vector drawn from the ball's centre.



On small element on this shell

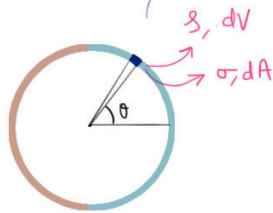
$$\begin{aligned} \sigma_r dA &= \rho dV \\ &= \rho dz dA \\ &= (a r \cos \theta) dz dA \end{aligned}$$

$$\Rightarrow \sigma_r = (a r dz) \cos \theta$$

This is similar to 1. So field inside due to elemental shell is constant. And its value is given by 2. That is..

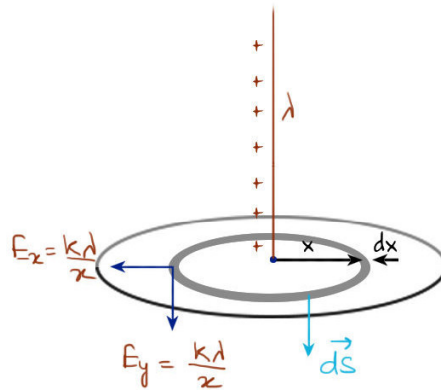
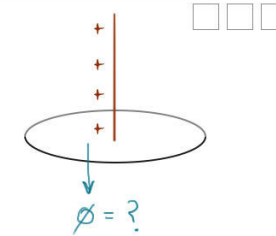
$$dE = \frac{a r dz}{3 \epsilon_0} \Rightarrow E = \frac{a}{3 \epsilon_0} \int_0^R r dr = \frac{a R^2}{6 \epsilon_0}$$

We will use this result for a shell (we derived it in problem 3.17)



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3.19. A very long uniformly charged thread oriented along the axis of a circle of radius R rests on its centre with one of the ends. The charge of the thread per unit length is equal to λ . Find the flux of the vector $\vec{E}$ across the circle area.



$$d\phi = \vec{E} \cdot d\vec{s}$$

$$= E_y ds$$

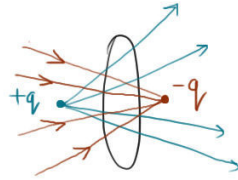
$$= \frac{k\lambda}{x} (2\pi x dx)$$

$$\Rightarrow \phi = k\lambda 2\pi \int_0^R dx$$

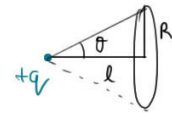
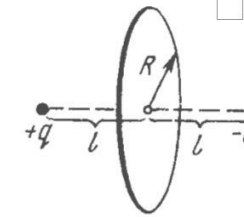
$$= \frac{\lambda R}{2 \epsilon_0}$$

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3.20. Two point charges q and $-q$ are separated by the distance $2l$ (Fig. 3.3). Find the flux of the electric field strength vector across a circle of radius R .



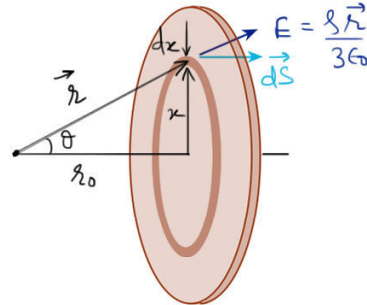
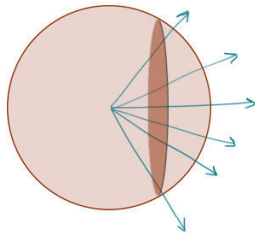
By symmetry, flux due to both charges passing through circle will be same



$$\begin{aligned} \therefore \phi &= 2 \times \frac{q}{\epsilon_0} \frac{2\pi(1-\cos\theta)}{4\pi} \longrightarrow \begin{array}{l} \text{Solid angle subtended by circle} \\ \text{Total solid angle} \end{array} \\ &= \frac{q}{\epsilon_0} \left(1 - \frac{l}{\sqrt{l^2 + R^2}}\right) \end{aligned}$$

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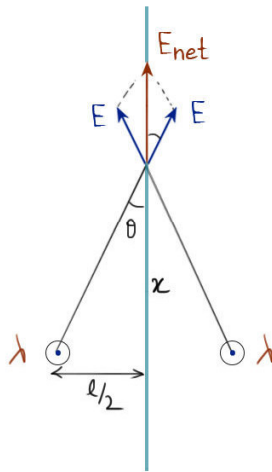
3.21. A ball of radius R is uniformly charged with the volume density ρ . Find the flux of the electric field strength vector across the ball's section formed by the plane located at a distance $r_0 < R$ from the centre of the ball.



$$\begin{aligned} d\phi &= \vec{E} \cdot d\vec{S} \\ &= \frac{\rho}{3\epsilon_0} (\vec{r} \cdot d\vec{S}) \\ &= \frac{\rho}{3\epsilon_0} (r \, dS \cos\theta) \\ &= \frac{\rho \sqrt{r_0^2 + r^2}}{3\epsilon_0} \cdot 2\pi r \, dr \cdot \frac{r_0}{\sqrt{r_0^2 + r^2}} \\ \phi &= \int d\phi = \frac{2\pi \rho r_0}{3\epsilon_0} \int_0^{\sqrt{R^2 - r_0^2}} r \, dr \\ &= \frac{\pi \rho r_0}{3\epsilon_0} (R^2 - r_0^2) \end{aligned}$$

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3.22. Each of the two long parallel threads carries a uniform charge λ per unit length. The threads are separated by a distance l . Find the maximum magnitude of the electric field strength in the symmetry plane of this system located between the threads.



$$\begin{aligned}
 E_{\text{net}} &= 2E \cos \theta \\
 &= 2 \frac{2k\lambda}{\sqrt{x^2 + \frac{l^2}{4}}} \cdot \frac{x}{\sqrt{x^2 + \frac{l^2}{4}}} \\
 &= 4k\lambda \frac{x}{(x^2 + \frac{l^2}{4})}
 \end{aligned}$$

$\Rightarrow$ 0 at $x=0$ $\Rightarrow$ maximum value in between 0 & ∞
 $\Rightarrow$ 0 at $x=\infty$

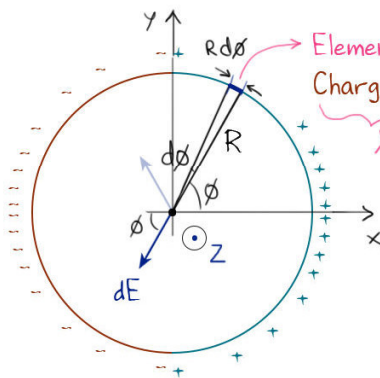
For maximum E

$$\frac{dE_{\text{net}}}{dx} = 0 \Rightarrow (x^2 + \frac{l^2}{4})(1) - x(2x) = 0$$

$$\Rightarrow x = \pm l/2 \Rightarrow E_{\text{max}} = \frac{4k\lambda \cdot l/2}{(\frac{l^2}{4} + \frac{l^2}{4})} = \frac{4k\lambda}{l}$$

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3.23. An infinitely long cylindrical surface of circular cross-section is uniformly charged lengthwise with the surface density $\sigma = \sigma_0 \cos \phi$, where ϕ is the polar angle of the cylindrical coordinate system whose z axis coincides with the axis of the given surface. Find the magnitude and direction of the electric field strength vector on the z axis.



Elemental wire going in the plane.

Charge on 1 m of elemental wire = σdA

$$= (\sigma_0 \cos \phi) (R d\phi (1))$$

$$\Rightarrow d\lambda = \sigma_0 R \cos \phi d\phi$$

Field due to elemental wire on a point on Z axis:

$$dE = \frac{2k d\lambda}{R} = \frac{2k}{R} \cdot \sigma_0 R \cos \phi d\phi = 2k\sigma_0 \cos \phi d\phi$$

Due to symmetry, field along Y axis is zero.

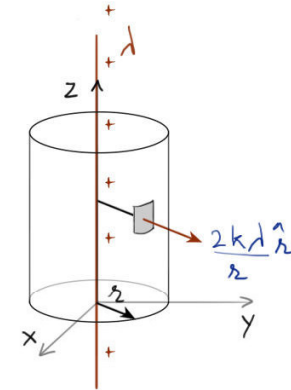
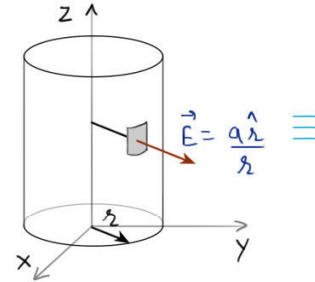
$$\therefore E_{\text{net}} = \int_0^{2\pi} dE \cos \phi = 2k\sigma_0 \int_0^{2\pi} \cos^2 \phi d\phi = 2k\pi\sigma_0 = \frac{\sigma_0}{2\epsilon_0}$$

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3.24. The electric field strength depends only on the x and y coordinates according to the law $\vec{E} = a(x\hat{i} + y\hat{j})/(x^2 + y^2)$, where a is a constant, $\hat{i}$ and $\hat{j}$ are the unit vectors of the x and y axes. Find the flux of the vector $\vec{E}$ through a sphere of radius R with its centre at the origin of coordinates.

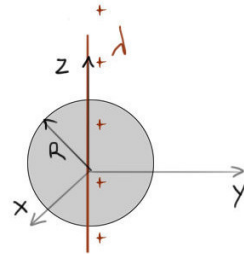
Method 1

$$\vec{E} = \frac{a(x\hat{i} + y\hat{j})}{x^2 + y^2} = \frac{a}{\underbrace{\sqrt{x^2 + y^2}}_r} \underbrace{\frac{x\hat{i} + y\hat{j}}{\sqrt{x^2 + y^2}}}_{\text{unit vector } \hat{r}} = \frac{a\hat{r}}{r} \equiv$$



$$\Rightarrow a = 2k\lambda$$

$$\Rightarrow \lambda = \frac{a}{2k}$$

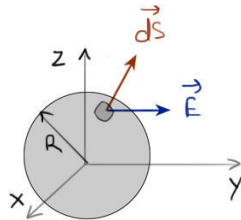


∴ By Gauss's law flux through sphere:

$$\phi = \frac{q_{in}}{\epsilon_0} = \frac{\lambda(2R)}{\epsilon_0} = \frac{a}{2k} \cdot \frac{2R}{\epsilon_0} = 4\pi a R //$$

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Method 2



$$\vec{E} = \frac{a(x\hat{i} + y\hat{j})}{x^2 + y^2}$$

$$\vec{dS} = dS \frac{(x\hat{i} + y\hat{j} + z\hat{k})}{\sqrt{x^2 + y^2 + z^2}} = \frac{dS (x\hat{i} + y\hat{j} + z\hat{k})}{R}$$

Flux through small area dS :

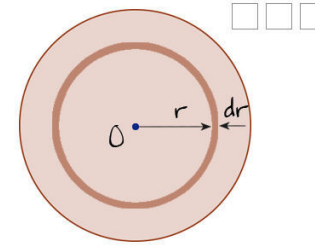
$$d\phi = \vec{E} \cdot \vec{dS} = \frac{a}{x^2 + y^2} \frac{dS (x^2 + y^2)}{R}$$

$$\phi = \int d\phi = \frac{a}{R} \int dS = \frac{a}{R} 4\pi R^2 = 4\pi a R //$$

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3.25. A ball of radius R carries a positive charge whose volume density depends only on a separation r from the ball's centre as $\rho = \rho_0 (1 - r/R)$, where ρ_0 is a constant. Assuming the permittivities of the ball and the environment to be equal to unity, find:

- (a) the magnitude of the electric field strength as a function of the distance r both inside and outside the ball;
 (b) the maximum intensity $E_{\max}$ and the corresponding distance r_m .



Charge on elemental shell:

$$dq = \rho_r dV$$

$$= \rho_0 \left(1 - \frac{r}{R}\right) 4\pi r^2 dr$$

Charge inside the area enclosed by shell:

$$q_r = \int dq = 4\pi \rho_0 \int_0^r r^2 \left(1 - \frac{r}{R}\right) dr$$

$$= 4\pi \rho_0 \left(\frac{r^3}{3} - \frac{r^4}{4R} \right)$$

$$= 4\pi \rho_0 r^3 \left(\frac{1}{3} - \frac{r}{4R} \right)$$

A :: E_r inside by gauss's law:

$$E_r \cdot 4\pi r^2 = \frac{q_r}{\epsilon_0}$$

$$\Rightarrow E_r = \frac{4\pi \rho_0 r^3 \left(\frac{1}{3} - \frac{r}{4R} \right)}{4\pi r^2 \epsilon_0}$$

$$\Rightarrow E_{in} = \frac{\rho_0 r}{\epsilon_0} \left(\frac{1}{3} - \frac{r}{4R} \right)$$

E_r outside:

$$E_{out} \cdot 4\pi r^2 = \frac{q_R}{\epsilon_0} \Rightarrow E_{out} = \frac{4\pi \rho_0 R^3 \left(\frac{1}{3} - \frac{R}{4R} \right)}{4\pi r^2 \epsilon_0} = \frac{\rho_0 R^3}{12 \epsilon_0 r^2}$$

B $\frac{dE_{in}}{dr} = 0$, For $E_{\max}$

$$\Rightarrow \frac{1}{3} - \frac{2r}{4R} = 0 \Rightarrow r = \frac{2R}{3}$$

$$\Rightarrow E_{\max} = \frac{\rho_0}{\epsilon_0} \frac{2R}{3} \left(\frac{1}{3} - \frac{1}{6} \right)$$

$$= \frac{\rho_0 R}{9 \epsilon_0}$$

3.26. A system consists of a ball of radius R carrying a spherically symmetric charge and the surrounding space filled with a charge of volume density $\rho = \alpha/r$, where α is a constant, r is the distance from the centre of the ball. Find the ball's charge at which the magnitude of the electric field strength vector is independent of r outside the ball. How high is this strength? The permittivities of the ball and the surrounding space are assumed to be equal to unity.

We wish to find E_p . So we find charge enclosed and use gauss's law.

Charge enclosed: $q_1 + q_2$ | $q_2 = \int dq_2 = \int \rho 4\pi r^2 dr = \int \frac{\alpha}{r} 4\pi r^2 dr$

$$= 4\pi \alpha \int_R^x r dr = 2\pi \alpha (x^2 - R^2)$$

Gauss's law on spherical surface at radius x :

$$E \cdot 4\pi x^2 = \frac{(q_1 + q_2)}{\epsilon_0}$$

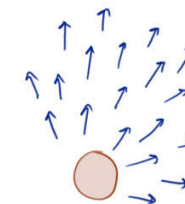
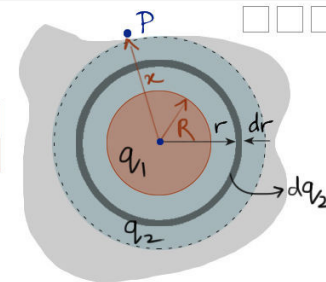
$$E = \frac{1}{4\pi \epsilon_0} \left(\frac{q_1 + q_2}{x^2} \right) = \frac{1}{4\pi \epsilon_0} \left(\frac{q_1}{x^2} + 2\pi \alpha \left(1 - \frac{R^2}{x^2} \right) \right)$$

$$E = \frac{\alpha}{2\epsilon_0} + \frac{1}{4\pi \epsilon_0} \frac{q_1 - 2\pi \alpha R^2}{x^2}$$

If E is independent of x , then

Constant E will then be:

$$E = \frac{\alpha}{2\epsilon_0}$$



Uniform radial E

3.27. A space is filled up with a charge with volume density $\rho = \rho_0 e^{-\alpha r^3}$, where ρ_0 and α are positive constants, r is the distance from the centre of this system. Find the magnitude of the electric field strength vector as a function of r . Investigate the obtained expression for the small and large values of r , i.e. at $\alpha r^3 \ll 1$ and $\alpha r^3 \gg 1$.

As charge distribution is symmetric, using Gauss's law:

$$\begin{aligned}
 E_r (4\pi r^2) &= \frac{\int dq}{\epsilon_0} \\
 E_r &= \frac{1}{4\pi r^2} \int \frac{\rho dv}{\epsilon_0} \\
 &= \frac{1}{4\pi r^2} \int_0^r \left(\frac{\rho_0 e^{-\alpha r^3}}{\epsilon_0} \right) 4\pi r^2 dr \\
 &\quad \downarrow \begin{array}{l} -\alpha r^3 \rightarrow t \\ \text{Integrating} \end{array} \\
 E_r &= \frac{\rho_0}{3\epsilon_0 \alpha r^2} (1 - e^{-\alpha r^3}) //
 \end{aligned}$$

$$\alpha r^3 \ll 1 \quad e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \dots$$

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} \dots$$

When x is small:

$$e^{-x} \approx 1 - x$$

$$\Rightarrow 1 - e^{-\alpha r^3} \approx 1 - (1 - \alpha r^3) = \alpha r^3$$

$$\Rightarrow E_r = \frac{\rho_0 r}{3\epsilon_0 \alpha} //$$

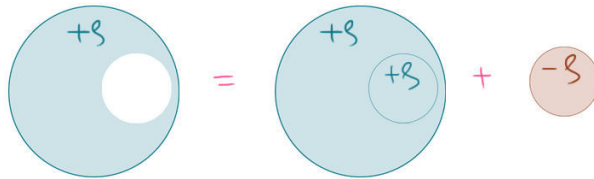
$$\alpha r^3 \gg 1$$

$$1 - e^{-\alpha r^3} \approx 1 \Rightarrow E_r = \frac{\rho_0}{3\epsilon_0 \alpha r^2} //$$

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Electric field inside a spherical cavity within a sphere.

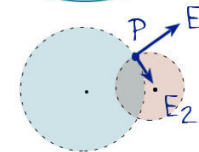
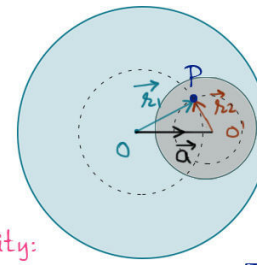
By principle of superposition:



∴ field at any point is now vector sum of both individual full spheres.

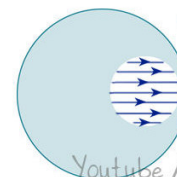
At a point P inside cavity:

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$



$$= \frac{\rho \vec{r}_1}{3\epsilon_0} + \frac{(-\rho) \vec{r}_2}{3\epsilon_0} = \frac{\rho (\vec{r}_1 - \vec{r}_2)}{3\epsilon_0} = \frac{\rho \vec{a}}{3\epsilon_0} //$$

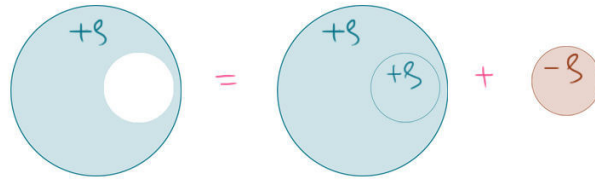
(A constant)



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Electric field inside a cylindrical cavity within a cylinder

By principle of superposition:



∴ field at any point is now vector sum of both individual full cylinders.

At a point P inside cavity:

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= \frac{2k\lambda_1}{r_1} \hat{r}_1 + \frac{2k\lambda_2}{r_2} \hat{r}_2$$

$\lambda = \rho \times (\text{Volume of 1m cylinder})$

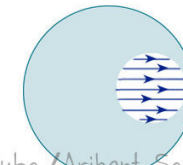
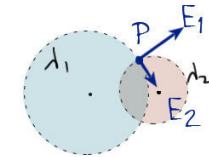
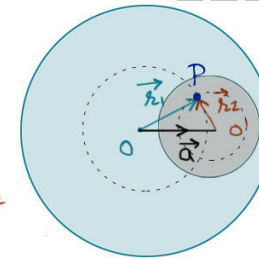
$$\lambda_1 = \rho A_1(l) \quad \lambda_2 = -\rho A_2(l)$$

$$= \rho \pi r_1^2 \quad = -\rho \pi r_2^2$$

$$E = \frac{2k(\rho \pi r_1^2)}{r_1} \hat{r}_1 + \frac{2k(-\rho \pi r_2^2)}{r_2} \hat{r}_2$$

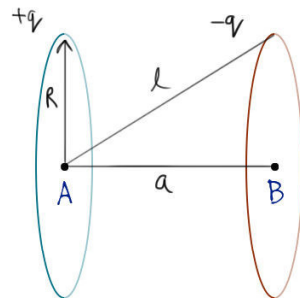
$$= 2k\rho\pi (\hat{r}_1 - \hat{r}_2)$$

$$= \frac{\rho \vec{a}}{2\epsilon_0} \quad (\text{A constant})$$



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3.30. There are two thin wire rings, each of radius R , whose axes coincide. The charges of the rings are q and $-q$. Find the potential difference between the centres of the rings separated by a distance a .



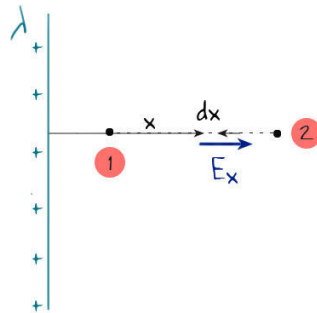
$$V_A - V_B$$

$$= \left[\frac{kq}{R} + \frac{k(-q)}{l} \right] - \left[\frac{k(-q)}{R} + \frac{kq}{l} \right]$$

$$= 2kq \left(\frac{1}{R} - \frac{1}{l} \right)$$

$$= 2kq \left(\frac{1}{R} - \frac{1}{\sqrt{R^2 + a^2}} \right)$$

3.31. There is an infinitely long straight thread carrying a charge with linear density $\lambda = 0.40 \mu\text{C/m}$. Calculate the potential difference between points 1 and 2 if point 2 is removed $\eta = 2.0$ times farther from the thread than point 1.



$$\Delta V = V_2 - V_1 = - \int_1^2 E \cdot dx$$

$$dV = - \vec{E}_x \cdot d\vec{x}$$

$$V = \int dV = - \int_a^{\eta a} \frac{2k\lambda}{x} dx$$

$$= -2k\lambda \ln \eta$$

$$\Rightarrow \Delta V = V_2 - V_1 = -2k\lambda \ln \eta //$$

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3.32. Find the electric field potential and strength at the centre of a hemisphere of radius R charged uniformly with the surface density σ .

$$V = \frac{kQ}{R} = \frac{k\sigma \cdot 2\pi R^2}{R} = \frac{\sigma R}{2\epsilon_0} //$$

On elemental ring:

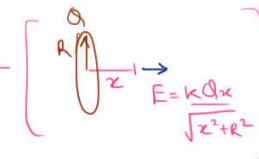
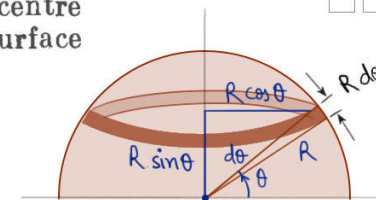
$$dq = \sigma \cdot 2\pi R \cos \theta \cdot R d\theta$$

dE at O due to elemental ring:

$$dE = \frac{k dq R \sin \theta}{(R^2 \sin^2 \theta + R^2 \cos^2 \theta)^{3/2}}$$

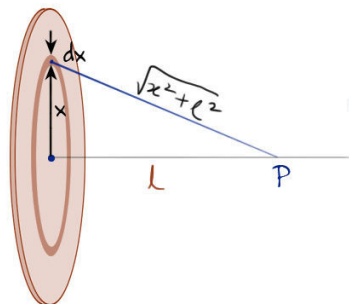
$$= \frac{k \sigma \cdot 2\pi R^3}{R^3} \sin \theta \cos \theta d\theta$$

$$E = \int dE = \sigma \pi k \int_0^{\pi/2} \sin 2\theta d\theta = \sigma \pi k = \frac{\sigma}{4\epsilon_0} //$$



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3.33. A very thin round plate of radius R carrying a uniform surface charge density σ is located in vacuum. Find the electric field potential and strength along the plate's axis as a function of a distance l from its centre. Investigate the obtained expression at $l \rightarrow 0$ and $l \gg R$.



$$dq = \sigma 2\pi x dx$$

$$dV \text{ at } P: dV = \frac{k dq}{\sqrt{x^2 + l^2}}$$

$$V = \int dV = k \sigma 2\pi \int_0^R \frac{x dx}{\sqrt{x^2 + l^2}}$$

$x^2 + l^2 \rightarrow t$
Integrating $\downarrow$

$$V = \frac{\sigma}{2\epsilon_0} (\sqrt{R^2 + l^2} - l)$$

$$dE \text{ at } P: dE = \frac{k dq l}{(x^2 + l^2)^{3/2}}$$

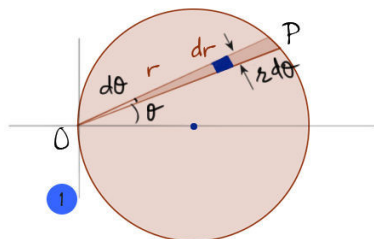
$$= k \sigma 2\pi l \int_0^R \frac{x dx}{(x^2 + l^2)^{3/2}}$$

$x^2 + l^2 \rightarrow t$
Integrating $\downarrow$

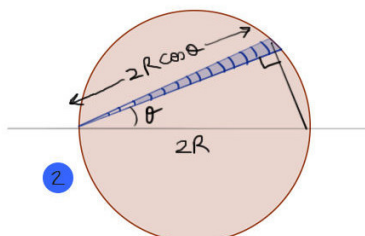
$$E = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{l}{\sqrt{R^2 + l^2}} \right)$$

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3.34. Find the potential ϕ at the edge of a thin disc of radius R carrying the uniformly distributed charge with surface density σ .



(dV_1 : V due to element inside a wedge at θ)



(dV_2 : V due to one wedge at θ)

$$dq = \sigma dA$$

$$= \sigma dr (r d\theta)$$

Potential due to element:

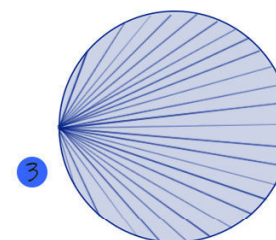
$$1 \quad dV_1 = \frac{k dq}{r} = \frac{k \sigma r dr d\theta}{r}$$

$$= k \sigma \int dr d\theta$$

For given $\theta \downarrow$

$$2 \quad dV_2 = k \sigma \int_0^{2R \cos \theta} dr d\theta$$

$$= k \sigma 2R \cos \theta d\theta$$



(V : Total V due to all wedges)

$$3 \quad V = \int dV = k \sigma 2R \int_{-\pi/2}^{+\pi/2} \cos \theta d\theta$$

$$= 4k \sigma R = \frac{\sigma R}{\epsilon_0}$$

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3.35. Find the electric field strength vector if the potential of this field has the form $\varphi = \mathbf{a} \cdot \mathbf{r}$, where $\mathbf{a}$ is a constant vector, and $\mathbf{r}$ is the radius vector of a point of the field.

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

Let $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ (α, β, γ are constants)

Given:

$$V = \vec{a} \cdot \vec{r} = \alpha x + \beta y + \gamma z$$

We know that:

$$\begin{aligned}\vec{E} &= -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k} \\ &= -\alpha\hat{i} - \beta\hat{j} - \gamma\hat{k} = -\vec{a}_{//}\end{aligned}$$

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3.36. Determine the electric field strength vector if the potential of this field depends on x, y coordinates as

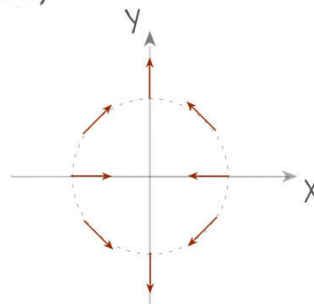
a) $\varphi = a(x^2 - y^2)$; b) $\varphi = axy$,

where a is a constant. Draw the approximate shape of these fields using lines of force (in the x, y plane).

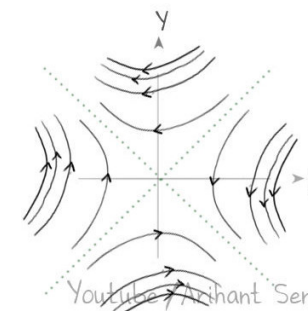
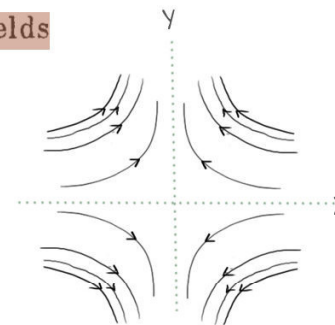
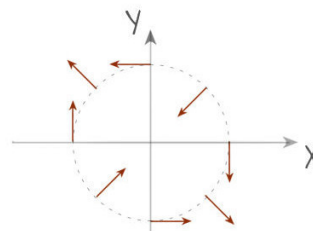
Using: $\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$

$$\begin{aligned}\vec{E} &= -2ax\hat{i} + 2ay\hat{j} \\ &= -2a(x\hat{i} - y\hat{j}) \quad \text{A}\end{aligned}$$

Plotting at critical points at same radius



$$\begin{aligned}\vec{E} &= -ay\hat{i} - ax\hat{j} \\ &= -a(y\hat{i} + x\hat{j}) \quad \text{B}\end{aligned}$$



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3.37. The potential of a certain electrostatic field has the form $\varphi = a(x^2 + y^2) + bz^2$, where a and b are constants. Find the magnitude and direction of the electric field strength vector. What shape have the equipotential surfaces in the following cases:

(a) $a > 0, b > 0$; (b) $a > 0, b < 0$?

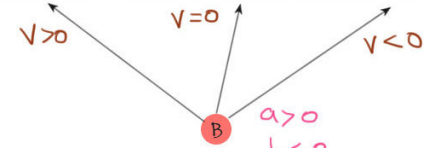
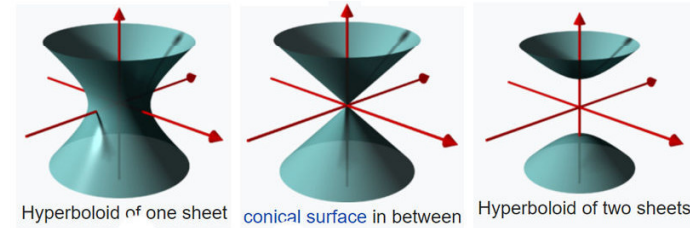
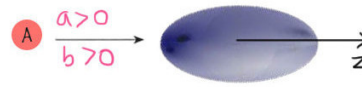
We know that: $\vec{E} = -\vec{\nabla} V$ where, $\vec{\nabla} = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k}$

$$\begin{aligned} \therefore \vec{E} &= -\vec{\nabla} [a(x^2 + y^2) + bz^2] \\ &= -2ax \hat{i} - 2ay \hat{j} - 2bz \hat{k} \end{aligned}$$

$$|\vec{E}| = 2\sqrt{a^2x^2 + a^2y^2 + b^2z^2}$$

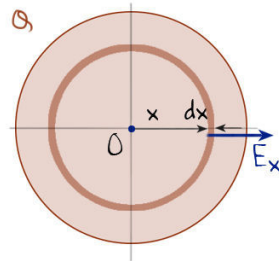
At equipotential surface, V is constant. That is:

$$\frac{x^2}{V/a} + \frac{y^2}{V/a} + \frac{z^2}{V/b} = 1$$



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Potential due to uniformly charged sphere.



We know,

$$E_{in} = \frac{kQx}{R^3}$$

$$E_{out} = \frac{kQ}{x^2}$$

For V_{in}

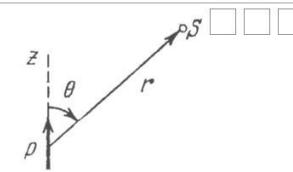
$$\begin{aligned} \int_0^V dV &= -\int_{\infty}^x E_x dx \\ &= -\int_{\infty}^R E_{out} dx - \int_R^x E_{in} dx \\ &= -\int_{\infty}^R \frac{kQ}{x^2} dx - \int_R^x \frac{kQx}{R^3} dx \\ &= \frac{kQ}{R} - \left[\frac{kQx^2}{2R^3} - \frac{kQ}{2R} \right] \\ &= \frac{3kQ}{2R} - \frac{kQx^2}{2R^3} \quad (x \leq R) \end{aligned}$$

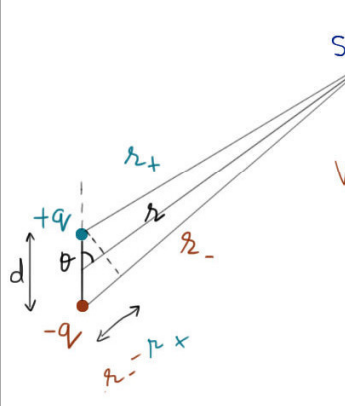
For V_{out}

$$\begin{aligned} \int_0^V dV &= -\int_{\infty}^x E_x dx \\ &= -\int_{\infty}^x \frac{kQ}{x^2} dx \\ &= \frac{kQ}{x} \quad (x > R) \end{aligned}$$

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3.39. Demonstrate that the potential of the field generated by a dipole with the electric moment $\mathbf{p}$ (Fig. 3.4) may be represented as $\varphi = \mathbf{p} \cdot \mathbf{r} / 4\pi\epsilon_0 r^3$, where $\mathbf{r}$ is the radius vector. Using this expression, find the magnitude of the electric field strength vector as a function of r and θ .





$$V_S = \frac{kq}{r_+} + \frac{k(-q)}{r_-}$$

$$= kq \left(\frac{r_- - r_+}{r_+ r_-} \right)$$

$$\approx kq \frac{d \cos \theta}{r^2}$$

$$\Rightarrow V_S = \frac{k \vec{p} \cdot \vec{r}}{r^3} = \frac{k p \cos \theta}{r^2}$$

$$\vec{E} = -\vec{\nabla} V, \quad \vec{\nabla} = \frac{\partial}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial}{\partial \theta} \hat{\theta}$$

$$\therefore E_r = -\frac{\partial}{\partial r} \left(\frac{k p \cos \theta}{r^2} \right) = \frac{2 k p \cos \theta}{r^3} \hat{r}$$

$$E_\theta = -\frac{1}{r} \frac{\partial}{\partial \theta} \left(\frac{k p \cos \theta}{r^2} \right) = \frac{k p \sin \theta}{r^3} \hat{\theta}$$

↓

$$E_{\text{total}} = \sqrt{E_r^2 + E_\theta^2} = \frac{k p}{r^3} \sqrt{1 + 3 \cos^2 \theta}$$

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3.40. A point dipole with an electric moment p oriented in the positive direction of the z axis is located at the origin of coordinates. Find the projections $E_{\parallel}$ and $E_{\perp}$ of the electric field strength vector (on the plane perpendicular to the z axis at the point S (see Fig. 3.4)). At which points is $\mathbf{E}$ perpendicular to $\mathbf{p}$?

$$E_{\perp} = E_\theta \cos \theta + E_r \sin \theta$$

$$= \frac{3 k p \sin \theta \cos \theta}{r^3}$$

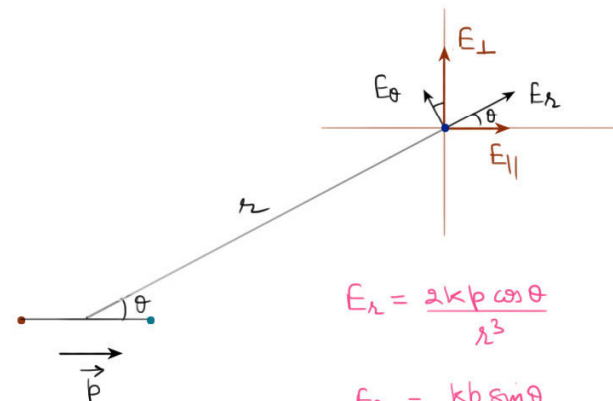
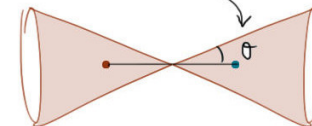
$$E_{\parallel} = E_r \cos \theta - E_\theta \sin \theta$$

$$= \frac{k p}{r^3} (3 \cos^2 \theta - 1)$$

$$\text{If } \vec{E} \perp \vec{p}, \quad E_{\parallel} = 0$$

$$\Rightarrow \theta = \pm \cos^{-1} \left(\frac{1}{\sqrt{3}} \right)$$

i.e. this conical surface,



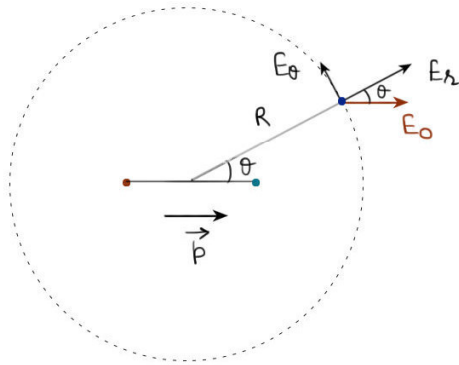
$$E_r = \frac{2 k p \cos \theta}{r^3}$$

$$E_\theta = \frac{k p \sin \theta}{r^3}$$

(Derived in 3.39)

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3.41. A point electric dipole with a moment $\vec{p}$ is placed in the external uniform electric field whose strength equals E_0 , with $\vec{p} \uparrow \uparrow \vec{E}_0$. In this case one of the equipotential surfaces enclosing the dipole forms a sphere. Find the radius of this sphere.



Let the radius of required sphere be R .

For spherical equipotential surface, net Electric field must be radial for all values of θ .

$$E_\theta = E_0 \sin \theta$$

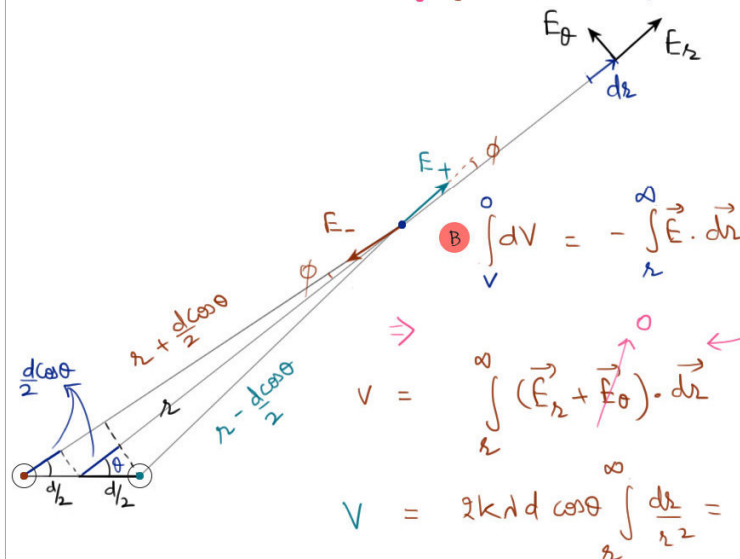
$$\frac{kps \sin \theta}{R^3} = E_0 \sin \theta$$

$$\Rightarrow R = \left(\frac{kp}{E_0} \right)^{1/3}$$

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3.42. Two thin parallel threads carry a uniform charge with linear densities λ and $-\lambda$. The distance between the threads is equal to l . Find the potential of the electric field and the magnitude of its strength vector at the distance $r \gg l$ at the angle θ to the vector l (Fig. 3.5).

We will find E first, then by $-\int \vec{E} \cdot d\vec{r}$ we shall find V .



$$E_\theta \approx E_+ - E_- \quad (\text{As } \theta \text{ is small})$$

$$= \frac{2k\lambda}{r - \frac{d \cos \theta}{2}} - \frac{2k\lambda}{r + \frac{d \cos \theta}{2}}$$

$$E_\theta = \frac{2k\lambda}{r^2} \cdot \frac{2d \cos \theta}{2} = \frac{2k\lambda d \cos \theta}{r^2} \quad (1)$$

$$E = -\frac{\partial V}{\partial r} \hat{r} - \frac{1}{r} \frac{\partial V}{\partial \theta} \hat{\theta}$$

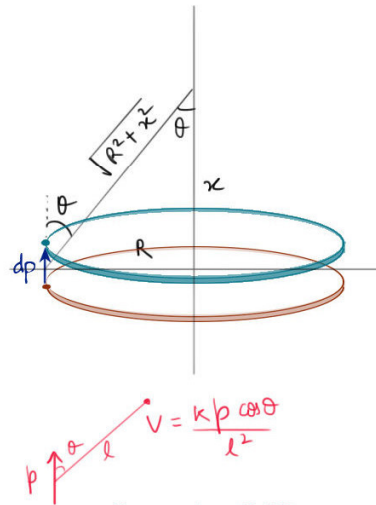
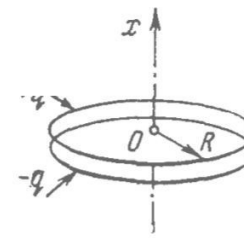
$$\Rightarrow E_\theta = \frac{2k\lambda d \sin \theta}{r^2}$$

$1/2 \Rightarrow$

$$E_{\text{net}} = \frac{2k\lambda d}{r^2}$$

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3.43. Two coaxial rings, each of radius R , made of thin wire are separated by a small distance l ($l \ll R$) and carry the charges q and $-q$. Find the electric field potential and strength at the axis of the system as a function of the x coordinate (Fig. 3.6). Show in the same drawing the approximate plots of the functions obtained. Investigate these functions at $|x| \gg R$.



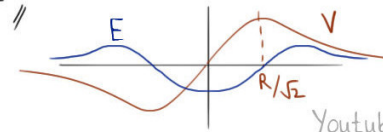
Derived in 3.39

$$dV = \frac{k dp \cos \theta}{(R^2 + x^2)}$$

$$\begin{aligned} V &= \frac{k \cos \theta}{(R^2 + x^2)} \int dp \\ &= \frac{kx}{(R^2 + x^2)^{3/2}} ql \\ &= \frac{kqlx}{(R^2 + x^2)^{3/2}} \end{aligned}$$

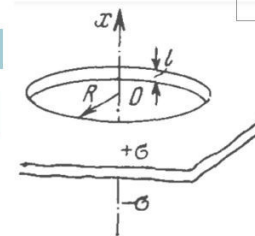
As potential is function of x only,

$$\begin{aligned} E &= E_x = -\frac{dV}{dx} \\ &= -kql \frac{d}{dx} \left(\frac{x}{(R^2 + x^2)^{3/2}} \right) \\ &= -kql \frac{(R^2 - 2x^2)}{(R^2 + x^2)^{5/2}} \end{aligned}$$



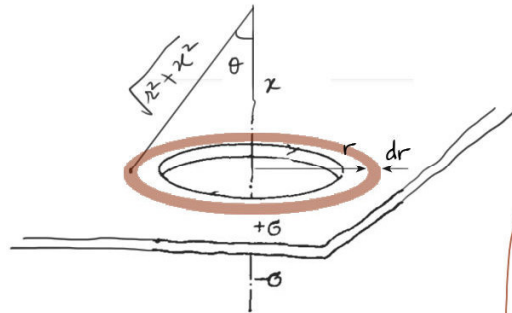
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3.44. Two infinite planes separated by a distance l carry a uniform surface charge of densities σ and $-\sigma$ (Fig. 3.7). The planes have round coaxial holes of radius R , with $l \ll R$. Taking the origin O and the x coordinate axis as shown in the figure, find the potential of the electric field and the projection of its strength vector E_x on the axes of the system as functions of the x coordinate. Draw the approximate plot $\varphi(x)$.



$$V = \frac{kqlx}{(R^2 + x^2)^{3/2}}$$

Derived in 3.43



$$dV = \frac{k \sigma (2\pi r dr) l x}{(x^2 + R^2)^{3/2}}$$

integrating
 $r \rightarrow x \tan \theta$
 $dr \rightarrow x \sec^2 \theta d\theta$

$$V = 2\pi k \sigma l x \int \frac{(x \tan \theta) x \sec^2 \theta d\theta}{x^3 \sec^3 \theta}$$

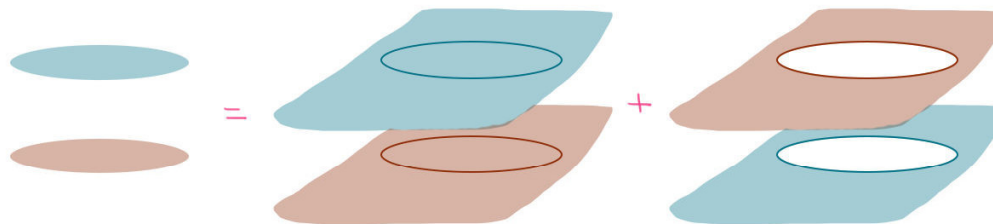
$$V = 2\pi k \sigma l \int_0^{\pi/2} \sin \theta d\theta \cos^2 \theta \frac{x}{\sqrt{R^2 + x^2}}$$

$$\Rightarrow V = \frac{2\pi k \sigma l x}{\sqrt{R^2 + x^2}}$$

$$\Rightarrow E = -\frac{dV}{dx} = -\frac{2\pi k \sigma l R^2}{(R^2 + x^2)^{3/2}}$$

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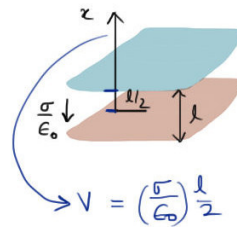
3.45. An electric capacitor consists of thin round parallel plates, each of radius R , separated by a distance l ($l \ll R$) and uniformly charged with surface densities σ and $-\sigma$. Find the potential of the electric field and the magnitude of its strength vector at the axes of the capacitor as functions of a distance x from the plates if $x \gg l$. Investigate the obtained expressions at $x \gg R$.



Two parallel sheets + flipped sheets with holes from 3.44

Due to full parallel plates, net field is zero

$$\therefore E = \frac{2\pi k \sigma l R^2}{(R^2 + x^2)^{3/2}}$$



Due to full parallel plates, only potential is due to field between the plates. Therefore:

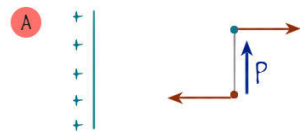
$$V = \pm \frac{\sigma l}{2\epsilon_0} - \frac{2\pi k \sigma l x}{\sqrt{R^2 + x^2}}$$

+ for $x > 0$
- for $x < 0$

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3.46. A dipole with an electric moment $\mathbf{p}$ is located at a distance r from a long thread charged uniformly with a linear density λ . Find the force $\mathbf{F}$ acting on the dipole if the vector $\mathbf{p}$ is oriented

- along the thread;
- along the radius vector $\mathbf{r}$;
- at right angles to the thread and the radius vector $\mathbf{r}$.

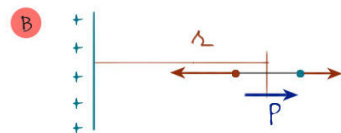


$$F = 0 //$$

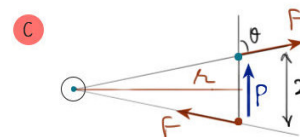
$$U = -\vec{p} \cdot \vec{E}$$

$$F = -\frac{dU}{dx} = -\left(\frac{d(-\vec{p} \cdot \vec{E})}{dx}\right)$$

$$= p \frac{dE}{dx}$$



$$F = p \frac{dE}{dr} = p \frac{d}{dr} \left(\frac{2k\lambda}{r} \right) = -\frac{2k\lambda p}{r^2} //$$

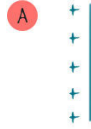


$$F = F_y = 2F \cos \theta = 2 \left(q \cdot \frac{2k\lambda}{\sqrt{r^2 + d^2}} \cdot \frac{d}{\sqrt{r^2 + d^2}} \right) = \frac{2k\lambda(2qd)}{(r^2 + d^2)} \approx \frac{2k\lambda p}{r^2} //$$

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- 3.46. A dipole with an electric moment $\vec{p}$ is located at a distance r from a long thread charged uniformly with a linear density λ . Find the force $\vec{F}$ acting on the dipole if the vector $\vec{p}$ is oriented
- along the thread;
 - along the radius vector $\vec{r}$;
 - at right angles to the thread and the radius vector $\vec{r}$.

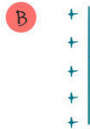
Method 2



Using 2 $\vec{p} = p \hat{k}$, $\vec{E} = \frac{2k\lambda}{r} \hat{k}$

$$(\vec{p} \cdot \vec{\nabla}) = p \frac{\partial}{\partial z}$$

$$\Rightarrow \vec{F} = p \frac{\partial}{\partial z} \left(\frac{2k\lambda}{r} \hat{k} \right) \text{ (As } E \text{ is independent of } z) = 0_{//}$$



Using 1 (As direction of E doesn't change along p) $\vec{p} = p \hat{i}$, $\vec{E} = \frac{2k\lambda}{r} \hat{i}$

$$(\vec{p} \cdot \vec{\nabla}) = p \frac{\partial}{\partial x} \Rightarrow F = p \frac{\partial}{\partial x} \left(\frac{2k\lambda}{r} \hat{i} \right) = 2k\lambda p \hat{i} \frac{\partial}{\partial x} \left(\frac{1}{r} \right) = -\frac{2k\lambda p}{r^2} \hat{i}$$

C



Using 2 $\vec{p} = p \hat{\theta}$, $\vec{E} = \frac{2k\lambda}{r} \hat{r} \Rightarrow \vec{p} \cdot \vec{\nabla} = \frac{1}{r} \frac{\partial}{\partial \theta}$

$$\Rightarrow F = \frac{p}{r} \frac{\partial}{\partial \theta} \left(\frac{2k\lambda}{r} \hat{r} \right) = \frac{2k\lambda p}{r^2} \frac{\partial \hat{r}}{\partial \theta} = \frac{2k\lambda p}{r^2} \hat{\theta}$$

$$\vec{F} = \nabla (\vec{p} \cdot \vec{E}) \text{ or } \vec{F} = (\vec{p} \cdot \nabla) \vec{E}$$

$$\vec{F} = (\vec{p} \cdot \nabla) \vec{E}$$

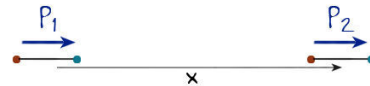
$$\nabla = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \quad 1$$

$$\nabla = \frac{\partial}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial}{\partial \theta} \hat{\theta} + \frac{\partial}{\partial z} \hat{k} \quad 2$$

$$\frac{\partial \hat{\theta}}{\partial \theta} = -\hat{r}, \quad \frac{d\hat{r}}{d\theta} = \hat{\theta}$$

Dipole-dipole interactions in non-uniform electric field

Case 1



$$\vec{p}_2 = p_2 \hat{i}, \quad \vec{E} = \frac{2kp_1}{x^3} \hat{i}$$

Using 1, as E doesn't change direction along p_2

$$\vec{p} \cdot \nabla = (p_2 \hat{i}) \cdot \left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right)$$

$$= p_2 \frac{\partial}{\partial x}$$

$$\Rightarrow \vec{F} = p_2 \frac{\partial}{\partial x} \left(\frac{2kp_1}{x^3} \hat{i} \right) = 2kp_1 p_2 \hat{i} \frac{\partial}{\partial x} \left(\frac{1}{x^3} \right)$$

$$= -\frac{6kp_1 p_2}{x^4} \hat{i}$$

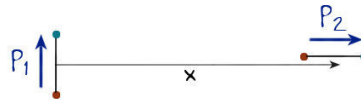
$$\vec{F} = (\vec{p} \cdot \nabla) \vec{E}$$

$$\nabla = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \quad 1$$

$$\nabla = \frac{\partial}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial}{\partial \theta} \hat{\theta} \quad 2$$

Force on a dipole in non-uniform electric field

Case 2 Approach 1



$$\vec{p}_2 = p_2 \hat{i}, \quad \vec{E} = -\frac{k p_1}{x^3} \hat{j}$$

Using ①, as E doesn't change direction along p_2

$$\begin{aligned} \vec{p} \cdot \nabla &= (p_2 \hat{i}) \cdot \left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) \\ &= p_2 \frac{\partial}{\partial x} \end{aligned}$$

$$\begin{aligned} \Rightarrow \vec{F} &= p_2 \frac{\partial}{\partial x} \left(-\frac{k p_1}{x^3} \hat{j} \right) = -k p_1 p_2 \hat{j} \frac{\partial}{\partial x} \left(\frac{1}{x^3} \right) \\ &= \frac{3 k p_1 p_2}{x^4} \hat{j} // \end{aligned}$$

$$\vec{F} = (\vec{p}_2 \cdot \nabla) \vec{E}$$

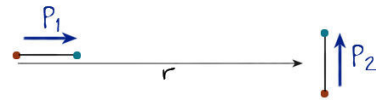
$$\nabla = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \quad ①$$

$$\nabla = \frac{\partial}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial}{\partial \theta} \hat{\theta} \quad ②$$

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Force on a dipole in non-uniform electric field

Case 2 Approach 2



$$\vec{p}_2 = p_2 \hat{\theta}, \quad \vec{E} = \frac{2 k p_1 \cos \theta}{r^3} \hat{r} + \frac{k p_1 \sin \theta}{r^3} \hat{\theta}$$

Using ②, as E changes direction along p_2

$$\vec{p} \cdot \nabla = (p_2 \hat{\theta}) \cdot \left(\frac{\partial}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial}{\partial \theta} \hat{\theta} \right) = \frac{p_2}{r} \frac{\partial}{\partial \theta}$$

$$\Rightarrow \vec{F} = \frac{p_2}{r} \frac{\partial}{\partial \theta} \left[\frac{k p_1}{r^3} (2 \cos \theta \hat{r} + \sin \theta \hat{\theta}) \right]$$

constant w.r.t θ

$$= \frac{k p_1 p_2}{r^4} \left[2(-\sin \theta) \hat{r} + 2 \cos \theta \hat{\theta} + \cos \theta \hat{\theta} + \sin \theta (-\hat{r}) \right] = \frac{k p_1 p_2}{r^4} (3 \cos \theta \hat{\theta} - 3 \sin \theta \hat{r}) = \frac{3 k p_1 p_2}{r^4} \hat{\theta}$$

$$\vec{F} = (\vec{p}_2 \cdot \nabla) \vec{E}$$

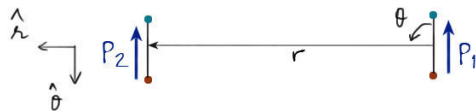
$$\nabla = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \quad ①$$

$$\nabla = \frac{\partial}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial}{\partial \theta} \hat{\theta} \quad ②$$

$$\frac{\partial \hat{\theta}}{\partial \theta} = -\hat{r}, \quad \frac{d \hat{r}}{d \theta} = \hat{\theta}$$

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Case 3



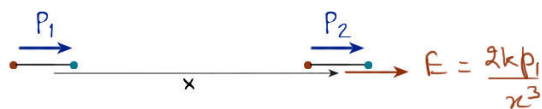
Same expression as in previous one. Except sign of P_2 as $\vec{P}_2 = -p_2 \hat{\theta}$

$$F = \frac{k p_1 (-p_2)}{r^4} (3 \cos \theta \hat{\theta} - 3 \sin \theta \hat{r}) = \frac{3 k p_1 p_2}{r^4} \hat{r}$$

∴ p_2 will be repelled left

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3.47. Find the interaction force between two water molecules separated by a distance $l = 10$ nm if their electric moments are oriented along the same straight line. The moment of each molecule equals $p = 0.62 \cdot 10^{-29}$ C·m.



$$\vec{F}_2 = p_2 \frac{dE}{dx} \quad (\text{As } E \text{ doesn't change direction along } p_2)$$

$$= p_2 \frac{d}{dx} \left(\frac{2kp_1}{x^3} \right)$$

$$= -\frac{6kp_1 p_2}{x^4} //$$

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3.48. Find the potential $\phi(x, y)$ of an electrostatic field $\mathbf{E} = a(y\mathbf{i} + x\mathbf{j})$, where a is a constant, $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors of the x and y axes.

3.49. $\mathbf{E} = 2axy\mathbf{i} + a(x^2 - y^2)\mathbf{j}$,

3.50. $\mathbf{E} = ay\mathbf{i} + (ax + bz)\mathbf{j} + by\mathbf{k}$,

$$\begin{aligned} dV &= -\vec{E} \cdot d\vec{r} \\ &= -\vec{E} \cdot (dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}) \\ &= -(E_x dx + E_y dy + E_z dz) \end{aligned}$$

$$\begin{aligned} 1 \quad dV &= -(ay dx + ax dy) \\ &= -a(y dx + x dy) \\ &= -a d(xy) \end{aligned}$$

$$\Rightarrow V = -a \int d(xy) = -axy + c$$

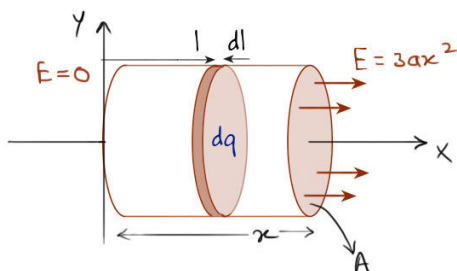
$$\begin{aligned} 2 \quad dV &= -[2axy dx + a(x^2 - y^2) dy] \\ &= -a[2xy dx + x^2 dy - y^2 dy] \\ &= -a[d(x^2 y) - y^2 dy] \Rightarrow V = -a\left[\int d(x^2 y) - \int y^2 dy\right] = -ax^2 y + \frac{ay^3}{3} + c \end{aligned}$$

$$\begin{aligned} 3 \quad dV &= -[ay dx + (ax + bz) dy + by dz] \\ &= -[a(y dx + x dy) + b(z dy + y dz)] \\ V &= -[a \int d(xy) + b \int d(yz)] \\ &= -axy - byz + c \end{aligned}$$

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3.51. The field potential in a certain region of space depends only on the x coordinate as $\phi = -ax^3 + b$, where a and b are constants. Find the distribution of the space charge $\rho(x)$.

$$\vec{E} = -\frac{\partial V}{\partial x} \mathbf{i} - \frac{\partial V}{\partial y} \mathbf{j} - \frac{\partial V}{\partial z} \mathbf{k} = 3ax^2 \mathbf{i}$$



Gauss's law on cylindrical closed surface:

$$EA = \frac{q_{in}}{\epsilon_0}$$

$$(3ax^2)A = \frac{\int dq}{\epsilon_0} = \frac{\int \rho_l A dl}{\epsilon_0}$$

$$\Rightarrow (3\epsilon_0 a) x^2 = \int_0^x \rho_l dl$$

differentiating
w.r.t x both sides..

$$(3\epsilon_0 a) 2x = \rho_x$$

$$\Rightarrow \rho_x = 6\epsilon_0 a x //$$

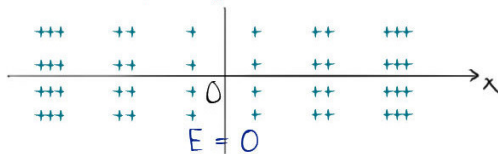
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3.51. The field potential in a certain region of space depends only on the x coordinate as $\varphi = -ax^3 + b$, where a and b are constants. Find the distribution of the space charge $\rho(x)$.

$$\varphi = -a|x|^3 + b$$

$$\begin{aligned} x > 0 & \Rightarrow V = -ax^3 + b \\ E &= 3ax^2 \Rightarrow \oint \vec{E} \cdot d\vec{l} = 6ax \quad (1) \\ x < 0 & \Rightarrow V = ax^3 + b \\ E &= -\frac{dV}{dx} = -3ax^2 \\ \Rightarrow \oint \vec{E} \cdot d\vec{l} &= -6ax \quad (2) \end{aligned}$$

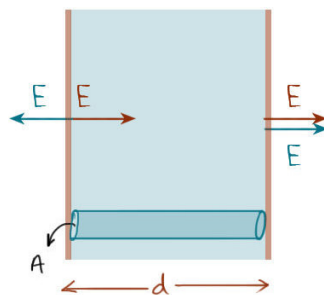
(1) & (2) give the correct distribution of charge that leads to $E=0$ at origin.



With original equation $\oint = 6ax$
 E can't be zero at origin as seen by charge distribution.
 Infact it will be maximum at origin.



3.52. A uniformly distributed space charge fills up the space between two large parallel plates separated by a distance d . The potential difference between the plates is equal to $\Delta\varphi$. At what value of charge density ρ is the field strength in the vicinity of one of the plates equal to zero? What will then be the field strength near the other plate?



Field between the plates without considering filled up charge: $E = \frac{\Delta\varphi}{d}$

If field at one end is zero. Then other end will be $2E$, as E due to distributed charge will be same on both ends.

Using Gauss's law on cylindrical surface for the inside charge:

$$\begin{aligned} 2EA &= \frac{q_{in}}{\epsilon_0} = \frac{\rho V}{\epsilon_0} = \frac{\rho (Ad)}{\epsilon_0} \\ \Rightarrow \rho &= \frac{2\epsilon_0 E}{d} = \frac{2\epsilon_0 \Delta\varphi}{d^2} \end{aligned}$$

Field at other plate: $2E = \frac{2\rho d}{2\epsilon_0} = \frac{\rho d}{\epsilon_0} //$

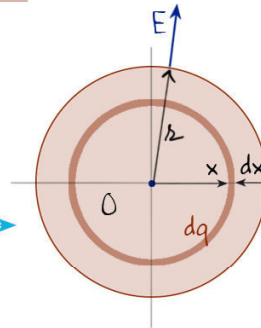
3.53. The field potential inside a charged ball depends only on the distance from its centre as $\phi = ar^2 + b$, where a and b are constants. Find the space charge distribution $\rho(r)$ inside the ball.

$$E_r = -\frac{\partial V}{\partial r} \hat{r} \quad (\text{Other terms not needed as } V \text{ only depends on } r, \text{ other derivatives will become zero})$$

$$= -2ar \hat{r}$$

$$= -2a\vec{r}$$

Method 2



As E is symmetrical about center, using gauss's law on a spherical surface at distance r .

$$E_r(4\pi r^2) = \frac{\int dq}{\epsilon_0}$$

$$\Rightarrow -2ar \left(\frac{4\pi r^2}{r^2}\right) = \frac{\int \rho 4\pi r^2 dr}{\epsilon_0}$$

$$\Rightarrow -2a\epsilon_0 r^3 = \int_0^r \rho 4\pi x^2 dx$$

differentiating w.r.t r both sides

$$-6a\epsilon_0 r^2 = \rho 4\pi r^2$$

$$\Rightarrow \rho = -6a\epsilon_0$$

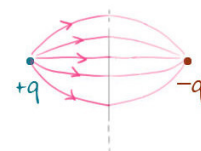
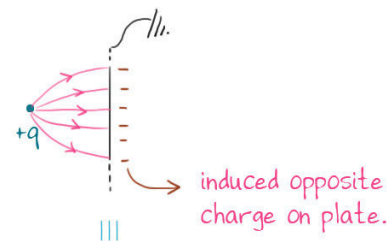
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Method 1

$$\vec{E} = \frac{\rho}{3\epsilon_0} \vec{r} \quad (\text{Similar to a negatively, uniformly charged sphere})$$

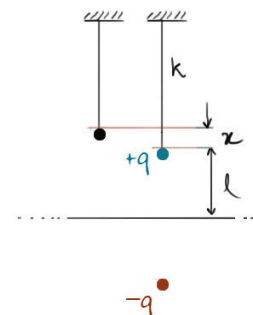
$$\Rightarrow -2a = \frac{\rho}{3\epsilon_0} \Rightarrow \rho = -6a\epsilon_0$$

3.54. A small ball is suspended over an infinite horizontal conducting plane by means of an insulating elastic thread of stiffness k . As soon as the ball was charged, it descended by x cm and its separation from the plane became equal to l . Find the charge of the ball.



Using the concept of electrical image (i.e. effect of infinite conducting sheet can be reproduced with a symmetric equal and opposite charge)

Sol:



Neglecting gravitational force compared to electrostatic force.

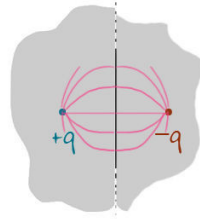
$$F_{up} = F_{down}$$

$$kx = \frac{1}{4\pi\epsilon_0} \frac{q^2}{(2l)^2}$$

$$\Rightarrow q = 2l \sqrt{4\pi\epsilon_0 kx}$$

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3.55. A point charge q is located at a distance l from the infinite conducting plane. What amount of work has to be performed in order to slowly remove this charge very far from the plane.



Method 1

Energy stored in the region if actual two charges were present:

$$U_{\text{pair}} = \frac{-kq^2}{2l}$$

Energy stored in half region:

$$U_{\text{single}} = \frac{U_{\text{pair}}}{2} = \frac{-kq^2}{4l}$$

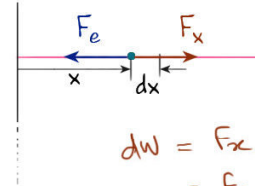


External work done to remove it:

$$W_{\text{ext}} = \Delta U = 0 - \left(\frac{-kq^2}{4l} \right) = \frac{kq^2}{4l} //$$

Method 2

If the charge is moved dx away from plane slowly:



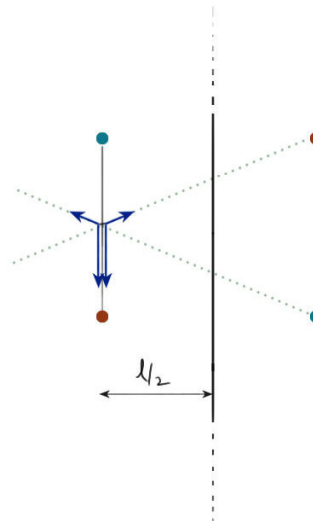
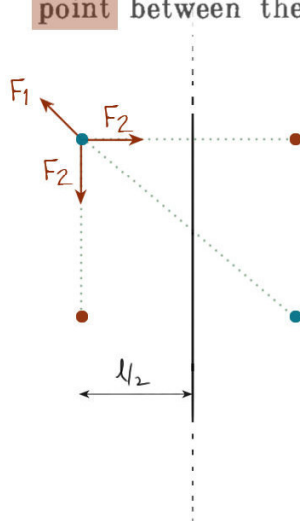
$$\begin{aligned} dW &= F_x dx \\ &= F_e dx \\ &= \frac{kq^2}{(2x)^2} dx \end{aligned}$$

$$\begin{aligned} W &= \int dW = \frac{kq^2}{4} \int_l^\infty \frac{dx}{x^2} \\ &= \frac{kq^2}{4l} // \end{aligned}$$

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3.56. Two point charges, q and $-q$, are separated by a distance l , both being located at a distance $l/2$ from the infinite conducting plane. Find:

- the modulus of the vector of the electric force acting on each charge;
- the magnitude of the electric field strength vector at the mid-point between these charges.

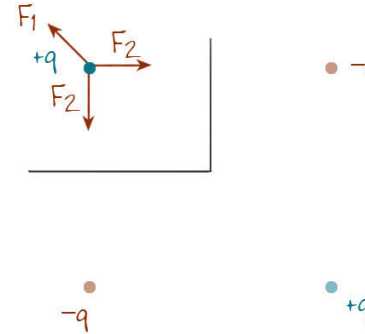
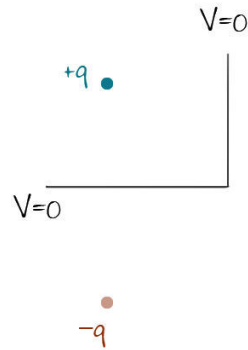


$$\begin{aligned} \text{A } F &= \sqrt{2}F_2 - F_1 \\ &= \sqrt{2} \frac{kq^2}{l^2} - \frac{kq^2}{(\sqrt{2}l)^2} \\ &= \frac{kq^2}{2l^2} (2\sqrt{2} - 1) \end{aligned}$$

$$\text{B } E = 2 \left(1 - \frac{1}{5\sqrt{5}} \right) \frac{q}{\pi \epsilon_0 l^2} //$$

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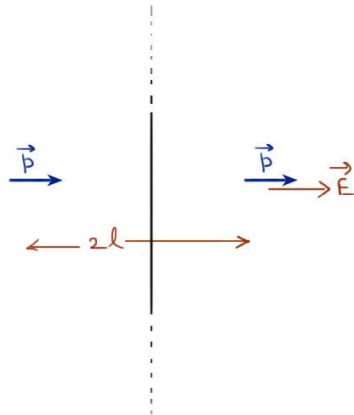
3.57. A point charge q is located between two mutually perpendicular conducting half-planes. Its distance from each half-plane is equal to l . Find the modulus of the vector of the force acting on the charge.



$$F = \sqrt{2}F_2 - F_1 = \sqrt{2} \frac{kq^2}{(2l)^2} - \frac{kq^2}{(\sqrt{2}2l)^2} = \frac{kq^2}{8l^2} (2\sqrt{2} - 1)$$

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3.58. A point dipole with an electric moment p is located at a distance l from an infinite conducting plane. Find the modulus of the vector of the force acting on the dipole if the vector p is perpendicular to the plane.



$$E_x = \frac{2kp}{r^3}$$

$$\text{Force on axial dipole: } F = p \frac{dE}{dr}$$

$$= p \frac{d}{dr} \left(\frac{2kp}{r^3} \right)$$

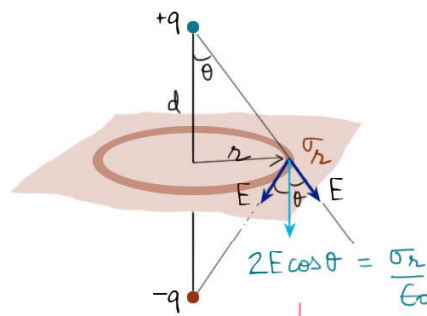
$$= 2kp^2 \frac{d}{dr} \left(\frac{1}{r^3} \right)$$

$$= -\frac{6kp^2}{r^4} = -\frac{6kp^2}{(2l)^4}$$

$$|\vec{F}| = \frac{3kp^2}{8l^4}$$

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3.59. A point charge q is located at a distance l from an infinite conducting plane. Determine the surface density of charges induced on the plane as a function of separation r from the base of the perpendicular drawn to the plane from the charge.



Field due to
 $+q$ and $-q$

Field due to
 $+q$ and charge on conducting plate

$$|\sigma_r| = 2\epsilon_0 E \cos\theta$$

$$= 2\epsilon_0 \frac{kq}{(r^2 + d^2)^{3/2}} \cdot \frac{d}{\sqrt{r^2 + d^2}}$$

$$\sigma_r = \frac{qd}{2\pi (r^2 + d^2)^{3/2}}$$

Total charge on plate:

$$|q| = \int dq = \int \sigma_r dA = \int \sigma_r \cdot 2\pi r dr$$

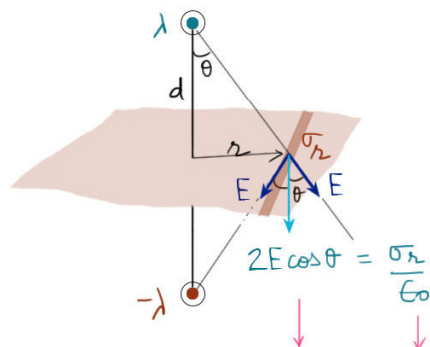
$$= \int_0^\infty \frac{qd \cdot 2\pi r dr}{2\pi (r^2 + d^2)^{3/2}} = q //$$

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3.60. A thin infinitely long thread carrying a charge λ per unit length is oriented parallel to the infinite conducting plane. The distance between the thread and the plane is equal to l . Find:

(a) the modulus of the vector of the force acting on a unit length of the thread;

(b) the distribution of surface charge density $\sigma(x)$ over the plane, where x is the distance from the plane perpendicular to the conducting surface and passing through the thread.



Field due to
 λ and $-\lambda$

Field due to
 λ and charge on conducting plate

A

$$F = qE$$

$$F/\lambda = \frac{q}{\lambda} E = \lambda E = \lambda \cdot \frac{2k\lambda}{2d} = \frac{k\lambda^2}{d}$$

B

$$|\sigma_r| = 2\epsilon_0 E \cos\theta$$

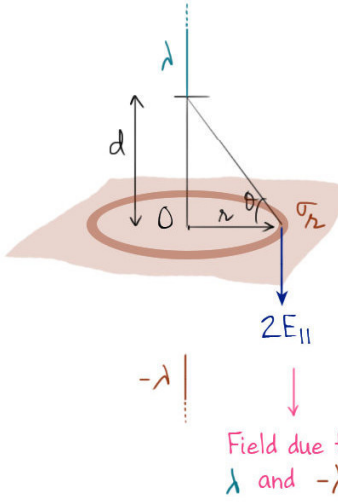
$$= 2\epsilon_0 \frac{2k\lambda}{\sqrt{r^2 + d^2}} \cdot \frac{d}{\sqrt{r^2 + d^2}}$$

$$= \frac{\lambda d}{\pi (r^2 + d^2)}$$

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3.61. A very long straight thread is oriented at right angles to an infinite conducting plane; its end is separated from the plane by a distance l . The thread carries a uniform charge of linear density λ . Suppose the point O is the trace of the thread on the plane. Find the surface density of the induced charge on the plane

- (a) at the point O ;
(b) as a function of a distance r from the point O .



$E_{\perp}$ will cancel out due to symmetry.

$$E_{||} = \frac{k\lambda}{r} (\cos\theta - \cos\frac{\pi}{2})$$

Using,

$$E_{||} = \frac{k\lambda}{r} (\cos\theta_1 - \cos\theta_2)$$

$$\Rightarrow \sigma_r = 2\epsilon_0 E_{||}$$

$$= 2\epsilon_0 \frac{k\lambda}{r} \left(\frac{r}{\sqrt{r^2 + d^2}} \right) = \frac{\lambda}{2\pi\sqrt{r^2 + d^2}} \quad \text{B}$$

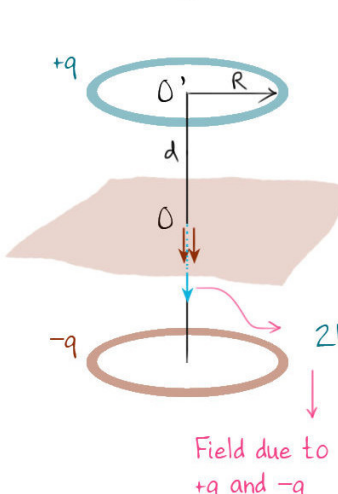
$$\Rightarrow \sigma_0 = \frac{\lambda}{2\pi d} \quad \text{A}$$

Field due to λ and $-\lambda$ Field due to λ and charge on conducting plate

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3.62. A thin wire ring of radius R carries a charge q . The ring is oriented parallel to an infinite conducting plane and is separated by a distance l from it. Find:

- (a) the surface charge density at the point of the plane symmetrical with respect to the ring;
(b) the strength and the potential of the electric field at the centre of the ring.



$$\sigma_0 = 2\epsilon_0 E$$

$$= 2\epsilon_0 \frac{kq d}{(d^2 + R^2)^{3/2}}$$

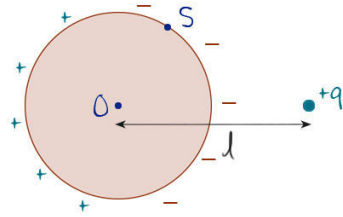
$$E_{O'} = \frac{kq(2d)}{((2d)^2 + R^2)^{3/2}} = \frac{2kq d}{(4d^2 + R^2)^{3/2}}$$

$$V_{O'} = \frac{k(+q)}{R} + \frac{k(-q)}{\sqrt{(2d)^2 + R^2}} = \frac{kq}{R} \left(1 - \frac{R}{\sqrt{4d^2 + R^2}} \right)$$

Field due to $+q$ and $-q$ Field due to $+q$ and charge on conducting plate

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3.63. Find the potential ϕ of an uncharged conducting sphere outside of which a point charge q is located at a distance l from the sphere's centre.



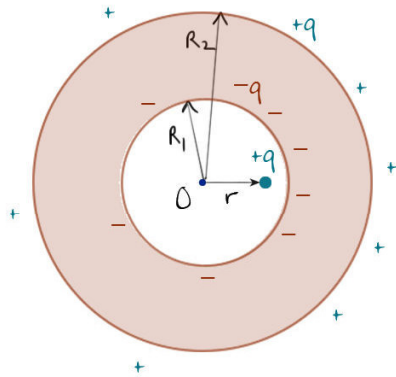
As the field inside the conducting sphere is zero, all the points on its surface and inside are at same potential.
Therefore $V_S = V_O$

$$V_O = \frac{kq_{\text{induced}}}{R} + \frac{kq}{l}$$

$$\Rightarrow V_O = \frac{kq}{l} = V_S$$

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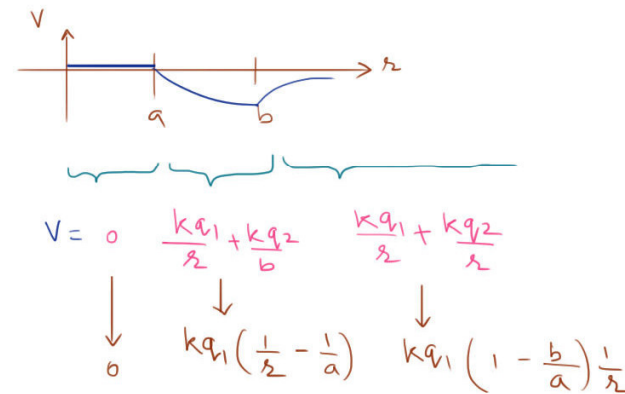
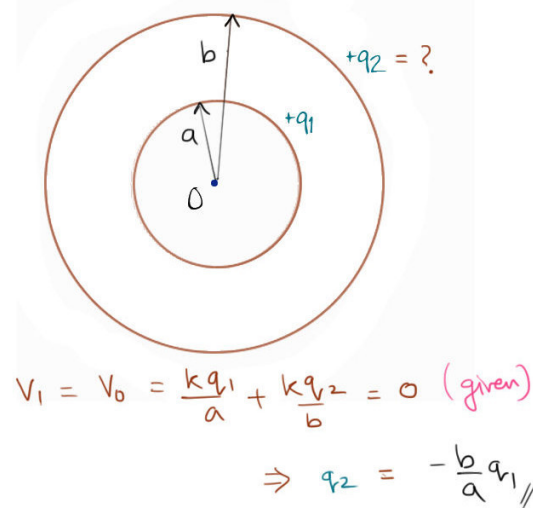
3.64. A point charge q is located at a distance r from the centre O of an uncharged conducting spherical layer whose inside and outside radii are equal to R_1 and R_2 respectively. Find the potential at the point O if $r < R_1$.



$$\begin{aligned} V_O &= V_{+q} + V_{-q \text{ induced}} + V_{+q \text{ induced}} \\ &= \frac{kq}{r} + \frac{k(-q)}{R_1} + \frac{k(+q)}{R_2} \end{aligned}$$

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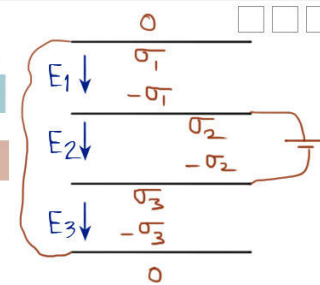
3.65. A system consists of two concentric conducting spheres, with the inside sphere of radius a carrying a positive charge q_1 . What charge q_2 has to be deposited on the outside sphere of radius b to reduce the potential of the inside sphere to zero? How does the potential ϕ depend in this case on a distance r from the centre of the system? Draw the approximate plot of this dependence.



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3.66. Four large metal plates are located at a small distance d from one another as shown in Fig. 3.8. The extreme plates are interconnected by means of a conductor while a potential difference $\Delta\phi$ is applied to internal plates. Find:

- the values of the electric field strength between neighbouring plates;
- the total charge per unit area of each plate.



Battery will give equal amount of +ve and -ve charge. $\Rightarrow \sigma_1 = \sigma_3$ ①

Charge on outward facing surfaces of outermost plates is half of total charge each. i.e zero.

Charge on adjacent inside surfaces is equal and opposite.

Potential of connected surfaces is equal. $\Rightarrow \frac{\sigma_1}{\epsilon_0} d + \frac{\sigma_2}{\epsilon_0} d + \frac{\sigma_3}{\epsilon_0} d = 0 \Rightarrow \sigma_1 + \sigma_2 + \sigma_3 = 0$ ②

Potential across battery is given $\Rightarrow \frac{\sigma_2}{\epsilon_0} d = \Delta V \Rightarrow \sigma_2 = \frac{\epsilon_0 \Delta V}{d}$ ③

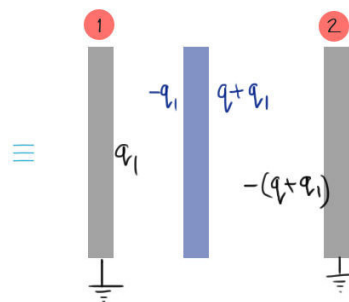
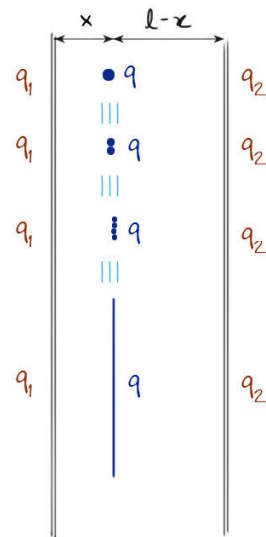
$1, 2, 3 \Rightarrow$

$\sigma_2 = \frac{\epsilon_0 \Delta V}{d}, \sigma_1 = \sigma_3 = -\frac{\epsilon_0 \Delta V}{2d}$

$E_2 = \frac{\sigma_2}{\epsilon_0}, E_1 = E_3 = \frac{\sigma_1}{\epsilon_0}$

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3.67. Two infinite conducting plates 1 and 2 are separated by a distance l . A point charge q is located between the plates at a distance x from plate 1. Find the charges induced on each plate.



Considering the charge spread over large area A .

Potential from 1 to 2 varies as:

$$V_1 + \frac{q_1}{\epsilon_0} x + \frac{q+q_1}{\epsilon_0} (l-x) = V_2 \quad (\text{as } V_1 = V_2)$$

$$\Rightarrow q_1 x + (q+q_1)(l-x) = 0$$

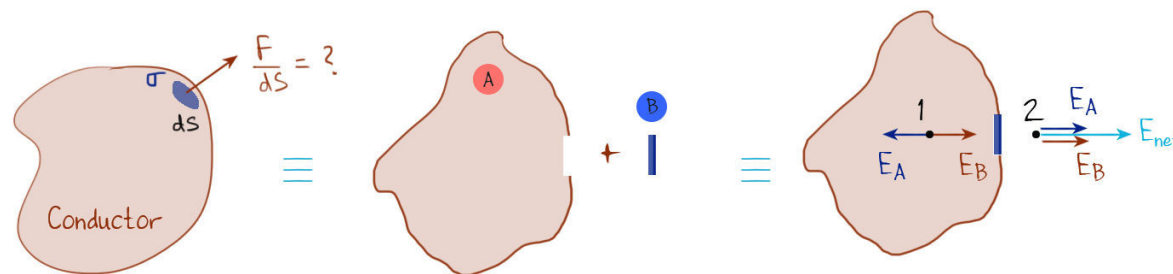
$$\Rightarrow q_1 = -q \left(\frac{l-x}{l} \right)$$

$$\& \quad q_2 = -(q+q_1)$$

$$= -q \frac{x}{l}$$

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Electrostatic Pressure: As experienced by surface element of a conductor with a given charge density σ .



We know that field at point 2 just outside the conductor surface is: σ/ϵ_0

Also field at point 1 just inside the conductor surface is zero.

$$\therefore E_A + E_B = \frac{\sigma}{\epsilon_0} \quad \& \quad E_A = E_B$$

$$\Rightarrow E_A = E_B = \frac{\sigma}{2\epsilon_0}$$

$\therefore$ Force on B due to A is:

$$dF = dq E_A$$

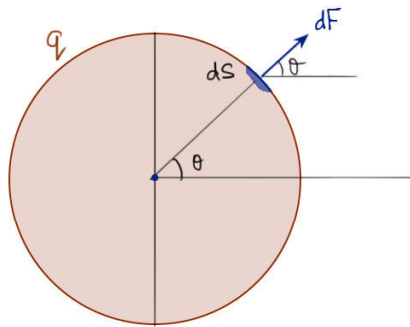
$$dF = \sigma dS \left(\frac{\sigma}{2\epsilon_0} \right)$$

$$\Rightarrow \frac{dF}{dS} = \frac{\sigma^2}{2\epsilon_0}$$

(Also known as electrostatic pressure)

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3.69. A metal ball of radius $R = 1.5 \text{ cm}$ has a charge $q = 10 \mu\text{C}$. Find the modulus of the vector of the resultant force acting on a charge located on one half of the ball.



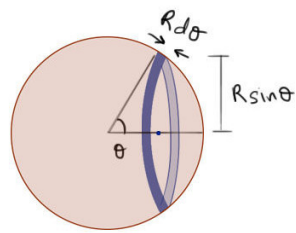
$$dF = \text{Electrostatic pressure} \times \text{Area} = \frac{\sigma^2}{2\epsilon_0} \cdot dS$$

By symmetry, only horizontal component will remain.

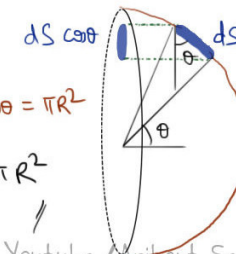
$$\therefore F = \int dF \cos \theta = \int \frac{\sigma^2}{2\epsilon_0} dS \cos \theta$$

Method 1

Method 2



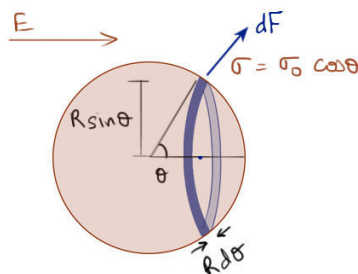
$$\begin{aligned} F &= \frac{\sigma^2}{2\epsilon_0} \int (2\pi R \sin \theta \cdot R d\theta) \cos \theta \\ &= \frac{\sigma^2}{2\epsilon_0} \pi R^2 \int_0^{\pi/2} \sin 2\theta d\theta \\ &= \frac{\sigma^2 \pi R^2}{2\epsilon_0} // \end{aligned}$$



$$\begin{aligned} \therefore \int dS \cos \theta &= \pi R^2 \\ \Rightarrow F &= \frac{\sigma^2}{2\epsilon_0} \pi R^2 // \end{aligned}$$

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3.70. When an uncharged conducting ball of radius R is placed in an external uniform electric field, a surface charge density $\sigma = \sigma_0 \cos \theta$ is induced on the ball's surface (here σ_0 is a constant, θ is a polar angle). Find the magnitude of the resultant electric force acting on an induced charge of the same sign.



$$dF = dq E = \sigma dS E = \frac{\sigma^2}{2\epsilon_0} dS$$

Note: In this question E is total field due to external uniform field and internal (rest of spherical surface) both.

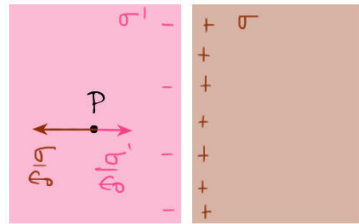
$$dF = \text{Electrostatic pressure} \times \text{Area} = \frac{\sigma^2}{2\epsilon_0} \cdot dS$$

By symmetry, only horizontal component will remain.

$$\begin{aligned} \therefore F &= \int dF \cos \theta = \int \frac{\sigma^2}{2\epsilon_0} dS \cos \theta \\ &= \int \frac{\sigma_0^2 \cos^2 \theta}{2\epsilon_0} (2\pi R \sin \theta \cdot R d\theta) \cos \theta \\ &= \frac{\sigma_0^2}{2\epsilon_0} \cdot 2\pi R^2 \int_0^{\pi/2} \sin \theta \cos^3 \theta d\theta \\ &= \frac{\sigma_0^2 \pi R^2}{\epsilon_0} - \frac{\cos^4 \theta}{4} \Big|_0^{\pi/2} = \frac{\sigma_0^2 \pi R^2}{4\epsilon_0} // \end{aligned}$$

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3.75. Demonstrate that at a dielectric-conductor interface the surface density of the dielectric's bound charge $\sigma' = -\sigma(\epsilon - 1)/\epsilon$, where ϵ is the permittivity, σ is the surface density of the charge on the conductor.



Dielectric (ϵ) Conductor

Prove that, $\sigma' = -\sigma(\frac{\epsilon-1}{\epsilon})$

Field at a point P kept in a dielectric in front of a charged conductor is:

$$E_p = \frac{\sigma}{\epsilon_0 \epsilon} \quad (1)$$

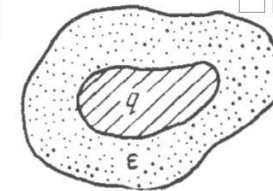
Rewriting Field at P, by pure electrostatic analysis of charged sheets:

$$E_p = \frac{\sigma}{\epsilon_0} - \frac{\sigma'}{\epsilon_0} \quad (2)$$

$$(1, 2) \Rightarrow \frac{\sigma}{\cancel{\epsilon_0} \epsilon} = \frac{\sigma}{\cancel{\epsilon_0}} - \frac{\sigma'}{\cancel{\epsilon_0}} \Rightarrow \sigma' = -\sigma(1 - \frac{1}{\epsilon}) //$$

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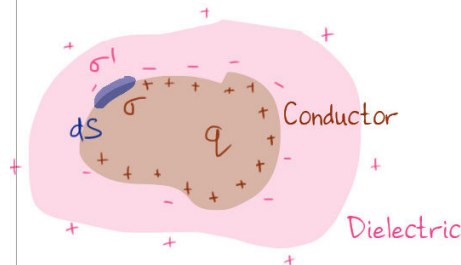
3.76. A conductor of arbitrary shape, carrying a charge q , is surrounded with uniform dielectric of permittivity ϵ (Fig. 3.9). Find the total bound charges at the inner and outer surfaces of the dielectric.



Using result from 3.75.

That is, on a dielectric conductor interface:

$$\sigma' = -\sigma(1 - \frac{1}{\epsilon})$$



$$\sigma' = -\sigma(1 - \frac{1}{\epsilon})$$

$$\Rightarrow \sigma' dS = -(1 - \frac{1}{\epsilon}) \sigma dS$$

$$\Rightarrow dq' = -(1 - \frac{1}{\epsilon}) dq$$

$$\Rightarrow \underbrace{\int dq'}_{q'_{in}} = -(1 - \frac{1}{\epsilon}) \underbrace{\int dq}_{q}$$

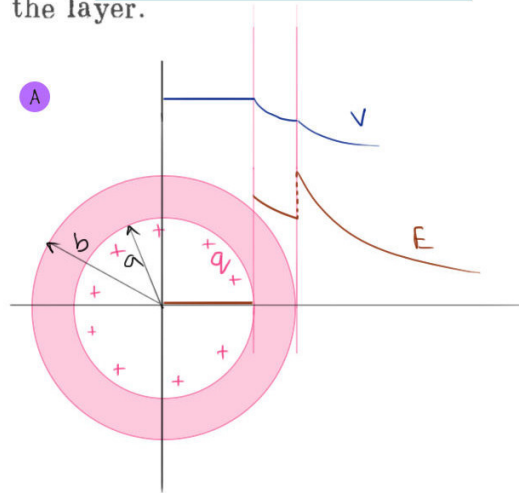
$$\Rightarrow q'_{in} = -(1 - \frac{1}{\epsilon}) q //$$

$$\Rightarrow q'_{out} = -q' = (1 - \frac{1}{\epsilon}) q //$$

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3.77. A uniform isotropic dielectric is shaped as a spherical layer with radii a and b . Draw the approximate plots of the electric field strength E and the potential φ vs the distance r from the centre of the layer if the dielectric has a certain positive extraneous charge distributed uniformly: $\rightarrow$ external

(a) over the internal surface of the layer; (b) over the volume of the layer.



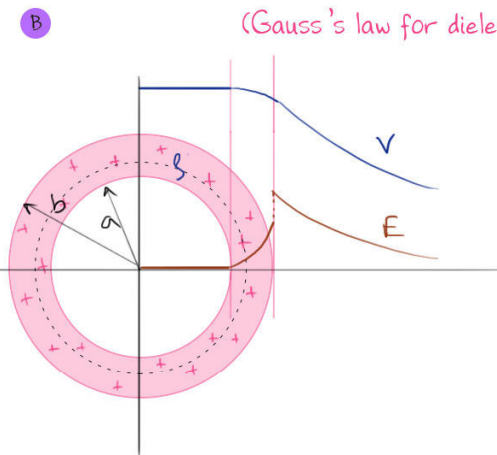
$$E_{in} = \frac{kq}{\epsilon r^2}, E_{out} = \frac{kq}{r^2} \Rightarrow V_{out} = \frac{kq}{r}$$

$$V_{in} = \int_b^r dv = - \int_b^r E \cdot dr$$

$$V - \frac{kq}{b} = - \frac{kq}{\epsilon} \left(-\frac{1}{r} \right)_b^r$$

$$V_{in} = \frac{kq}{b} \left(1 - \frac{1}{\epsilon} \right) + \frac{kq}{\epsilon r}$$

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(Gauss's law for dielectric) E_{in} : $E \left(\frac{4\pi}{3} r^3 \right) = \frac{\rho}{\epsilon_0} \left(\frac{4\pi}{3} (r^3 - a^3) \right) \Rightarrow E_{in} = \frac{\rho}{3\epsilon_0} \left(\frac{r^3 - a^3}{r^2} \right)$

$$E_{out} = \frac{kq}{r^2} \Rightarrow V_{out} = \frac{kq}{r}$$

$$V_{in} = \int_b^r dv_{in} = - \int_b^r E_{in} \cdot dr$$

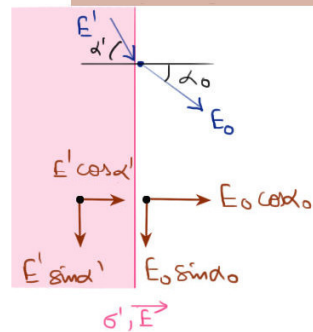
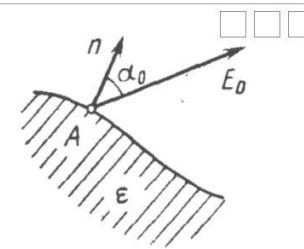
$$V - \frac{kq}{b} = - \frac{\rho}{3\epsilon_0} \int_b^r \left(r - \frac{a^3}{r^2} \right) dr$$

$$V = \frac{kq}{b} - \frac{\rho}{3\epsilon_0} \left(\frac{r^2 - b^2}{2} + a^3 \left(\frac{1}{r} - \frac{1}{b} \right) \right)$$

$$= C_1 - C_2 r^2 - \frac{C_3}{r}$$

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3.78. Near the point A (Fig. 3.10) lying on the boundary between glass and vacuum the electric field strength in vacuum is equal to $E_0 = 10.0 \text{ V/m}$, the angle between the vector E_0 and the normal n of the boundary line being equal to $\alpha_0 = 30^\circ$. Find the field strength E in glass near the point A, the angle α between the vector E and n , as well as the surface density of the bound charges at the point A.



Field parallel to dielectric surface doesn't change because induced charges effect field in normal direction only.

$$\therefore E' \sin \alpha' = E_0 \sin \alpha_0 \quad (1)$$

Field perpendicular to surface changes by formula:

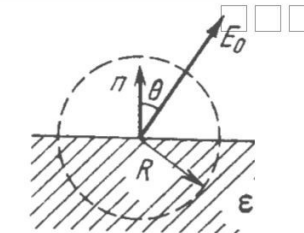
$$E' \cos \alpha' = \frac{E_0 \cos \alpha_0}{\epsilon} \quad (2)$$

$$\begin{aligned} E + \frac{\sigma'}{\epsilon_0} &= E_0 \cos \alpha_0 & E - \frac{\sigma'}{\epsilon_0} &= E' \cos \alpha' \\ \Rightarrow E_0 \cos \alpha_0 - E' \cos \alpha' &= \frac{\sigma'}{\epsilon_0} & \text{1,2} \Rightarrow \tan \alpha' &= \epsilon \tan \alpha_0 \\ \Rightarrow \sigma' &= \epsilon_0 E_0 \cos \alpha_0 \left(1 - \frac{1}{\epsilon}\right) & \& \quad E' &= E_0 \sqrt{\sin^2 \alpha_0 + \frac{\cos^2 \alpha_0}{\epsilon^2}} \end{aligned}$$

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3.79. Near the plane surface of a uniform isotropic dielectric with permittivity ϵ the electric field strength in vacuum is equal to E_0 , the vector E_0 forming an angle θ with the normal of the dielectric's surface (Fig. 3.11). Assuming the field to be uniform both inside and outside the dielectric, find:

(a) the flux of the vector E through a sphere of radius R with centre located at the surface of the dielectric;



From problem 3.78

$$\sigma' = \epsilon_0 E_0 \cos \theta \left(1 - \frac{1}{\epsilon}\right)$$

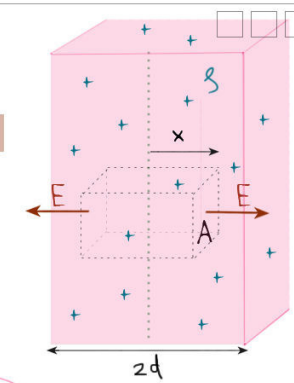
$$\therefore \text{flux} = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\sigma' (\pi R^2)}{\epsilon_0} = \pi R^2 E_0 \cos \theta \left(1 - \frac{1}{\epsilon}\right)$$

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3.80. An infinite plane of uniform dielectric with permittivity ϵ is uniformly charged with extraneous charge of space density ρ .

The thickness of the plate is equal to $2d$. Find:

(a) the magnitude of the electric field strength and the potential as functions of distance l from the middle point of the plane (where the potential is assumed to be equal to zero); having chosen the x coordinate axis perpendicular to the plate, draw the approximate plots of the projection $E_x(x)$ of the vector $\mathbf{E}$ and the potential $\varphi(x)$;

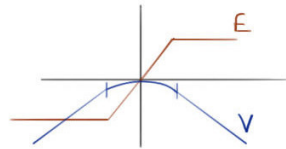


Gauss's law inside dielectric:

$$E(2A) = \frac{\rho(2x)A}{\epsilon \epsilon_0} \Rightarrow E_{in} = \frac{\rho x}{\epsilon \epsilon_0} \Rightarrow V_{in} = -\int E \cdot dx = -\frac{\rho x^2}{2\epsilon \epsilon_0}$$

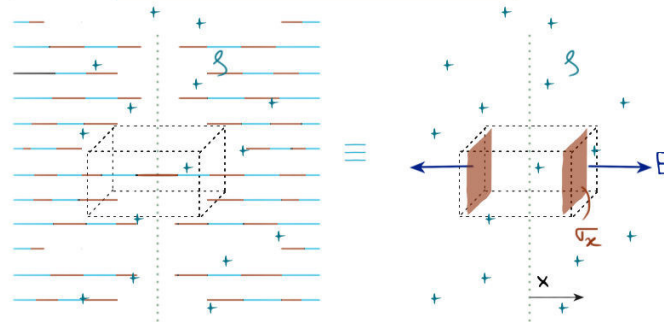
$$E_{out}(2A) = \frac{\rho(2d)A}{\epsilon_0} \Rightarrow E_{out} = \frac{\rho d}{\epsilon_0} \Rightarrow \int_{V_{surface}} dV = -\int E \cdot dx = -\frac{\rho d}{\epsilon_0} x \Big|_d^x$$

$$\Rightarrow V_{out} = -\frac{\rho d}{\epsilon_0} (x-d) + V_{surface} = -\frac{\rho d}{\epsilon_0} \left(x-d - \frac{d}{2} \right) = -\frac{\rho d^2}{2\epsilon_0}$$



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(b) the surface and space densities of the bound charge.



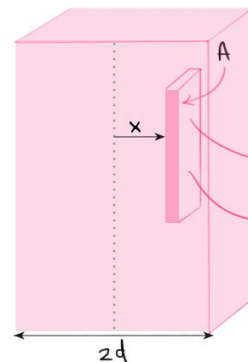
using Gauss's law without dielectric:

$$E 2A = \frac{q_{net}}{\epsilon_0}$$

$$\frac{\rho x 2A}{\epsilon_0} = \frac{\rho(2xA) + 2\sigma_x A}{\epsilon_0}$$

$$\Rightarrow \sigma_x = -\rho x \left(1 - \frac{1}{\epsilon}\right) \Rightarrow \sigma_s = +\rho d \left(1 - \frac{1}{\epsilon}\right)$$

$$\therefore \rho_s = \frac{dq_s}{dV} = \frac{A d\sigma_s}{A dx} = -\rho \left(1 - \frac{1}{\epsilon}\right)$$



Space density: ρ_s
(Due to bound charges)

$$dq_b = \sigma_{x+dx} A - \sigma_x A = A d\sigma$$

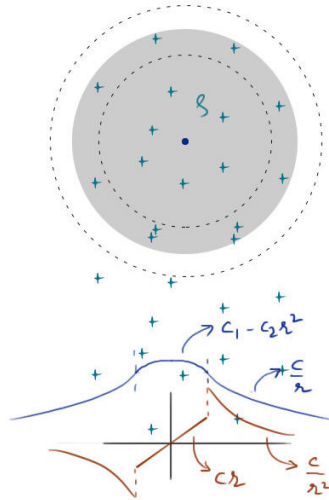
$$dV = A dx$$

Youtube / Arihant Senior

3.81. Extraneous charges are uniformly distributed with space density $\rho > 0$ over a ball of radius R made of uniform isotropic dielectric with permittivity ϵ . Find: (Filling the ball completely)

(a) the magnitude of the electric field strength as a function of distance r from the centre of the ball; draw the approximate plots $E(r)$ and $\varphi(r)$;

(b) the space and surface densities of the bound charges.



A E_{in} (Gauss's law for dielectric:)

$$\frac{\rho \left(\frac{4}{3} \pi r^3 \right)}{\epsilon \epsilon_0} = E \left(4 \pi r^2 \right) \Rightarrow E_{in} = \frac{\rho r}{3 \epsilon \epsilon_0}$$

E_{out} (Gauss's law)

$$\frac{\rho \left(\frac{4}{3} \pi R^3 \right)}{\epsilon_0} = E \left(4 \pi r^2 \right) \Rightarrow E_{out} = \frac{\rho R^3}{3 \epsilon_0 r^2}$$

V_{out}

$$\int_0^V dV = \int_{\infty}^r -E_{out} \cdot dr$$

$$V = -\frac{\rho R^3}{3 \epsilon_0} \int_{\infty}^r \frac{1}{r^2} dr$$

$$\Rightarrow V_{out} = \frac{\rho R^3}{3 \epsilon_0 r}$$

V_{in}

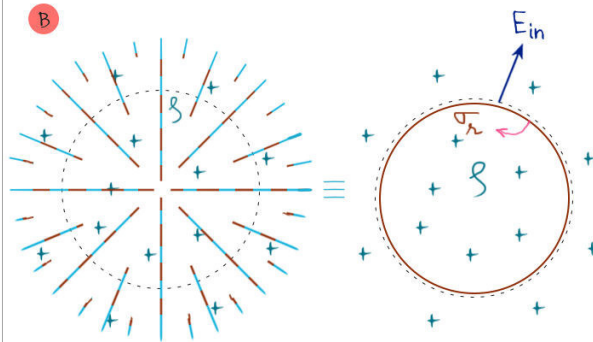
$$\int_{V_R}^V dV = \int_R^r -E_{in} \cdot dr$$

$$V - \frac{\rho R^3}{3 \epsilon_0 R} = -\frac{\rho}{3 \epsilon_0} \int_R^r r dr$$

$$\Rightarrow V_{in} = \frac{\rho R^2}{3 \epsilon_0} \left[1 + \frac{1}{2\epsilon} - \frac{r^2}{2\epsilon R^2} \right]$$

Youtube / Arihant Senior

B



Method 1 Gauss's law without dielectric:

$$E_{in} (4 \pi r^2) = \frac{q_{net}}{\epsilon_0}$$

$$\frac{\rho r}{3 \epsilon_0} (4 \pi r^2) = \frac{\rho \cdot \frac{4}{3} \pi r^3 + \sigma_r 4 \pi r^2}{\epsilon_0}$$

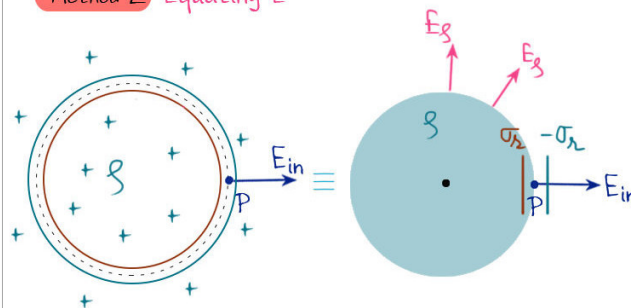
$$\Rightarrow \sigma_r = -\frac{\rho r}{3} \left(1 - \frac{1}{\epsilon} \right)$$

(At general radius r)

$$\sigma_R = +\frac{\rho R}{3} \left(1 - \frac{1}{\epsilon} \right)$$

(At balls outer surface)

Method 2 Equating E

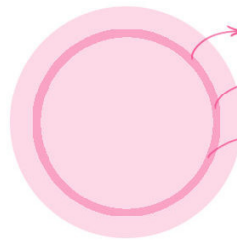


$$\frac{\rho r}{3 \epsilon_0} + \frac{\sigma_r}{2 \epsilon_0} + \frac{\sigma_r}{2 \epsilon_0} = E_{in} \Rightarrow \sigma_r = -\frac{\rho r}{3 \epsilon_0} \left(1 - \frac{1}{\epsilon} \right)$$

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Space density: ρ
(Due to bound charges)

$$\sigma_r = -\frac{\rho r}{3\epsilon_0} \left(1 - \frac{1}{\epsilon}\right)$$



$$dq = \sigma_{r+dr} A_{r+dr} - \sigma_r A_r = d(\sigma A)$$

$$dV = 4\pi r^2 dr$$

$\rho = ?$

$$\therefore \rho = \frac{dq}{dV} = \frac{d(\sigma A)}{dV} = \frac{-\frac{\rho}{3} \left(\frac{\epsilon-1}{\epsilon}\right) 4\pi r^2 dr}{4\pi r^2 dr} = \frac{-\rho (\epsilon-1)}{\epsilon} //$$

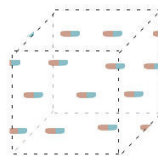
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Polarization vector

Its the property of a polarized dielectric
Defined as dipole moment per unit volume of dielectric.

$$\text{Polarization } \vec{P} = \frac{\text{Dipole moment}}{\text{Volume}}$$

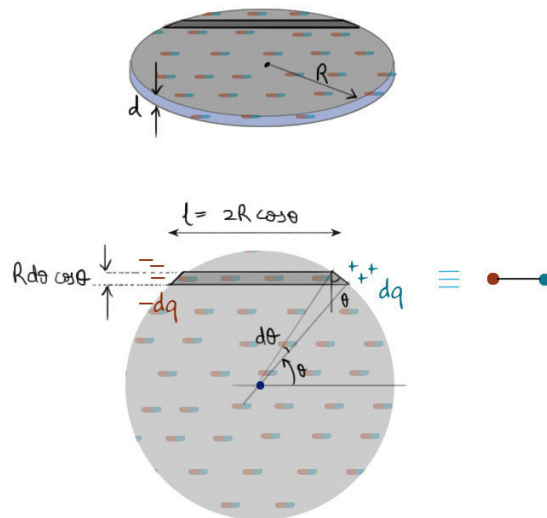
∴ A volume of dielectric can be replaced with equivalent dipole.



$$\vec{P} = \vec{P} V$$

Youtube / Arihant Senior

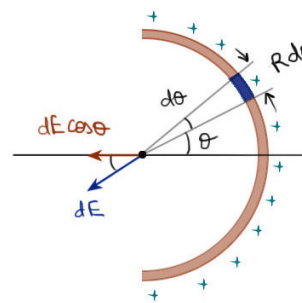
3.82. A round dielectric disc of radius R and thickness d is statically polarized so that it gains the uniform polarization $\mathbf{P}$, with the vector $\mathbf{P}$ lying in the plane of the disc. Find the strength $\mathbf{E}$ of the electric field at the centre of the disc if $d \ll R$.



$$P \times dV = dq \cdot l$$

$$P (R d \cos \theta \cdot d) = dq \cdot l$$

$$\Rightarrow dq = PR d \cos \theta d\theta$$



At center, $E = \int dE \cos \theta$

$$= \int \frac{dq}{4\pi\epsilon_0 R^2} \cos \theta$$

$$= \frac{PRd}{4\pi\epsilon_0 R^2} \int_0^\pi \cos \theta d\theta$$

$$= \frac{Pd}{4\epsilon_0 R} = -\frac{\vec{P}d}{4\epsilon_0 R}$$

Youtube / Arihant Senior

3.83. Under certain conditions the polarization of an infinite uncharged dielectric plate takes the form $\mathbf{P} = \mathbf{P}_0 (1 - x^2/d^2)$, where $\mathbf{P}_0$ is a vector perpendicular to the plate, x is the distance from the middle of the plate, d is its half-thickness. Find the strength $\mathbf{E}$ of the electric field inside the plate and the potential difference between its surfaces.

1 Writing dipole moment of small cylindrical volume at distance x :

A

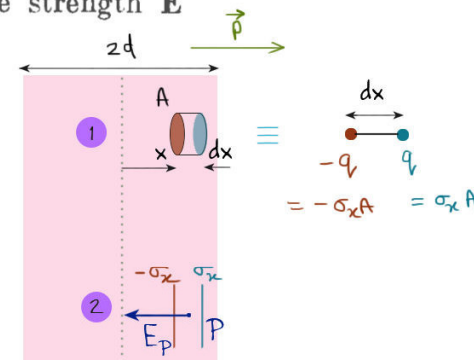
$$P dV = q dx = (\sigma_x A) dx$$

$$P_0 \left(1 - \frac{x^2}{d^2}\right) (A \cdot dx) = \sigma_x A \cdot dx$$

$$\sigma_x = P_0 \left(1 - \frac{x^2}{d^2}\right)$$

2

$$E_p = \frac{\sigma_x}{2\epsilon_0} + \frac{\sigma_x}{2\epsilon_0} = \frac{\sigma_x}{\epsilon_0} = -\frac{P_0}{\epsilon_0} \left(1 - \frac{x^2}{d^2}\right)$$



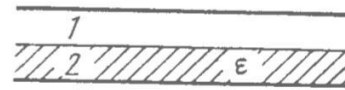
As there is no extraneous charge, Field at point P is only due to charges in immediate surrounding.

B

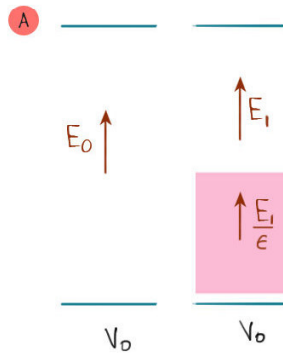
$$\Delta V = - \int_{-d}^{+d} E \cdot dx = - \int_{-d}^{+d} -\frac{P_0}{\epsilon_0} \left(1 - \frac{x^2}{d^2}\right) dx = \frac{4P_0 d}{3\epsilon_0}$$

Youtube / Arihant Senior

3.84. Initially the space between the plates of the capacitor is filled with air, and the field strength in the gap is equal to E_0 . Then half the gap is filled with uniform isotropic dielectric with permittivity ϵ as shown in Fig. 3.12. Find the moduli of the vectors $\mathbf{E}$ and $\mathbf{D}$ in both parts of the gap (1 and 2) if the introduction of the dielectric



- (a) does not change the voltage across the plates;
(b) leaves the charges at the plates constant.

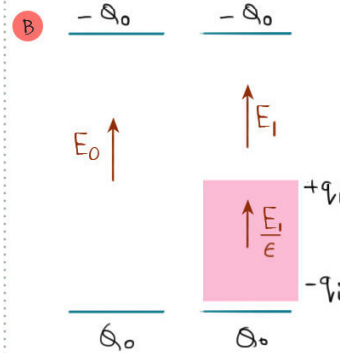


$$V_0 = V_0$$

$$E_0 = E_1 = \frac{E_0}{\epsilon}$$

$$E_1 = 2E_0 \frac{\epsilon}{\epsilon + 1}$$

$$E_2 = \frac{E_1}{\epsilon} = \frac{2E_0}{\epsilon + 1}$$



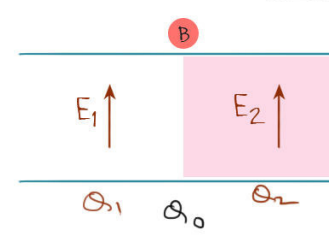
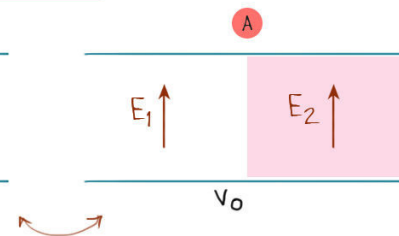
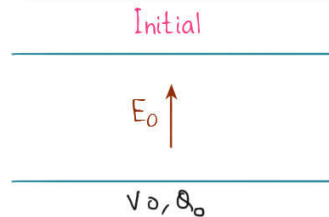
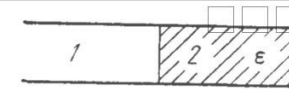
$$E_0 = E_1 = \frac{Q_0}{A\epsilon_0}$$

$$E_1 = E_0 \parallel$$

$$E_2 = \frac{E_0}{\epsilon \parallel}$$

Youtube / Arihant Senior

3.85. Solve the foregoing problem for the case when half the gap is filled with the dielectric in the way shown in Fig. 3.13.



A

$$V_0 = V_0$$

$$E_0 = E_1 = E_2$$

$$\Rightarrow E_1 = E_2 = E_0 \parallel$$

B

$$V_1 = V_2$$

$$\Rightarrow E_1 = E_2 \quad (1)$$

$$E_0 = \frac{Q_0}{2A\epsilon_0}$$

$$E_1 = \frac{Q_1}{A\epsilon_0}$$

$$E_2 = \frac{Q_2}{A\epsilon_0}$$

$$Q_0 = Q_0$$

$$Q_0 = Q_1 + Q_2$$

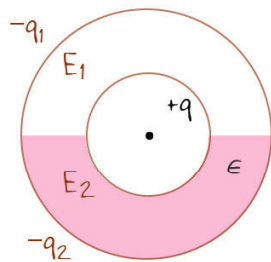
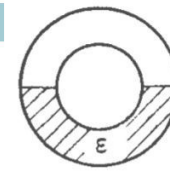
$$E_0 2A\epsilon_0 = E_1 A\epsilon_0 + E_2 A\epsilon_0$$

From 1,

$$E_1 = E_2 = \frac{2E_0}{1 + \epsilon}$$

Youtube / Arihant Senior

3.86. Half the space between two concentric electrodes of a spherical capacitor is filled, as shown in Fig. 3.14, with uniform isotropic dielectric with permittivity ϵ . The charge of the capacitor is q . Find the magnitude of the electric field strength between the electrodes as a function of distance r from the curvature centre of the electrodes.



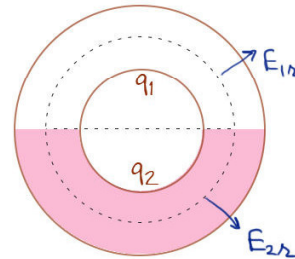
$$V_1 = V_2$$

$$-\int E_{1r} dr = -\int E_{2r} dr$$

$$\Rightarrow E_{1r} = E_{2r} \quad (1)$$

$$q = q_1 + q_2 \quad (2)$$

Gauss's law on hemispherical surface:



$$E_{1r} (2\pi r^2) = \frac{q_1}{\epsilon_0} \quad (3)$$

$$E_{2r} (2\pi r^2) = \frac{q_2}{\epsilon \epsilon_0}$$

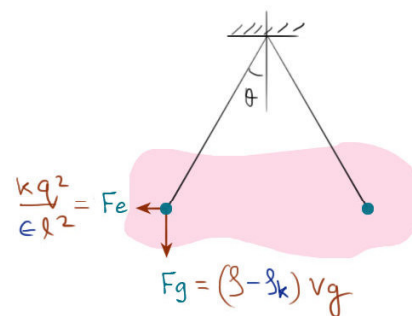
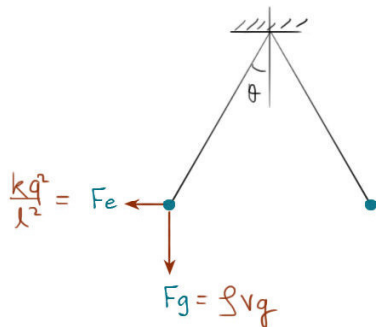
From 1, 2 and 3:

$$q = E_{1r} \epsilon_0 (2\pi r^2) + E_{1r} (2\pi r^2) \epsilon$$

$$\Rightarrow q = E_{1r} \epsilon_0 2\pi r^2 (1 + \epsilon) \Rightarrow E_{1r} = E_{2r} = \frac{q}{2\pi r^2 \epsilon_0 (1 + \epsilon)}$$

Youtube / Arihant Senior

3.87. Two small identical balls carrying the charges of the same sign are suspended from the same point by insulating threads of equal length. When the surrounding space was filled with kerosene the divergence angle between the threads remained constant. What is the density of the material of which the balls are made?



$$\tan \theta = \frac{F_e}{F_{g1}} = \frac{F_e}{F_{g2}}$$

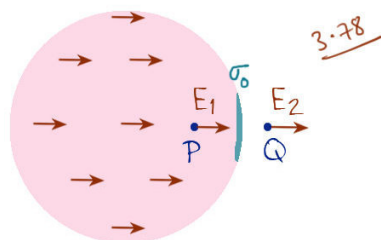
$$\Rightarrow \frac{\frac{kq^2}{l^2}}{S\sqrt{g}} = \frac{\frac{kq^2}{l^2}}{(S - S_k)\sqrt{g}}$$

$$\Rightarrow \epsilon (S - S_k) = S$$

$$\Rightarrow S = \frac{\epsilon S_k}{\epsilon - 1}$$

Youtube / Arihant Senior

3.88. A uniform electric field of strength $E = 100 \text{ V/m}$ is generated inside a ball made of uniform isotropic dielectric with permittivity $\epsilon = 5.00$. The radius of the ball is $R = 3.0 \text{ cm}$. Find the maximum surface density of the bound charges and the total bound charge of one sign.



Maximum surface charge density will be at the end of ball in the direction of E_1 . As maximum polarized molecules will fall on it.

Consider two points P and Q just inside and outside the dielectric.

Only difference in the fields at those two points is due to the surface charge between them.

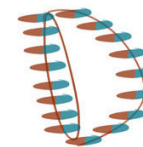
$$\therefore E_2 - E_1 = \frac{\sigma}{\epsilon_0} \quad (1)$$

Also,

$$\frac{E_2}{E_1} = \epsilon \quad (2)$$

Eliminating E_2 from 1 and 2

$$E_1 \epsilon - E_1 = \frac{\sigma}{\epsilon_0} \Rightarrow \sigma = E_1 \epsilon_0 (\epsilon - 1)$$



$$\text{Total charge: } \sigma \cdot 4\pi R^2 = E_1 \epsilon_0 (\epsilon - 1) 4\pi R^2$$

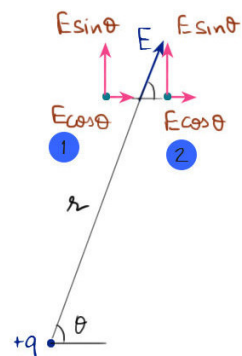
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3.89. A point charge q is located in vacuum at a distance l from the plane surface of a uniform isotropic dielectric filling up all the half-space. The permittivity of the dielectric equals ϵ . Find:

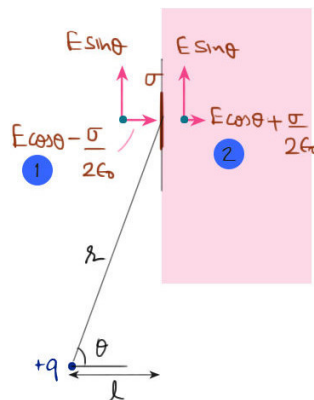
- (a) the surface density of the bound charges as a function of distance r from the point charge q ; analyse the obtained result at $l \rightarrow 0$;
(b) the total bound charge on the surface of the dielectric.

Let, $E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$ at two points 1 and 2 near each other.

A Without dielectric



With dielectric



$$\frac{E_1}{E_2} = \epsilon \Rightarrow \frac{E \cos \theta - \frac{\sigma}{2\epsilon_0}}{E \cos \theta + \frac{\sigma}{2\epsilon_0}} = \epsilon$$

$$\Rightarrow \frac{2E \cos \theta}{-2\frac{\sigma}{2\epsilon_0}} = \frac{\epsilon + 1}{\epsilon - 1}$$

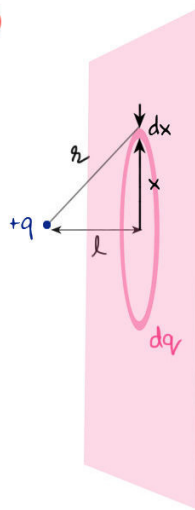
$$\Rightarrow \sigma = -2E \epsilon_0 \cos \theta \left(\frac{\epsilon - 1}{\epsilon + 1} \right)$$

$$\frac{q}{4\pi\epsilon_0 r^2} \cdot \frac{1}{2}$$

$$\Rightarrow \sigma = \frac{-q l (\epsilon - 1)}{2\pi l^3 (\epsilon + 1)}$$

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B



Elemental charge induced on dA :

$$dq = \sigma dA$$

∴ Total charge induced on dielectric

$$\Rightarrow q_i = \int dq = \int_0^{\infty} \frac{-ql(\epsilon-1)}{2\pi r^3(\epsilon+1)} \cdot 2\pi x dx$$

$r = \sqrt{l^2 + x^2}$

$$= -ql \frac{(\epsilon-1)}{(\epsilon+1)} \int_0^{\infty} \frac{x dx}{(x^2 + l^2)^{3/2}}$$

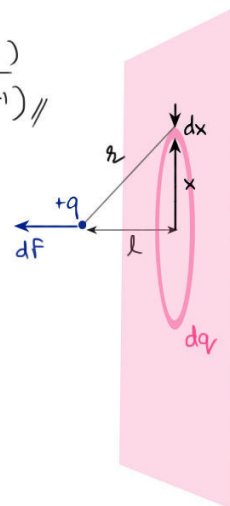
$$= -q \frac{(\epsilon-1)}{(\epsilon+1)} // \quad \frac{1}{l}$$

Youtube / Arihant Senior

3.90. Making use of the formulation and the solution of the foregoing problem, find the magnitude of the force exerted by the charges bound on the surface of the dielectric on the point charge q .

From 3.89:

$$\sigma = \frac{-ql(\epsilon-1)}{2\pi r^3(\epsilon+1)} //$$



$$dF = k q dq(l) // \sigma dA$$

$$= k q \left(\frac{-ql(\epsilon-1)}{2\pi r^3(\epsilon+1)} \right) \cdot 2\pi x dx \cdot \frac{1}{r^2}$$

$$F = \int dF = -k q^2 l^2 \frac{(\epsilon-1)}{(\epsilon+1)} \int_0^{\infty} \frac{x dx}{(x^2 + l^2)^3} // \frac{1}{4l^2}$$

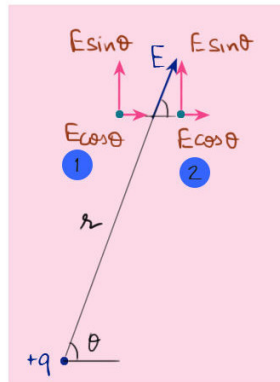
$$= -\frac{k q^2 (\epsilon-1)}{4l^2 (\epsilon+1)} //$$

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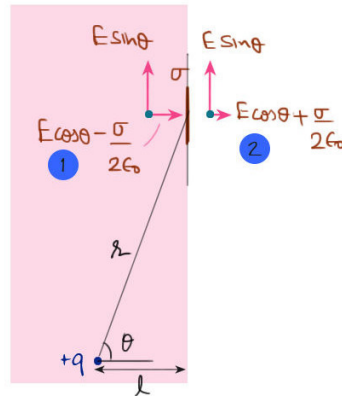
3.92. A small conducting ball carrying a charge q is located in a uniform isotropic dielectric with permittivity ϵ at a distance l from an infinite boundary plane between the dielectric and vacuum. Find the surface density of the bound charges on the boundary plane as a function of distance r from the ball. Analyse the obtained result for $l \rightarrow 0$.

Let, $E = \frac{1}{4\pi\epsilon_0\epsilon} \frac{q}{r^2}$ at two points 1 and 2 near each other.

Full dielectric



Half dielectric



$$\frac{E_1}{E_2} = \frac{1}{\epsilon} \Rightarrow \frac{E \cos \theta - \frac{\sigma}{2\epsilon_0}}{E \cos \theta + \frac{\sigma}{2\epsilon_0}} = \frac{1}{\epsilon}$$

$$\Rightarrow \frac{2E \cos \theta}{-2\frac{\sigma}{2\epsilon_0}} = \frac{1+\epsilon}{1-\epsilon}$$

$$\Rightarrow \sigma = 2E \epsilon_0 \cos \theta \left(\frac{\epsilon-1}{\epsilon+1} \right)$$

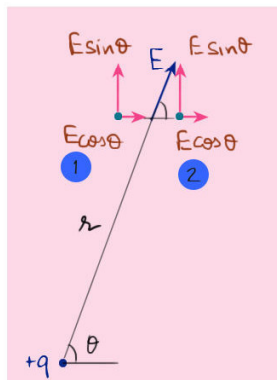
$$\Rightarrow \sigma = \frac{q l (\epsilon-1)}{2\pi r^3 \epsilon (\epsilon+1)}$$

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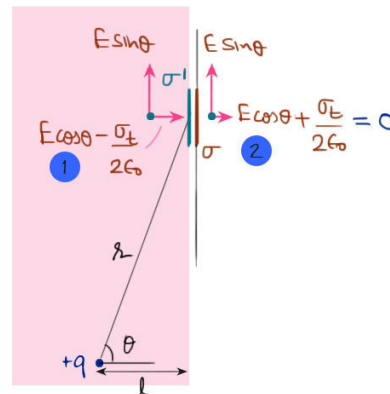
3.93. A half-space filled with uniform isotropic dielectric with permittivity ϵ has the conducting boundary plane. Inside the dielectric, at a distance l from this plane, there is a small metal ball possessing a charge q . Find the surface density of the bound charges at the boundary plane as a function of distance r from the ball.

Let, $E = \frac{1}{4\pi\epsilon_0\epsilon} \frac{q}{r^2}$ at two points 1 and 2 near each other.

Full dielectric



Half dielectric



Field perpendicular to conducting sheet on right side is zero.

$$E \cos \theta + \left(\frac{\sigma + \sigma'}{2\epsilon_0} \right) = 0 \quad \sigma' = -\sigma \frac{\epsilon-1}{\epsilon}$$

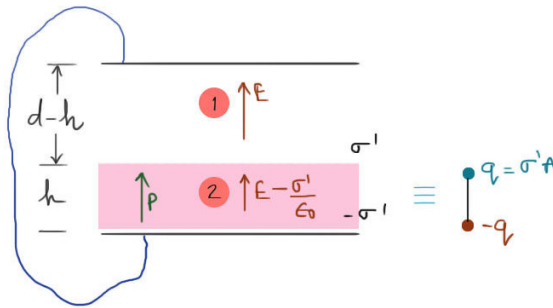
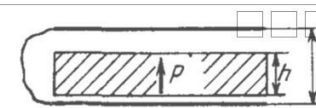
(Derived in problem 3.75)

$$\Rightarrow \frac{q}{4\pi\epsilon_0\epsilon} \frac{l}{r^2} = \frac{-1}{2\epsilon_0} \left(\frac{-\sigma' \epsilon}{\epsilon-1} + \sigma \right)$$

$$\Rightarrow \sigma' = \frac{q l (\epsilon-1)}{2\pi r^3 \epsilon}$$

Youtube / Arihant Senior

3.94. A plate of thickness h made of uniform *statically* polarized dielectric is placed inside a capacitor whose parallel plates are interconnected by a conductor. The polarization of the dielectric is equal to $\mathbf{P}$ (Fig. 3.15). The separation between the capacitor plates is d . Find the strength and induction vectors for the electric field both inside and outside the plates.



$$P V = q h$$

$$\Rightarrow P (A h) = (\sigma' A) h$$

$$\Rightarrow P = \sigma'$$

Potential difference across plates is zero

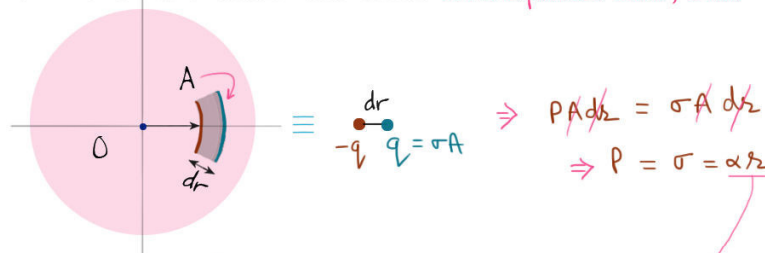
$$\therefore (E - \frac{\sigma'}{\epsilon_0}) h + E (d - h) = 0$$

$$1 \Rightarrow E = \frac{P h}{d \epsilon_0}$$

$$2 \Rightarrow E_2 = E - \frac{\sigma'}{\epsilon_0} = \frac{P h}{d \epsilon_0} - \frac{P}{\epsilon_0} = -\frac{P}{\epsilon_0} (1 - \frac{h}{d})$$

Youtube / Arihant Senior

3.95. A long round dielectric cylinder is polarized so that the vector $\mathbf{P} = \alpha \mathbf{r}$, where α is a positive constant and $\mathbf{r}$ is the distance from the axis. Find the space density ρ' of bound charges as a function of distance r from the axis. (Prerequisite: 3.80, 3.81)



$$\Rightarrow P A d z = \sigma A d z$$

$$\Rightarrow P = \sigma = \alpha r$$



$$d q = \sigma_{r+d r} A_{r+d r} - \sigma_r A_r = d(-\sigma A)$$

$$d V = 2 \pi r d r l$$

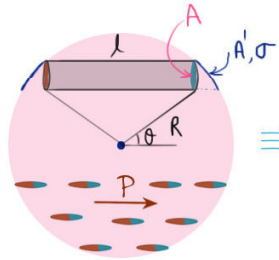
$$\rho = ?$$

$$\therefore \rho_r = \frac{d q}{d V} = -\frac{d(\sigma A)}{d V} = -\frac{\alpha \cdot 2 \pi l d r^2}{2 \pi r d r l} = -2 \alpha$$

Youtube / Arihant Senior

3.96. A dielectric ball is polarized uniformly and statically. Its polarization equals $\mathbf{P}$. Taking into account that a ball polarized in this way may be represented as a result of a small shift of all positive charges of the dielectric relative to all negative charges,

- find the electric field strength $\mathbf{E}$ inside the ball;
- demonstrate that the field outside the ball is that of a dipole located at the centre of the ball, the potential of that field being equal to $\varphi = \mathbf{p}_0 \mathbf{r} / 4\pi \epsilon_0$, where $\mathbf{p}_0$ is the electric moment of the ball, and $\mathbf{r}$ is the distance from its centre.



$$-q \quad q = \sigma A' = \sigma A \cos \theta$$

$$PV = qL$$

$$P(A \cos \theta) = \sigma A \cos \theta$$

$$\Rightarrow \sigma = P \cos \theta$$

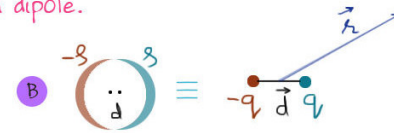
(Surface charge density on dielectric sphere)

From 3.17 we know that for such charge distribution, field inside is constant.

And its value is:

$$E_{in} = -\frac{P}{3\epsilon_0}$$

This field can also be generated by overlap of two uniformly and oppositely charged spheres separated by a small distance d . i.e a dipole.

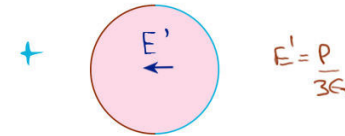
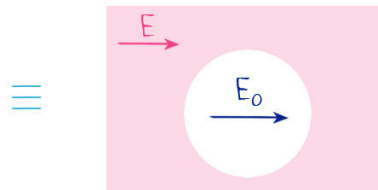
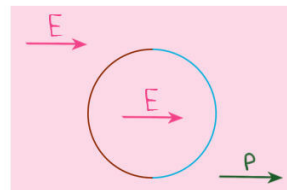


And we know that potential due to a dipole is:

$$V = k \frac{\vec{p} \cdot \vec{r}}{r^3} \quad (\vec{p} = \frac{4}{3}\pi R^3 \vec{P})$$

Youtube / Arihant Senior

3.97. Utilizing the solution of the foregoing problem, find the electric field strength $\mathbf{E}_0$ in a spherical cavity in an infinite statically polarized uniform dielectric if the dielectric's polarization is $\mathbf{P}$, and far from the cavity the field strength is $\mathbf{E}$.



$$E' = \frac{P}{3\epsilon_0}$$

Equating field in the spherical space:

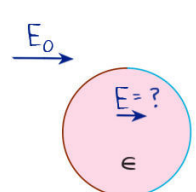
$$\therefore \vec{E} = \vec{E}_0 - \frac{\vec{P}}{3\epsilon_0}$$

$$\Rightarrow \vec{E}_0 = \vec{E} + \frac{\vec{P}}{3\epsilon_0}$$

Note: In book answer is different. I think mine is correct, as field inside vacuum must be greater than field inside dielectric)

Youtube / Arihant Senior

3.98. A uniform dielectric ball is placed in a uniform electric field of strength E_0 . Under these conditions the dielectric becomes polarized uniformly. Find the electric field strength E inside the ball and the polarization P of the dielectric whose permittivity equals ϵ . Make use of the result obtained in Problem 3.96.



$$\vec{E} = \vec{E}_0 - \frac{\vec{P}}{3\epsilon_0} \quad (3)$$

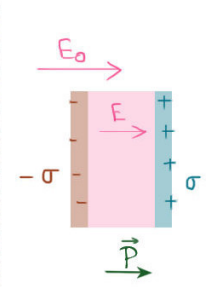
$$\Rightarrow E = E_0 - \frac{\epsilon_0(\epsilon-1)E}{3\epsilon_0}$$

$$\Rightarrow E = \frac{E_0}{1 + \frac{\epsilon-1}{3}}$$

$$\Rightarrow E = \frac{3E_0}{\epsilon+2} \quad (4)$$

From 3,4: $\frac{P}{\epsilon_0(\epsilon-1)} = \frac{3E_0}{\epsilon+2} \Rightarrow P = \frac{3E_0\epsilon_0(\epsilon-1)}{(\epsilon+2)}$

Relation between E and P



$$E_0 - \frac{\sigma}{\epsilon_0} = E$$

$$\Rightarrow \sigma = \epsilon_0(\epsilon-1)E \quad (1)$$

Also,

$$P \cancel{A} = (\sigma \cancel{A}) \cancel{l}$$

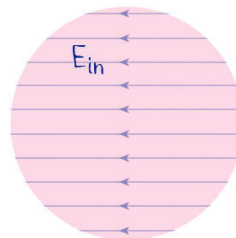
$$P = \sigma \quad (2)$$

From 1 and 2,

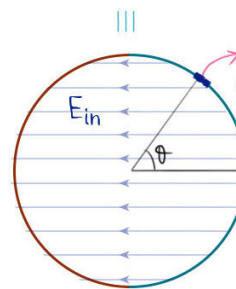
$$P = \epsilon_0(\epsilon-1)E \quad (3)$$

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3.99. An infinitely long round dielectric cylinder is polarized uniformly and statically, the polarization P being perpendicular to the axis of the cylinder. Find the electric field strength E inside the dielectric.

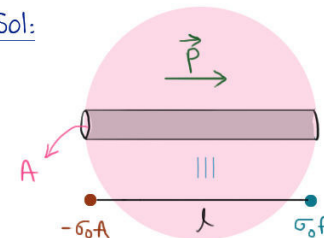


We know that for a uniform polarized cylinder, surface charge density of bounded charge will be of form $\sigma = \sigma_0 \cos \theta$. Then, field inside will be constant, and its value will be: $E_{in} = \frac{\sigma_0}{2\epsilon_0} \quad (1)$



$\sigma = \sigma_0 \cos \theta$
(Surface bound charge density)

Sol:



$$P \cdot V = q \cdot l$$

$$\Rightarrow P \cancel{A} \cancel{l} = \sigma_0 \cancel{A} \cancel{l}$$

$$\Rightarrow P = \sigma_0 \quad (2)$$

From 1 and 2,

$$E_{in} = \frac{P}{2\epsilon_0} = \frac{-\vec{P}}{2\epsilon_0}$$

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3.100. A long round cylinder made of uniform dielectric is placed in a uniform electric field of strength E_0 . The axis of the cylinder is perpendicular to vector E_0 . Under these conditions the dielectric becomes polarized uniformly. Making use of the result obtained in the foregoing problem, find the electric field strength E in the cylinder and the polarization P of the dielectric whose permittivity is equal to ϵ .

Diagram: A cylinder of dielectric ϵ is placed in a uniform electric field E_0 . Inside the cylinder, the electric field is E .

$$\vec{P} = \epsilon_0(\epsilon - 1)\vec{E} \quad (\text{As derived in 3.98})$$

$$\vec{E} = \vec{E}_0 - \frac{\vec{P}}{2\epsilon_0}$$

$$\Rightarrow E = E_0 - \frac{\epsilon_0(\epsilon - 1)E}{2\epsilon_0}$$

$$\Rightarrow E = \frac{E_0}{1 + \frac{\epsilon - 1}{2}}$$

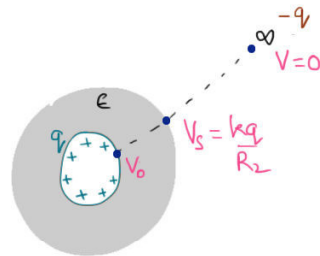
$$\Rightarrow E = \frac{2E_0}{\epsilon + 1} \quad (4)$$

From, 3, 4:

$$\frac{P}{\epsilon_0(\epsilon - 1)} = \frac{2E_0}{\epsilon + 1} \Rightarrow P = \frac{2E_0\epsilon_0(\epsilon - 1)}{(\epsilon + 1)}$$

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3.101. Find the capacitance of an isolated ball-shaped conductor of radius R_1 surrounded by an adjacent concentric layer of dielectric with permittivity ϵ and outside radius R_2 .



Capacitance of isolated ball = Capacitance between ball and infinity

$$\therefore C = \frac{q}{V_0 - 0} = \frac{q}{V_0}$$

$$C = \frac{q}{\frac{kq}{R_2} + \frac{kq}{\epsilon} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)}$$

$$= \frac{4\pi\epsilon_0\epsilon R_1 R_2}{R_2 + (\epsilon - 1)R_1} //$$

$$V_0 \int_{V_s}^{\infty} dV = - \int_{R_2}^{R_1} E \cdot dr$$

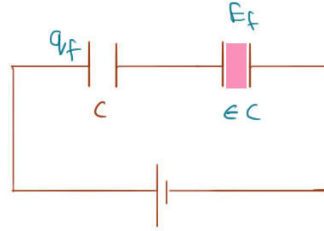
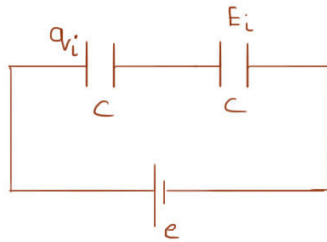
$$V_0 - V_s = - \int_{R_2}^{R_1} \frac{kq}{\epsilon r^2} dr$$

$$V_0 - \frac{kq}{R_2} = \frac{kq}{\epsilon} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$V_0 = \frac{kq}{R_2} + \frac{kq}{\epsilon} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

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3.102. Two parallel-plate air capacitors, each of capacitance C , were connected in series to a battery with emf $\mathcal{E}$. Then one of the capacitors was filled up with uniform dielectric with permittivity ϵ . How many times did the electric field strength in that capacitor decrease? What amount of charge flows through the battery?



$$\frac{E_i}{E_f} = ? , \Delta q = ?$$

B $q_i = C_{eq} \mathcal{E} = \frac{C}{2} \mathcal{E}$

$$q_f = C_{eq} \mathcal{E} = \frac{\epsilon C}{\epsilon + 1} \mathcal{E}$$

$$\Delta q = q_f - q_i = C \left(\frac{\epsilon}{\epsilon + 1} - \frac{1}{2} \right) \mathcal{E} = \frac{C \mathcal{E} (\epsilon - 1)}{2(\epsilon + 1)}$$

A $V = E d$

$$\Rightarrow \frac{E_i}{E_f} = \frac{V_i}{V_f} = \frac{\mathcal{E}/2}{\mathcal{E}/2}$$

$$= \frac{\epsilon + 1}{2}$$

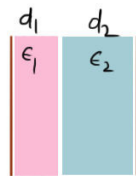
$$\Rightarrow \text{Falls by } \frac{\epsilon + 1}{2} \text{ times.}$$

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3.103. The space between the plates of a parallel-plate capacitor is filled consecutively with two dielectric layers 1 and 2 having the thicknesses d_1 and d_2 and the permittivities ϵ_1 and ϵ_2 respectively. The area of each plate is equal to S . Find:

(a) the capacitance of the capacitor;

(b) the density σ' of the bound charges on the boundary plane if the voltage across the capacitor equals V and the electric field is directed from layer 1 to layer 2.

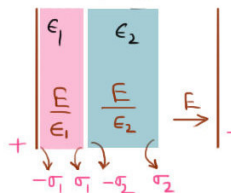


A Its equivalent of two capacitors in series.

$$C_1 = \frac{S \epsilon_0 \epsilon_1}{d_1}, \quad C_2 = \frac{S \epsilon_0 \epsilon_2}{d_2}$$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = S \epsilon_0 \frac{\epsilon_1 \epsilon_2}{d_2 \epsilon_1 + d_1 \epsilon_2}$$

B $\sigma_1 - \sigma_2 =$ Density of bound charges on boundary plane



Inside 1: $E - \frac{\sigma_1}{\epsilon_0} = \frac{E}{\epsilon_1}$ subtracting

Inside 2: $E - \frac{\sigma_2}{\epsilon_0} = \frac{E}{\epsilon_2}$

$$\frac{q}{S \epsilon_0} = \frac{C_{eq} V}{S \epsilon_0}$$

$$\sigma_1 - \sigma_2 = \frac{E (\epsilon_1 - \epsilon_2) \epsilon_0}{\epsilon_1 \epsilon_2}$$

$$= \left[\frac{S \epsilon_0 \epsilon_1 \epsilon_2}{d_2 \epsilon_1 + d_1 \epsilon_2} \frac{V}{S \epsilon_0} \right] \frac{(\epsilon_1 - \epsilon_2)}{\epsilon_1 \epsilon_2} \epsilon_0$$

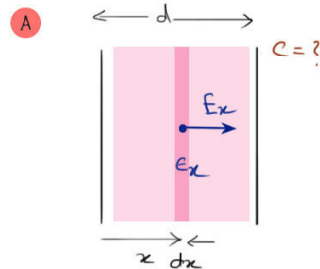
$$= \frac{\epsilon_0 (\epsilon_1 - \epsilon_2) V}{\epsilon_1 d_2 + \epsilon_2 d_1}$$

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3.104. The gap between the plates of a parallel-plate capacitor is filled with isotropic dielectric whose permittivity ϵ varies linearly from ϵ_1 to ϵ_2 ($\epsilon_2 > \epsilon_1$) in the direction perpendicular to the plates. The area of each plate equals S , the separation between the plates is equal to d . Find:

(a) the capacitance of the capacitor;

(b) the space density of the bound charges as a function of ϵ if the charge of the capacitor is q and the field $\mathbf{E}$ in it is directed toward the growing ϵ values.



Lets put q charge on capacator.

$$E_x = \frac{E}{\epsilon_x} = \frac{q}{S \epsilon_0 \epsilon_x}$$

$$dV = -\int E_x dx$$

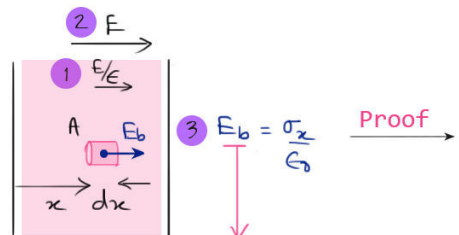
$$= -\frac{q}{S \epsilon_0} \int_0^d \frac{dx}{\epsilon_1 + (\epsilon_2 - \epsilon_1) \frac{x}{d}}$$

$$V = \frac{-q}{S \epsilon_0} \left(\ln \frac{\epsilon_2}{\epsilon_1} \right) \frac{d}{\epsilon_2 - \epsilon_1} \Rightarrow C = \frac{q}{V} = \frac{S \epsilon_0 (\epsilon_2 - \epsilon_1)}{d \ln \epsilon_2 / \epsilon_1}$$

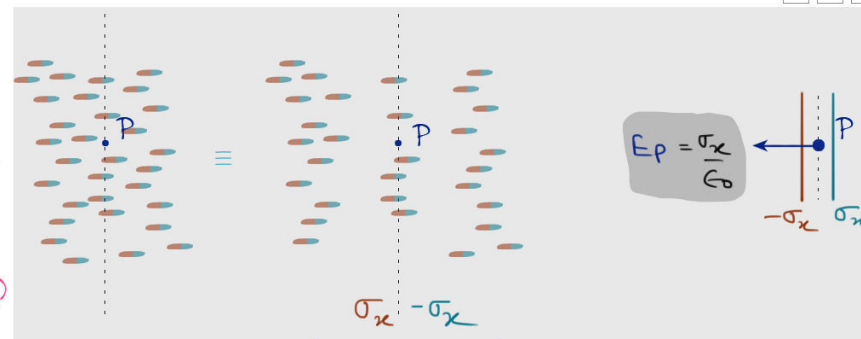
$\Rightarrow$

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B Finding space charge density.



(Due to bound charges only)



$$dq = \sigma_{x+dx} A_{x+dx} - \sigma_x A_x$$

$$= d(\sigma A)$$

$$\rho_x = \frac{dq}{dV} = \frac{d(\sigma A)}{d(A dx)} = \frac{A d\sigma}{A dx} = \frac{d\sigma}{dx}$$

$$\Rightarrow \rho_x = \frac{-q}{S} \frac{d}{dx} \left(-\frac{1}{\epsilon} \right) = \frac{-q}{S \epsilon^2} \frac{d\epsilon}{dx}$$

$$= \frac{-q}{S \epsilon^2} \frac{(\epsilon_2 - \epsilon_1)}{d}$$

$$\epsilon = \epsilon_1 + (\epsilon_2 - \epsilon_1) \frac{x}{d}$$

$$E_{net} = E_{ext} + E_{bound}$$

$$\Rightarrow \frac{E}{\epsilon} = E + \frac{\sigma_x}{\epsilon_0}$$

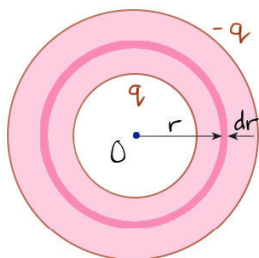
$$\Rightarrow \sigma_x = -E \epsilon_0 \left(1 - \frac{1}{\epsilon} \right)$$

$$\Rightarrow \sigma_x = \frac{-q}{S \epsilon_0} \epsilon_0 \left(1 - \frac{1}{\epsilon} \right)$$

$$\sigma_x = \frac{-q}{S} \left(1 - \frac{1}{\epsilon} \right)$$

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3.105. Find the capacitance of a spherical capacitor whose electrodes have radii R_1 and $R_2 > R_1$ and which is filled with isotropic dielectric whose permittivity varies as $\epsilon = a/r$, where a is a constant, and r is the distance from the centre of the capacitor.



$$E_{in} = \frac{kq}{\epsilon r^2} = \frac{kq}{ar}$$

$$\epsilon = \frac{a}{r}$$

$$\therefore dV = -\int E_{in} \cdot dr$$

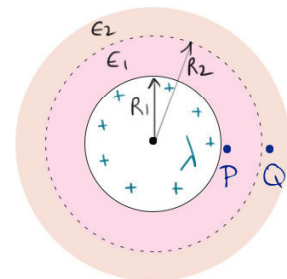
$$\Delta V = -\int_{R_1}^{R_2} \frac{kq}{ar} \cdot dr$$

$$= -\frac{kq}{a} \ln \frac{R_2}{R_1}$$

$$\Rightarrow C = \frac{q}{|V|} = \frac{a}{k \ln R_2/R_1} = \frac{4\pi\epsilon_0 a}{\ln R_2/R_1} //$$

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3.106. A cylindrical capacitor is filled with two cylindrical layers of dielectric with permittivities ϵ_1 and ϵ_2 . The inside radii of the layers are equal to R_1 and $R_2 > R_1$. The maximum permissible values of electric field strength are equal to E_{1m} and E_{2m} for these dielectrics. At what relationship between ϵ , R , and E_m will the voltage increase result in the field strength reaching the breakdown value for both dielectrics simultaneously?



At breakdown, electric field inside the dielectric reach its maximum values E_m

Outside the cylinder, $E = \frac{2kd}{s} \Rightarrow$ In given dielectrics, breakdown will happen at points P and Q as minimum r there implies maximum Field.

Condition simultaneous breakdown:

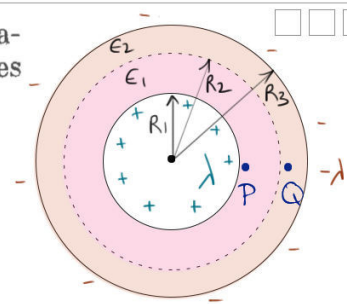
$$E_{1m} = \frac{2kd}{\epsilon_1 R_1}$$

$$E_{2m} = \frac{2kd}{\epsilon_2 R_2}$$

$$\Rightarrow E_{1m} \epsilon_1 R_1 = E_{2m} \epsilon_2 R_2 //$$

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3.107. There is a double-layer cylindrical capacitor whose parameters are shown in Fig. 3.16. The breakdown field strength values for these dielectrics are equal to E_1 and E_2 respectively. What is the breakdown voltage of this capacitor if $\epsilon_1 R_1 E_1 < \epsilon_2 R_2 E_2$? Prerequisite: 3.106:



$$E_P = \frac{2k\lambda}{\epsilon_1 R_1} \Rightarrow E_P \epsilon_1 R_1 = E_Q \epsilon_2 R_2 = 2k\lambda$$

$$E_Q = \frac{2k\lambda}{\epsilon_2 R_2}$$

As λ increases, E_P will reach its maximum value E_1 first, as it is given that: $\epsilon_1 R_1 E_1 < \epsilon_2 R_2 E_2$

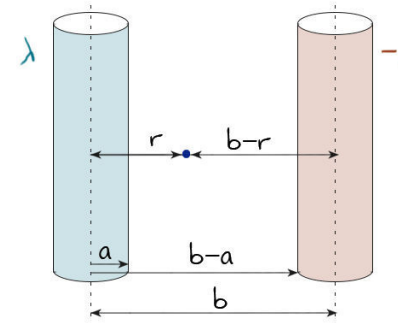
When that happens: $2k\lambda = E_1 \epsilon_1 R_1$

$$\therefore \text{Breakdown voltage, } \Delta V = \int_{R_1}^{R_2} \frac{2k\lambda}{\epsilon_1 r} dr + \int_{R_2}^{R_3} \frac{2k\lambda}{\epsilon_2 r} dr$$

$$= E_1 \epsilon_1 R_1 \left(\ln \frac{R_2}{R_1} + \frac{\epsilon_1}{\epsilon_2} \ln \frac{R_3}{R_2} \right) //$$

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3.108. Two long straight wires with equal cross-sectional radii a are located parallel to each other in air. The distance between their axes equals b . Find the mutual capacitance of the wires per unit length under the condition $b \gg a$.



At a general point between wires:

$$E = \frac{2k\lambda}{r} + \frac{2k\lambda}{(b-r)}$$

$$V = \int_a^{b-a} E \cdot dr = 2k\lambda \left[\ln \frac{r}{b-r} \right]_a^{b-a}$$

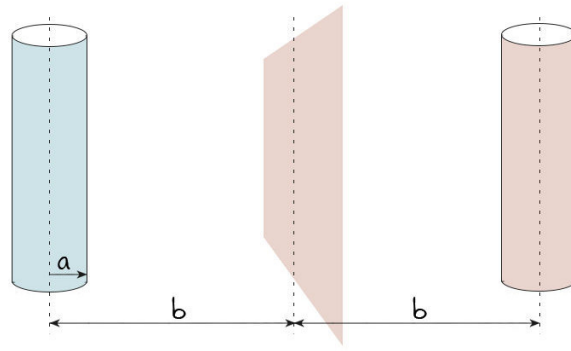
$$= 4k\lambda \ln \left(\frac{b-a}{a} \right) \approx 4k\lambda \ln \frac{b}{a}$$

Finding capacitance

$$Q = CV \xrightarrow{\text{Per unit length:}} \lambda = CV \Rightarrow C = \frac{\lambda}{V} = \frac{1}{4k \ln b/a} = \frac{\pi \epsilon_0}{\ln b/a} //$$

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3.109. A long straight wire is located parallel to an infinite conducting plate. The wire cross-sectional radius is equal to a , the distance between the axis of the wire and the plane equals b . Find the mutual capacitance of this system per unit length of the wire under the condition $a \ll b$.



But potential of the wire-plane capacitor is half of two wires.

And also, $C \propto \frac{1}{V}$

$\Rightarrow$ Capacitance of wire-plane system:

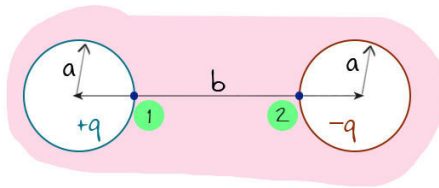
$$C_{WP} = 2 C_{WW} \\ = \frac{2\pi\epsilon_0}{\ln \frac{2b}{a}} //$$

From previous problem 3.108

Capacitance of two mirrored wires will be: $C_{WW} = \frac{\pi\epsilon_0}{\ln \frac{2b}{a}}$

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3.110. Find the capacitance of a system of two identical metal balls of radius a if the distance between their centres is equal to b , with $b \gg a$. The system is located in a uniform dielectric with permittivity ϵ .



$$V_1 = \frac{kq}{\epsilon a} - \frac{kq}{\epsilon(b-a)}$$

$$V_2 = -\frac{kq}{\epsilon a} + \frac{kq}{\epsilon(b-a)}$$

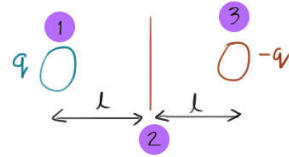
$$V_1 - V_2 = \frac{kq}{\epsilon} \left(\frac{1}{a} - \frac{1}{b-a} + \frac{1}{a} - \frac{1}{b-a} \right) \\ = \frac{2kq}{\epsilon} \left(\frac{1}{a} - \frac{1}{b-a} \right) \approx \frac{2kq}{\epsilon a}$$

$$\therefore C = \frac{q}{V_1 - V_2} = \frac{\epsilon a}{2k} = 2\pi\epsilon_0\epsilon a //$$

(Independent of b when $b \gg a$)

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3.111. Determine the capacitance of a system consisting of a metal ball of radius a and an infinite conducting plane separated from the centre of the ball by the distance l if $l \gg a$. Prerequisite: 3.108, 3.109, 3.110



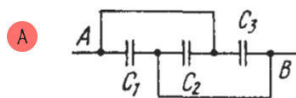
$$V_{12} = \frac{1}{2} V_{13} \quad \& \quad C \propto \frac{1}{V}$$

$$\Rightarrow \frac{C_{12}}{C_{13}} = \frac{V_{13}}{V_{12}} = 2$$

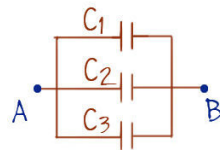
$$\Rightarrow C_{12} = 2C_{13} = 2[2\pi\epsilon_0 a] \\ = 4\pi\epsilon_0 a //$$

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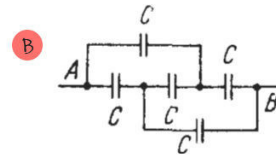
3.112. Find the capacitance of a system of identical capacitors between points A and B shown in



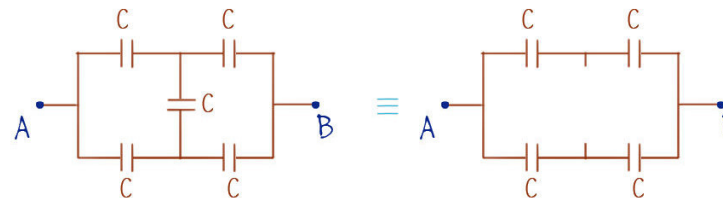
|||



$$\Rightarrow C_{AB} = C_1 + C_2 + C_3 //$$



|||

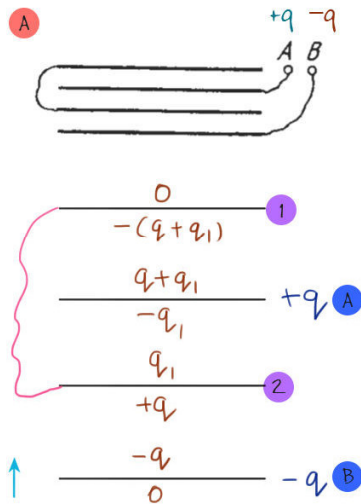


Its a balanced wheatstone bridge of capacitors.
So we can remove the middle capacitor.

$$\Rightarrow C_{AB} = \frac{C}{2} + \frac{C}{2} = C //$$

Youtube / Arihant Senior

3.113. Four identical metal plates are located in air at equal distances d from one another. The area of each plate is equal to S . Find the capacitance of the system between points A and B if the plates are interconnected as shown



Step 1 Outer side of outermost plates get half of total charge each. In this case zero.

Step 2 ↑ We start writing charges from bottom plate and proceed to top. by giving opposite charges to adjacent sides. Introducing variables when needed.

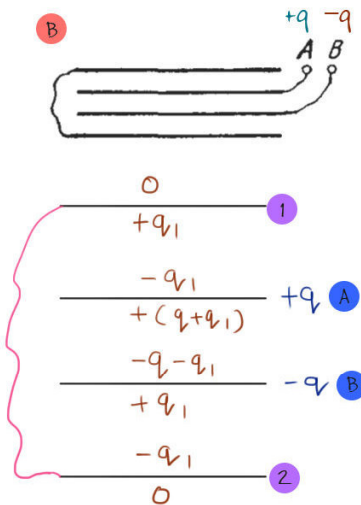
Step 3 Potential difference between 1 and 2 is zero.

$$\therefore \cancel{V_2} + \frac{q_1}{A\epsilon_0} d + \frac{q+q_1}{A\epsilon_0} d = \cancel{V_1} \Rightarrow q+2q_1=0 \Rightarrow q_1 = -q/2$$

Step 4 Find net potential between A and B . And then capacitance.

$$V_B - \frac{q}{A\epsilon_0} d + \frac{q_1}{A\epsilon_0} d = V_A \Rightarrow V_B - V_A = \frac{3q}{2} \frac{d}{A\epsilon_0} \Rightarrow C = \frac{q}{V_B - V_A} = \frac{2A\epsilon_0}{3d}$$

Youtube / Arihant Senior



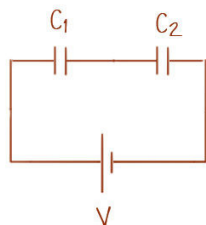
Step 3 Potential difference between 1 and 2 is zero.

$$\therefore q_1 + (q+q_1) + q_1 = 0 \Rightarrow q_1 = -q/3$$

Step 4 Find net potential between A and B . And then capacitance.

$$V_B - (q+q_1) \frac{d}{A\epsilon_0} = V_A \Rightarrow V_B - V_A = \frac{2q}{3} \frac{d}{A\epsilon_0} \Rightarrow C = \frac{q}{V_B - V_A} = \frac{3A\epsilon_0}{2d}$$

3.114. A capacitor of capacitance $C_1 = 1.0 \mu\text{F}$ withstands the maximum voltage $V_1 = 6.0 \text{ kV}$ while a capacitor of capacitance $C_2 = 2.0 \mu\text{F}$, the maximum voltage $V_2 = 4.0 \text{ kV}$. What voltage will the system of these two capacitors withstand if they are connected in series?



1 breaks when

$$V_1 = \frac{C_2 V}{C_1 + C_2}$$

ie, $V = \frac{3 \cdot 6}{2} = 9 \text{ kV}$

2 breaks when

$$V_2 = \frac{C_1 V}{C_1 + C_2}$$

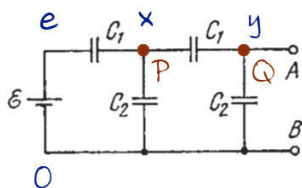
ie $V = \frac{4(3)}{1} = 12 \text{ kV}$

As $9 \text{ kV} < 12 \text{ kV}$

Maximum voltage of system = 9 kV //

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3.115. Find the potential difference between points A and B of the system shown in Fig. 3.19 if the emf is equal to $\mathcal{E} = 110 \text{ V}$ and the capacitance ratio $C_2/C_1 = \eta = 2.0$.



Net charge zero at P:

$$C_1(x-e) + C_2(x-0) + C_1(x-y) = 0$$

$$\Rightarrow (x-e) + \eta x + (x-y) = 0 \quad (1)$$

Net charge zero at Q:

$$C_1(y-x) + C_2(y-0) = 0$$

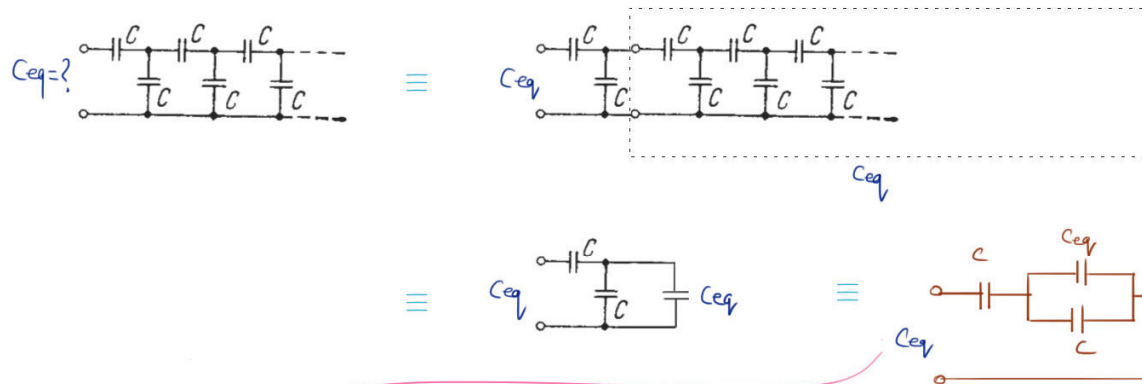
$$\Rightarrow y-x + \eta y = 0 \quad (2)$$

Eliminating x from 1 and 2:

$$y = \frac{e}{1+3\eta+\eta^2} //$$

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3.116. Find the capacitance of an infinite circuit formed by the repetition of the same link consisting of two identical capacitors, each with capacitance C

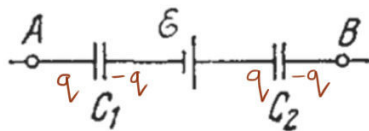


$$\therefore C_{eq} = \frac{C(C + C_{eq})}{C + (C + C_{eq})} \Rightarrow C_{eq}^2 + CC_{eq} - C^2 = 0$$

$$\Rightarrow C_{eq} = \frac{(\sqrt{5}-1)C}{2} //$$

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3.117. A circuit has a section AB shown in Fig. 3.21. The emf of the source equals $\mathcal{E} = 10$ V, the capacitor capacitances are equal to $C_1 = 1.0 \mu\text{F}$ and $C_2 = 2.0 \mu\text{F}$, and the potential difference $\varphi_A - \varphi_B = 5.0$ V. Find the voltage across each capacitor.



Using KVL from A to B

$$V_A - \frac{q}{C_1} + \mathcal{E} - \frac{q}{C_2} = V_B$$

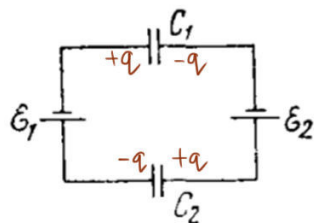
$$\Rightarrow q = \left[(V_A - V_B) + \mathcal{E} \right] \frac{C_1 C_2}{C_1 + C_2}$$

$$\Rightarrow V_1 = \frac{q}{C_1}, V_2 = \frac{q}{C_2}$$

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3.118. In a circuit shown in Fig. 3.22 find the potential difference between the left and right plates of each capacitor.

3.119. Find the charge of each capacitor in the circuit shown



Assuming $E_2 > E_1$, right plate of C_2 will get $+q$ charge.

For the system, both capacitors are in series.

$$\therefore C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \Rightarrow q = C_{eq} V_{eq} = (E_2 - E_1) \frac{C_1 C_2}{C_1 + C_2} \quad \text{B}$$

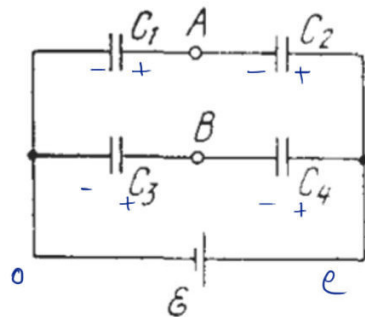
And, $E_{eq} = E_2 - E_1$

(Between left and right plate) A $V_1 = \frac{q}{C_1} = \frac{C_2 (E_2 - E_1)}{C_1 + C_2} \quad //$

$$V_2 = \frac{-q}{C_2} = \frac{-C_1 (E_2 - E_1)}{C_1 + C_2} \quad //$$

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3.120. Determine the potential difference $\varphi_A - \varphi_B$ between points A and B of the circuit shown in Fig. 3.23. Under what condition is it equal to zero?



$$V_A = e - V_2 = e - \frac{e C_1}{C_1 + C_2} = \frac{e C_2}{C_1 + C_2}$$

$$V_B = e - V_4 = e - \frac{e C_3}{C_3 + C_4} = \frac{e C_4}{C_3 + C_4}$$

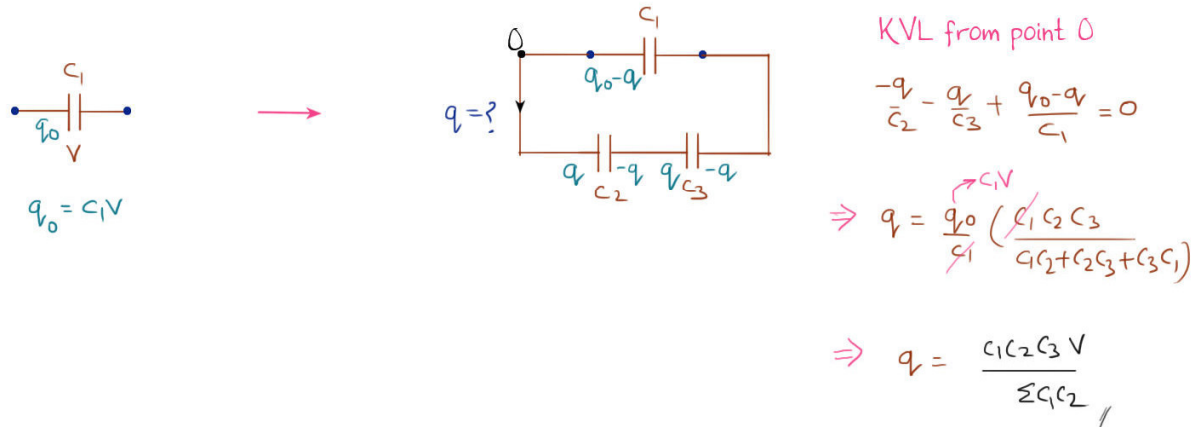
$$\Rightarrow V_A - V_B = e \left[\frac{C_2}{C_1 + C_2} - \frac{C_3}{C_3 + C_4} \right] = e \frac{C_2 C_3 - C_1 C_4}{(C_1 + C_2)(C_3 + C_4)}$$

$V_A = V_B$, When

$$V_A - V_B = 0 \Rightarrow C_2 C_3 = C_1 C_4 \Rightarrow \frac{C_1}{C_2} = \frac{C_3}{C_4}$$

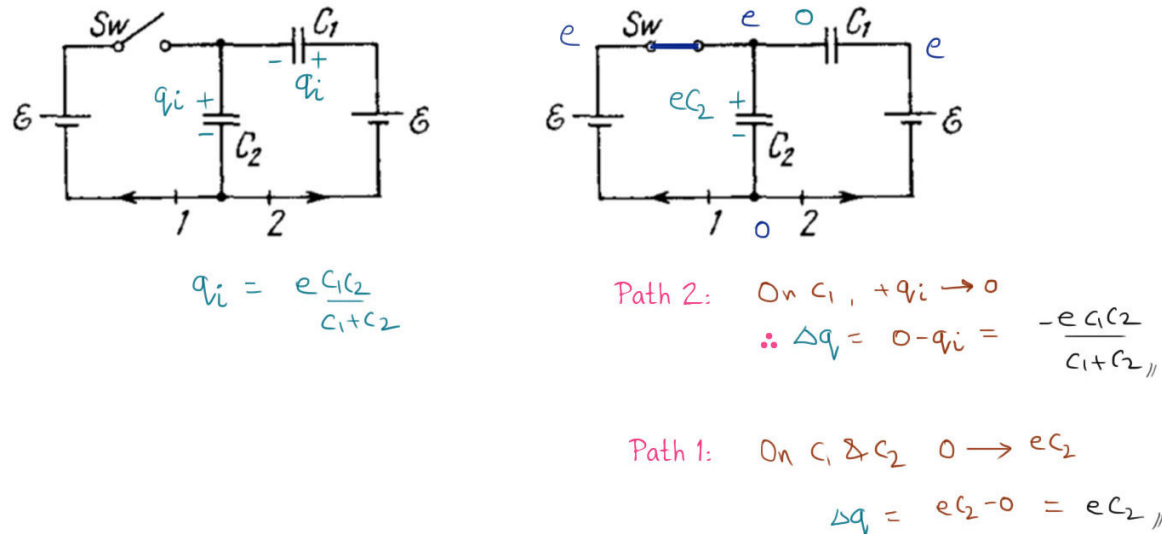
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3.121. A capacitor of capacitance $C_1 = 1.0 \mu\text{F}$ charged up to a voltage $V = 110 \text{ V}$ is connected in parallel to the terminals of a circuit consisting of two uncharged capacitors connected in series and possessing the capacitances $C_2 = 2.0 \mu\text{F}$ and $C_3 = 3.0 \mu\text{F}$. What charge will flow through the connecting wires?



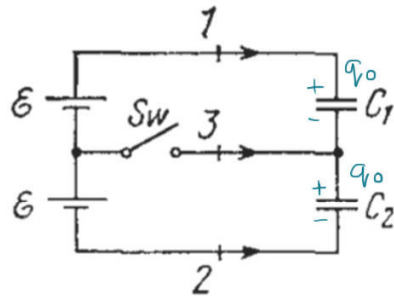
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3.122. What charges will flow after the shorting of the switch Sw in the circuit illustrated in Fig. 3.24 through sections 1 and 2 in the directions indicated by the arrows?

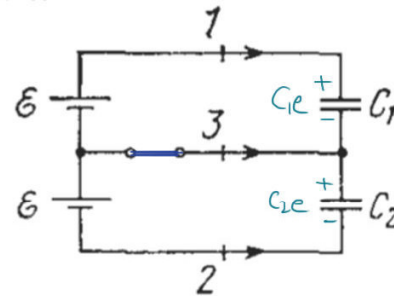


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3.123. In the circuit shown in Fig. 3.25 the emf of each battery is equal to $\mathcal{E} = 60$ V, and the capacitor capacitances are equal to $C_1 = 2.0 \mu\text{F}$ and $C_2 = 3.0 \mu\text{F}$. Find the charges which will flow after the shorting of the switch Sw through sections 1, 2 and 3 in the directions indicated by the arrows.



$$q_0 = \left(\frac{C_1 C_2}{C_1 + C_2} \right) 2e$$



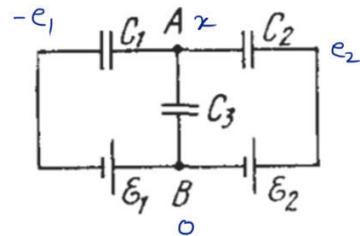
$$\begin{aligned} \text{Path 2: } \Delta q &= -C_2 e - (-q_0) \\ &= -C_2 e + \frac{2C_1 C_2 e}{C_1 + C_2} \\ &= C_2 \frac{(C_1 - C_2)}{C_1 + C_2} e \end{aligned}$$

$$\begin{aligned} \text{Path 1: } \Delta q &= C_1 e - q_0 \\ &= C_1 e - \frac{2C_1 C_2 e}{C_1 + C_2} \\ &= \frac{C_1 (C_1 - C_2)}{C_1 + C_2} e \end{aligned}$$

$$\begin{aligned} \text{Path 3: } \Delta q &= (C_2 e - C_1 e) - 0 \\ &= (C_2 - C_1) e \end{aligned}$$

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3.124. Find the potential difference $\varphi_A - \varphi_B$ between points A and B of the circuit shown in Fig. 3.26.



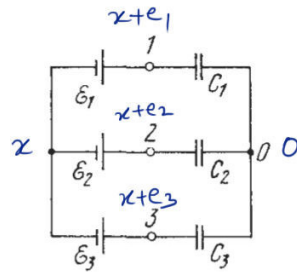
Net charge on node A is zero.

$$\therefore C_1(x + e_1) + C_3(x - 0) + C_2(x - e_2) = 0$$

$$\Rightarrow x = \frac{e_2 C_2 - e_1 C_1}{C_1 + C_2 + C_3}$$

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3.125. Determine the potential at point 1 of the circuit shown in Fig. 3.27, assuming the potential at the point O to be equal to zero. Using the symmetry of the formula obtained, write the expressions for the potentials at points 2 and 3.



Net charge on node O is zero.

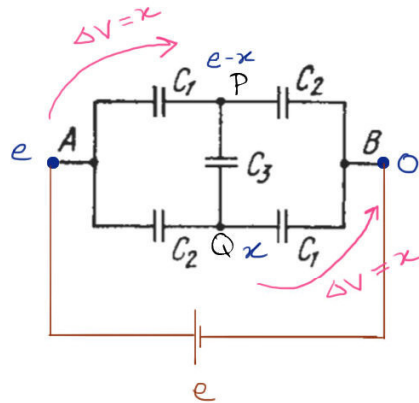
$$\therefore C_1(x+e_1-0) + C_2(x+e_2-0) + C_3(x+e_3-0) = 0$$

$$\Rightarrow x = \frac{-C_1 e_1 - C_2 e_2 - C_3 e_3}{C_1 + C_2 + C_3}$$

$$\Rightarrow V_1 = x + e_1 = \frac{C_2(e_1 - e_2) + C_3(e_1 - e_3)}{C_1 + C_2 + C_3} //$$

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3.126. Find the capacitance of the circuit shown in Fig. 3.28 between points A and B.



By symmetric nature of circuit, potential drop from A to P = potential drop from Q to B.

$$\therefore V_{AP} = V_{QB}$$

Net charge on Node P is zero.

$$\therefore C_1(e-x-e) + C_3(e-x-x) + C_2(e-x-0) = 0$$

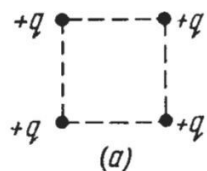
$$\Rightarrow x = \frac{e(C_3 + C_2)}{C_1 + C_2 + 2C_3}$$

$$\text{Also, } C_{eq} = \frac{q}{e} = \frac{C_2(e-x) + C_1(e-(e-x))}{e} \quad (\text{Charge on left side})$$

$$\Rightarrow C_{eq} = \frac{2C_1C_2 + C_3(C_1 + C_2)}{C_1 + C_2 + C_3} //$$

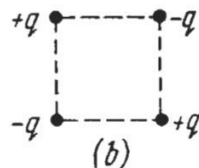
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3.127. Determine the interaction energy of the point charges located at the corners of a square with the side a in the circuits shown in Fig. 3.29.



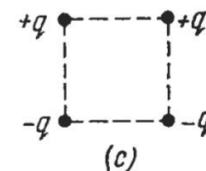
$$E = 4 \times \frac{kq^2}{a} + 2 \times \frac{kq^2}{\sqrt{2}a}$$

$$= (4 + \sqrt{2}) \frac{kq^2}{a} //$$



$$E = 4 \times \frac{-kq^2}{a} + 2 \times \frac{kq^2}{\sqrt{2}a}$$

$$= - (4 - \sqrt{2}) \frac{kq^2}{a} //$$



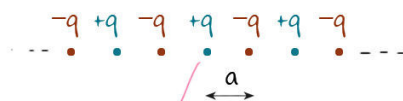
$$E = 0 + 2 \times \frac{-kq^2}{\sqrt{2}a}$$

$$= - \frac{\sqrt{2}kq^2}{a} //$$

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3.128. There is an infinite straight chain of alternating charges q and $-q$. The distance between the neighbouring charges is equal to a . Find the interaction energy of each charge with all the others.

Instruction. Make use of the expansion of $\ln(1 + \alpha)$ in a power series in α .



$$F_{+q} = 2 \left[\frac{-kq^2}{a} + \frac{kq^2}{2a} - \frac{kq^2}{3a} + \dots \right]$$

$$= - \frac{2kq^2}{a} \left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots \right)$$

$$= - \frac{2kq^2}{a} \ln 2 //$$

Using

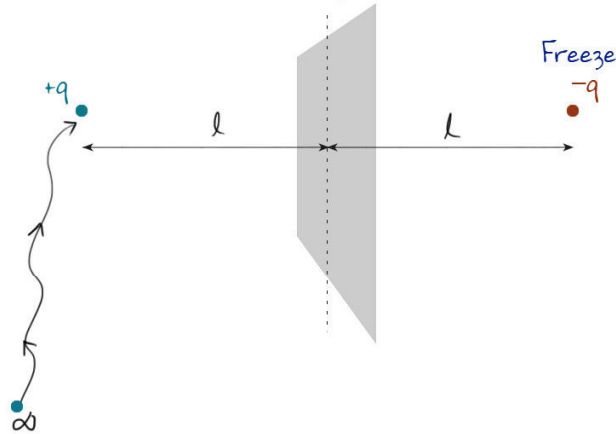
$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

↓

$$\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$$

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3.129. A point charge q is located at a distance l from an infinite conducting plane. Find the interaction energy of that charge with charge induced on the plane.



By mirror method.

Induced charges on plate can be replaced with $-q$ charge on symmetrically opposite side of plate.

To find Interaction energy of system, we need to 'freeze' one of the charges and move the other one from infinity to its position.

Freezing $-q$ charge, W/D by E to bring q to its location: $W_e = \frac{kq^2}{2l}$

∴ Actual energy of the system $= -W_e = -\frac{kq^2}{2l}$

Note: In problem 3.55, W/D to bring $+q$ near the plate, is calculated by keeping the charges on plate 'flexible', and not frozen. It came less as 'unfrozen and flexible' $-q$ would be further away from $+q$ during motion.

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3.130. Calculate the interaction energy of two balls whose charges q_1 and q_2 are spherically symmetrical. The distance between the centres of the balls is equal to l .

Instruction. Start with finding the interaction energy of a ball and a thin spherical layer.

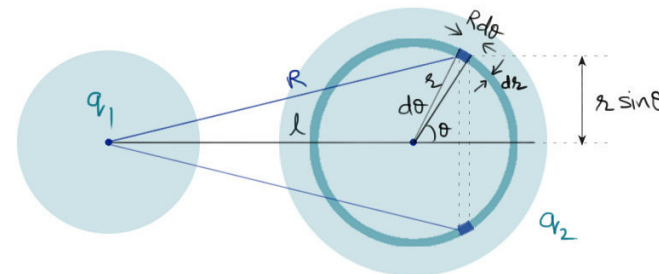
Bringing a small charge dq and depositing it on a ring element within a shell element on the sphere.

$$\begin{aligned} dq \text{ deposited on ring} &= \rho_r dv \\ &= \rho_r (2\pi r \sin\theta) (r d\theta) dr \end{aligned}$$

Approach

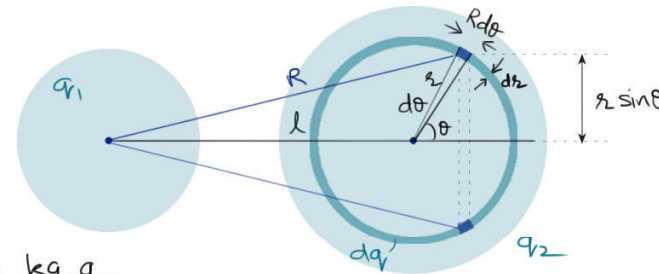
- Energy between q_1 and ring $dE_r = \frac{kq_1 dq}{R}$
- Energy between q_1 and shell $dE_s = \int dE_r$
- (Energy between q_1 and sphere) $E = \int dE_s$

Spherically symmetrical charge distribution is radially symmetrical (need not be uniform)



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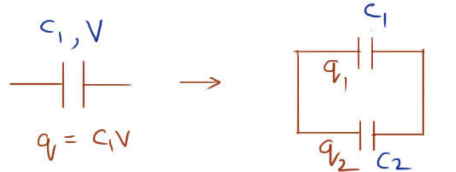
$$\therefore dE_s = \int dE_r = kq_1(q_2 2\pi r^2 dr) \int_0^{\frac{2}{l}} \frac{\sin\theta d\theta}{\sqrt{l^2 + r^2 + 2lr\cos\theta}}$$



$$E = \int dE_s = \frac{kq_1}{\ell} \underbrace{\int (4\pi r^2 dr)}_{q_2} \Big|_r = \frac{kq_1 q_2}{\ell}$$

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3.131. A capacitor of capacitance $C_1 = 1.0 \text{ }\mu\text{F}$ carrying initially a voltage $V = 300 \text{ V}$ is connected in parallel with an uncharged capacitor of capacitance $C_2 = 2.0 \text{ }\mu\text{F}$. Find the increment of the electric energy of this system by the moment equilibrium is reached. Explain the result obtained.



$$Q = Q_1 + Q_2 \quad (1) \quad \left| \quad \begin{aligned} V_1 &= V_2 \\ \frac{Q_1}{C_1} &= \frac{Q_2}{C_2} \quad (2) \end{aligned} \right.$$

From 1 and 2,

$$q_1 = \frac{qC_1}{C_1 + C_2}, \quad q_2 = \frac{qC_2}{C_1 + C_2}$$

Initial energy: $E_i = \frac{1}{2} \frac{q^2}{C_1}$

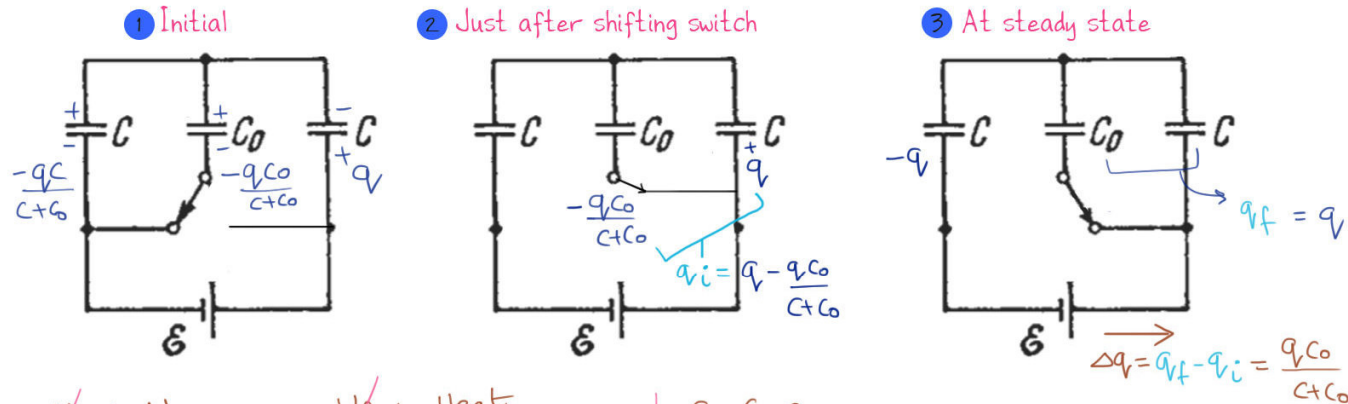
$$\begin{aligned}\text{Final energy: } E_f &= \frac{1}{2} \frac{q_1^2}{c_1} + \frac{1}{2} \frac{q_2^2}{c_2} \\ &= \frac{1}{2} q^2 \left[\frac{c_1}{(c_1+c_2)^2} + \frac{c_2}{(c_1+c_2)^2} \right] \\ &= \frac{1}{2} \frac{q^2}{(c_1+c_2)}\end{aligned}$$

Increment: $\Delta E = E_f - E_i = \frac{1}{2} q V^2 \left(\frac{1}{c_1 + c_2} - \frac{1}{c_1} \right)$

$\swarrow$ $\frac{q^2}{2 c_1 (c_1 + c_2)} = - \frac{c_1 c_2 V^2}{2 (c_1 + c_2)}$

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3.132. What amount of heat will be generated in the circuit shown in Fig. 3.30 after the switch S is shifted from position 1 to position 2?



$$U_i + W_{\text{battery}} = U_f + \text{Heat}$$

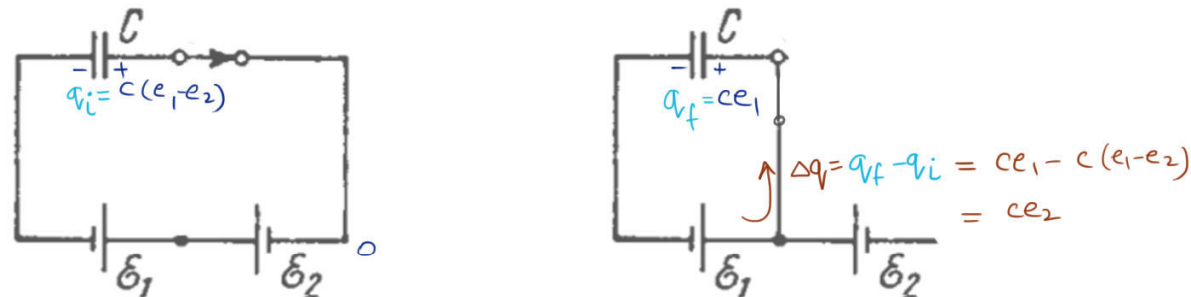
$$\Rightarrow \text{Heat} = W_{\text{battery}} = (\Delta q) e$$

$$= \frac{qC_0 e}{C+C_0} = \frac{C C_0 e^2}{2C+C_0} //$$

$$q = C_{\text{eq}} e = \frac{C(C_0+C)}{C+(C_0+C)} e = \frac{C(C_0+C)e}{2C+C_0}$$

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3.133. What amount of heat will be generated in the circuit shown in Fig. 3.31 after the switch S is shifted from position 1 to position 2?



$$U_i + W_{\text{battery}} = U_f + \text{Heat}$$

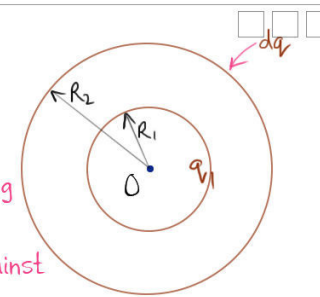
$$\Rightarrow \text{Heat} = U_i - U_f + W_{\text{battery}}$$

$$= \frac{1}{2} C (e_1 - e_2)^2 - \frac{1}{2} C e_1^2 + \Delta q e_1$$

$$= \frac{1}{2} C (e_1^2 + e_2^2 - 2e_1 e_2 - e_1^2 + 2e_1 e_2) = \frac{1}{2} C e_2^2 // \rightarrow \text{its independent of } e_1 !$$

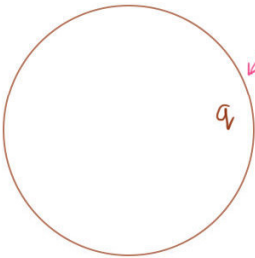
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3.134. A system consists of two thin concentric metal shells of radii R_1 and R_2 with corresponding charges q_1 and q_2 . Find the self-energy values W_1 and W_2 of each shell, the interaction energy of the shells W_{12} , and the total electric energy of the system.



Self energy: W_{ext} to form the system.

A For a shell, depositing dq on already existing q :



$$dW_{ext} = \frac{kq dq}{R}$$

$$W_{ext} = \frac{k}{R} \int_0^{q_0} q dq$$

Shell: Self energy $= W_{ext} = \frac{kq_0^2}{2R} \Rightarrow W_1 = \frac{kq_1^2}{2R_1}, W_2 = \frac{kq_2^2}{2R_2}$

B For concentric shells, depositing dq on outermost shell:

Interaction energy is W_{ext} against inner shell.

$$dW_{ext} = \frac{kq_1 dq}{R_2}$$

$$\Rightarrow W_{12} = W_{ext} = \frac{kq_1}{R_2} \int_0^{q_2} dq = \frac{kq_1 q_2}{R_2}$$

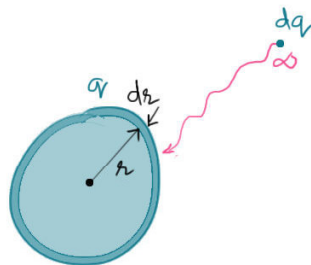
C Total energy of system $= W_1 + W_2 + W_{12}$

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3.135. A charge q_0 is distributed uniformly over the volume of a ball of radius R . Assuming the permittivity to be equal to unity, find:

(a) the electrostatic self-energy of the ball;

(b) the ratio of the energy W_1 stored in the ball to the energy W_2 pervading the surrounding space.



$$dW_{ext} = dqV$$

$$= dq \cdot \frac{kq}{r}$$

$$\oint = \oint = \oint$$

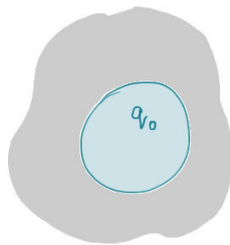
$$\frac{dq}{\cancel{4\pi r^2} dr} = \frac{q}{\frac{4}{3}\pi r^3} = \frac{q_0}{\frac{4}{3}\pi R^3}$$

$$dW_{ext} = \left(\frac{3q_0}{R^3} r^2 dr \right) \cdot k \frac{q_0 r^3}{R^3} \frac{1}{r}$$

Self energy, $\Delta U = W_{ext} = \int dW_{ext} = \frac{3kq_0^2}{R^6} \int_0^R r^4 dr = \frac{3kq_0^2}{5R}$

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(b) the ratio of the energy W_1 stored in the ball to the energy W_2 pervading the surrounding space.



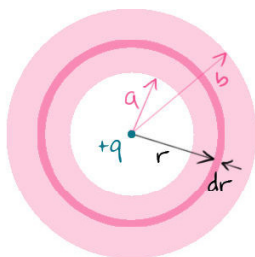
Electrostatic energy W_2 stored in outside region alone =
Total electrostatic energy stored in space for a shell = $W_2 = \frac{kq_0^2}{2R}$ ①

$$W_1 + W_2 = \frac{3kq_0^2}{5R} \text{ ②}$$

$$\text{From 1 and 2, } \frac{W_1}{W_2} = \frac{\frac{3}{5} - \frac{1}{2}}{\frac{1}{2}} = \frac{1}{5} //$$

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3.136. A point charge $q = 3.0 \mu\text{C}$ is located at the centre of a spherical layer of uniform isotropic dielectric with permittivity $\epsilon = 3.0$. The inside radius of the layer is equal to $a = 250 \text{ mm}$, the outside radius is $b = 500 \text{ mm}$. Find the electrostatic energy inside the dielectric layer.



Electrostatic energy stored in elemental shell = $dV = \left(\frac{1}{2} \epsilon_0 \epsilon E^2\right) dV$

$$= \frac{1}{2} \epsilon_0 \epsilon \left(\frac{kq}{r^2}\right)^2 4\pi r^2 dr$$

$$= \frac{1}{2} \cancel{\epsilon_0 \epsilon} \frac{k^2 q^2}{\cancel{\epsilon^2} r^4} \cancel{4\pi} r^2 dr$$

$$= \frac{kq^2}{2\epsilon} \int_a^b \frac{dr}{r^2} = \frac{kq^2}{2\epsilon} \left(\frac{1}{a} - \frac{1}{b}\right) //$$

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3.137. A spherical shell of radius R_1 with uniform charge q is expanded to a radius R_2 . Find the work performed by the electric forces in this process.

$$W_E = -\Delta U$$

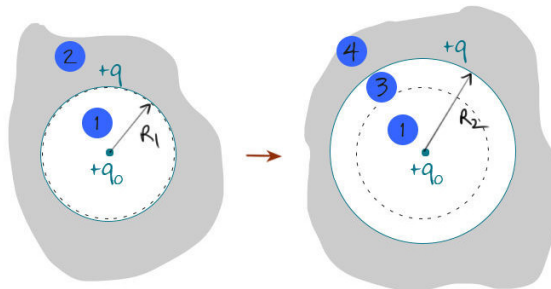
$$= -\left(\frac{kq^2}{2R_f} - \frac{kq^2}{2R_i} \right)$$

$$= \frac{kq^2}{2} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) //$$

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3.138. A spherical shell of radius R_1 with a uniform charge q has a point charge q_0 at its centre. Find the work performed by the electric forces during the shell expansion from radius R_1 to radius R_2 .

(Dividing the entire region in parts)



$$-W_E = \Delta U$$

$$= U_f - U_i$$

$$= (E_1 + E_3 + E_4) - (E_1 + E_2)$$

$$= (E_3 + E_4) - E_2$$

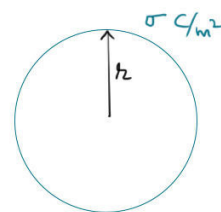
$$= \frac{kq_0^2}{2} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) + \frac{k(q+q_0)^2}{2R_2} - \frac{k(q+q_0)^2}{2R_1}$$

$$-W_E = \frac{k}{2} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) (q_0^2 - (q+q_0)^2)$$

$$\Rightarrow W_E = \frac{k}{2} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) (q^2 + 2qq_0) = kq \left(\frac{q}{2} + q_0 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) //$$

Youtube / Arihant Senior

3.139. A spherical shell is uniformly charged with the surface density σ . Using the energy conservation law, find the magnitude of the electric force acting on a unit area of the shell.



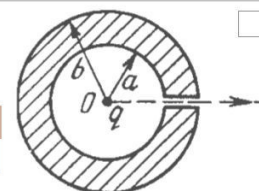
Self energy of the shell $E = \frac{kq^2}{2r}$

∴ Radial force on shell element: $F = -\frac{dE}{dr} = -\frac{kq^2}{2} \frac{d}{dr} \left(\frac{1}{r} \right) = \frac{kq^2}{2r^2}$
(Which is same as force on all radial elements)

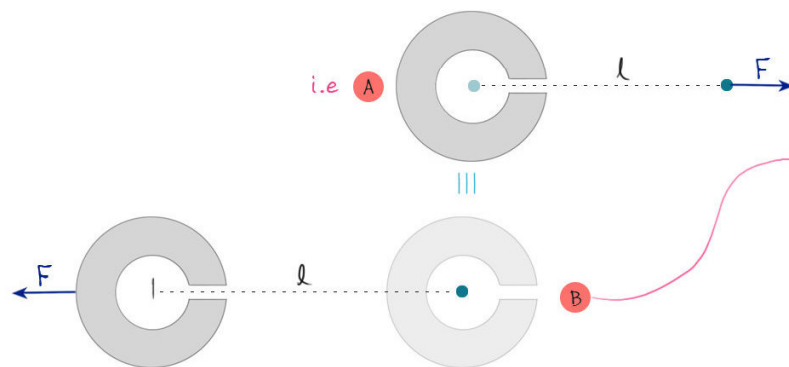
∴ Radial force on shell element per unit area: (Electric pressure) $\frac{F}{4\pi r^2} = \frac{kq^2}{2r^2 \cdot 4\pi r^2} = \frac{1}{2} \frac{1}{4\pi\epsilon_0} \cdot \frac{q^2}{r^2 \cdot 4\pi r^2}$
 $= \frac{1}{2\epsilon_0} \left(\frac{q}{4\pi r^2} \right) \left(\frac{q}{4\pi r^2} \right)$
 $= \frac{\sigma^2}{2\epsilon_0}$

Youtube / Arihant Senior

3.140. A point charge q is located at the centre O of a spherical uncharged conducting layer provided with a small orifice (Fig. 3.32). The inside and outside radii of the layer are equal to a and b respectively. What amount of work has to be performed to slowly transfer the charge q from the point O through the orifice and into infinity?



Work required to remove q keeping conductor fixed is same as work needed to remove conductor, keeping q fixed. (As forces are action reaction pair.)



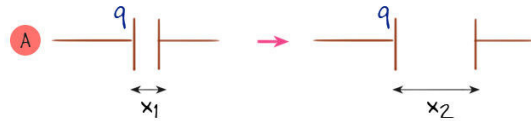
$$\begin{aligned} W_{\text{ext}} &= \Delta U \\ &= U_f - U_i \\ &= (E_1 + E_4 + E_3) - (E_1 + E_2 + E_3) \\ &= E_4 \\ &= \frac{kq^2}{2} \left(\frac{1}{a} - \frac{1}{b} \right) \end{aligned}$$

(Inside conductor)

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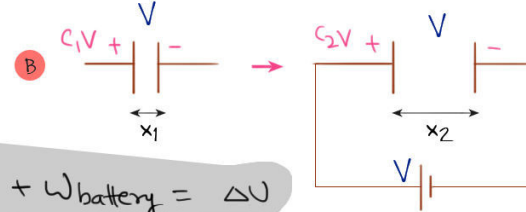
3.141. Each plate of a parallel-plate air capacitor has an area S . What amount of work has to be performed to slowly increase the distance between the plates from x_1 to x_2 if

(a) the charge of the capacitor, which is equal to q , or (b) the voltage across the capacitor, which is equal to V , is kept constant in the process?



$$W_{\text{ext}} = \Delta U$$

$$\begin{aligned} &= \frac{1}{2} \frac{q^2}{C_2} - \frac{1}{2} \frac{q^2}{C_1} \\ &= \frac{1}{2} q^2 \left(\frac{x_2}{\epsilon_0 S} - \frac{x_1}{\epsilon_0 S} \right) \\ &= \frac{q^2 (x_2 - x_1)}{2 \epsilon_0 S} \end{aligned}$$



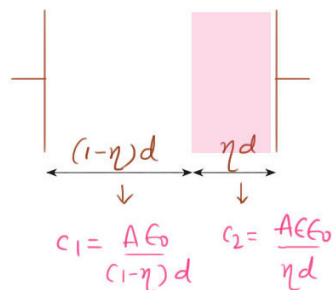
$$W_{\text{ext}} + W_{\text{battery}} = \Delta U$$

$$\begin{aligned} W_{\text{ext}} &= \Delta U - W_{\text{battery}} \\ &= \frac{1}{2} (C_2 - C_1) V^2 - \Delta q V \\ &= \frac{1}{2} (C_2 - C_1) V^2 - (C_2 - C_1) V^2 \\ &= -\frac{1}{2} (C_2 - C_1) V^2 \\ &= -\frac{1}{2} \epsilon_0 S V^2 \left(\frac{1}{x_2} - \frac{1}{x_1} \right) = \frac{\epsilon_0 S V^2}{2} \left(\frac{1}{x_1} - \frac{1}{x_2} \right) \end{aligned}$$

Youtube / Arihant Senior

3.142. Inside a parallel-plate capacitor there is a plate parallel to the outer plates, whose thickness is equal to $\eta = 0.60$ of the gap width. When the plate is absent the capacitor capacitance equals $c = 20$ nF. First, the capacitor was connected in parallel to a constant voltage source producing $V = 200$ V, then it was disconnected from it, after which the plate was slowly removed from the gap. Find the work performed during the removal, if the plate is

(a) made of metal; (b) made of glass.



$$C_1 = \frac{A \epsilon_0}{(1-\eta)d}, \quad C_2 = \frac{A \epsilon_0}{\eta d}$$

$$C = A \epsilon_0 / d$$

$$C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2} = \frac{C \epsilon}{\eta + \epsilon - \eta \epsilon}$$

When dielectric is removed, $q = C_{\text{eq}} V$

$$W_{\text{ext}} = \Delta U = \frac{q^2}{2} \left(\frac{1}{C} - \frac{1}{C_{\text{eq}}} \right)$$

$$= \frac{1}{2} C_{\text{eq}}^2 V^2 \left(\frac{1}{C} - \frac{1}{C_{\text{eq}}} \right)$$

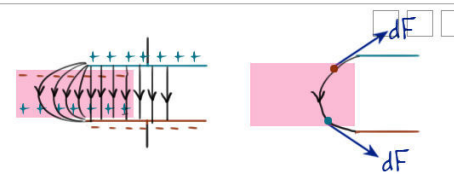
$$= \frac{C V^2 \eta \epsilon (\epsilon - 1)}{2 (\eta + \epsilon - \eta \epsilon)^2} \quad \text{B} \quad \xrightarrow[\text{Conductor}]{\epsilon \rightarrow \infty} \frac{C V^2 \eta}{2 (1-\eta)^2} \quad \text{A}$$

Glass

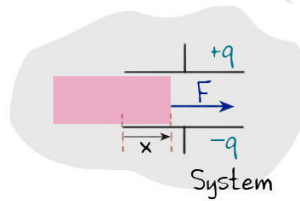
Youtube / Arihant Senior

Force on dielectric in a capacitor

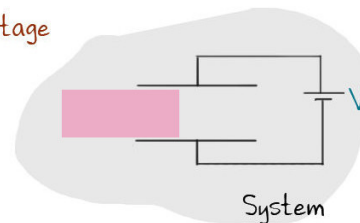
Force on dielectric is due to the fringe electric field lines at the edge of capacitor, that are not perpendicular to capacitor plates.



A Fixed charge



B Fixed Voltage

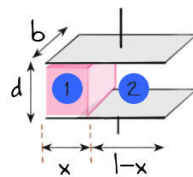


Approach for finding force

Step 1 $U = f(x)$

Step 2 $F = \left| \frac{dU}{dx} \right|$

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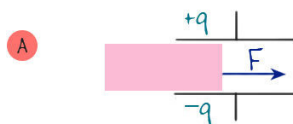
$$C_1 = \frac{(bx)\epsilon\epsilon_0}{d}, \quad C_2 = \frac{b(l-x)\epsilon_0}{d} \Rightarrow C_{eq} = C_1 + C_2$$

$$\Rightarrow C_{eq} = \frac{\epsilon_0 b}{d} (l - x + \epsilon x)$$

Calculating force

Step 1 $U = f(x)$

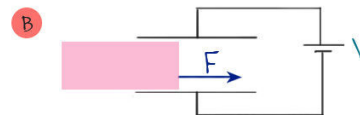
Step 2 $F = \left| \frac{dU}{dx} \right|$



$$U = \frac{q^2}{2C_{eq}} = \frac{q^2}{2} \left(\frac{d}{\epsilon_0 b (l - x + \epsilon x)} \right)$$

$$F = \left| \frac{dU}{dx} \right| = \frac{q^2 d (\epsilon - 1)}{2 \epsilon_0 b (l - x + \epsilon x)^2}$$

(Depends on x)



$$U = \frac{1}{2} C_{eq} V^2 = \frac{V^2}{2} \frac{\epsilon_0 b}{d} (l - x + \epsilon x)$$

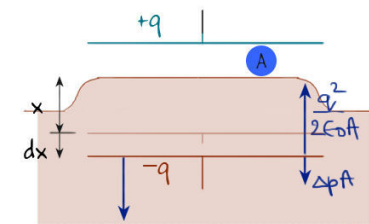
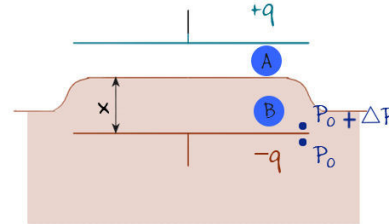
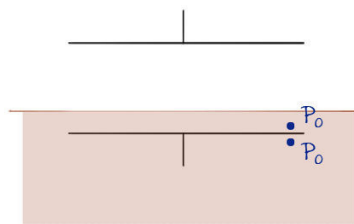
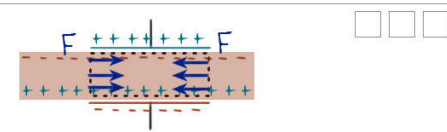
$$F = \left| \frac{dU}{dx} \right| = \frac{\epsilon_0 b V^2}{2d} (\epsilon - 1)$$

(Constant)

Youtube/Arihant Senior

Pressure rise, of liquid dielectric, inside a capacitor

Pressure inside the liquid rises, as the liquid is being pushed from both sides between the plates of capacitor.



$$dU = F_{\text{ext}} dx$$

$$U = U_A + U_B$$

$$= U_A + \frac{q^2}{2C_B}$$

$$U = U_A + \frac{q^2 x}{2A\epsilon_0\epsilon}$$

$$F_{\text{ext}} = \frac{q^2}{2A\epsilon_0\epsilon} \quad (1)$$

$$F_{\text{ext}} = \frac{q^2}{2\epsilon_0 A} - \Delta P A \quad (2)$$

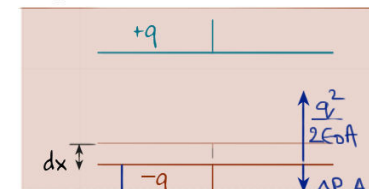
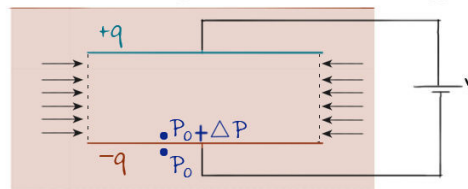
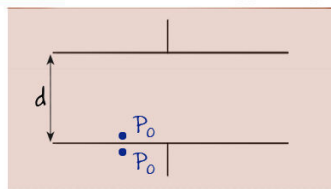
Due to top plate

Due to excess pressure

From 1,2, $\Delta P = \frac{q^2}{2\epsilon_0 A^2} \left(1 - \frac{1}{\epsilon}\right)$

Youtube/Arihant Senior

3.143. A parallel-plate capacitor was lowered into water in a horizontal position, with water filling up the gap between the plates $d = 1.0 \text{ mm}$ wide. Then a constant voltage $V = 500 \text{ V}$ was applied to the capacitor. Find the water pressure increment in the gap.



$$dU = \left(\epsilon_0 \epsilon \frac{E^2}{2}\right) dV$$

(Field energy density)

$$= \frac{\epsilon_0 \epsilon}{2} \left(\frac{\sigma}{\epsilon_0}\right)^2 A dx$$

$$= \frac{\epsilon_0 \epsilon}{2} \frac{q^2}{A^2 \epsilon_0^2} A dx$$

$$\Rightarrow dU = \frac{q^2 dx}{2A\epsilon_0\epsilon}$$

$$\Rightarrow F_{\text{ext}} = \frac{q^2}{2A\epsilon_0\epsilon} \quad (2)$$

$$F_{\text{ext}} = \frac{q^2}{2\epsilon_0 A} - \Delta P A \quad (1)$$

Due to top plate

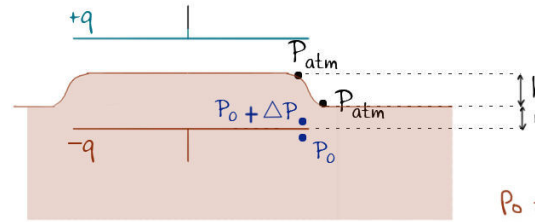
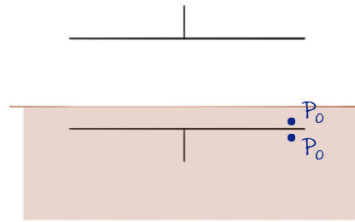
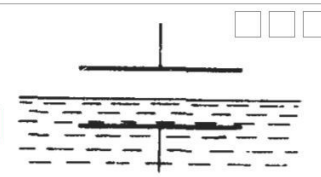
Due to excess pressure

From 1 and 2, $\Delta P = \frac{q^2}{2A^2\epsilon_0} \left(1 - \frac{1}{\epsilon}\right) = \frac{\epsilon_0 (\epsilon - 1) V^2}{2d^2}$

CV = $\frac{A\epsilon_0\epsilon V}{d}$

Youtube/Arihant Senior

3.144. A parallel-plate capacitor is located horizontally so that one of its plates is submerged into liquid while the other is over its surface (Fig. 3.33). The permittivity of the liquid is equal to ϵ , its density is equal to ρ . To what height will the level of the liquid in the capacitor rise after its plates get a charge of surface density σ ?



$$P_0 = P_{atm} + \rho g y$$

$$P_0 + \Delta P = P_{atm} + \rho g (h + y)$$

Subtracting

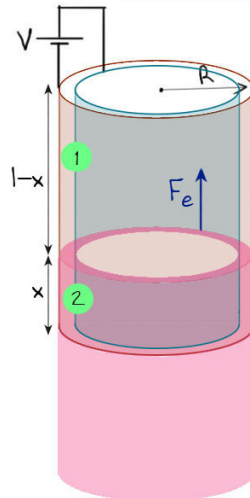
$$\Delta P = \rho g h$$

$$\Delta P = \frac{q^2}{2\epsilon A^2} \left(1 - \frac{1}{\epsilon}\right)$$

$$\therefore h = \frac{q^2}{2\epsilon A^2 \rho g} \left(1 - \frac{1}{\epsilon}\right)$$

Youtube / Arihant Senior

3.145. A cylindrical layer of dielectric with permittivity ϵ is inserted into a cylindrical capacitor to fill up all the space between the electrodes. The mean radius of the electrodes equals R , the gap between them is equal to d , with $d \ll R$. The constant voltage V is applied across the electrodes of the capacitor. Find the magnitude of the electric force pulling the dielectric into the capacitor.



$$C_{eq} = C_1 + C_2$$

$$= \frac{2\pi\epsilon_0(l-x)}{\ln \frac{R+d/2}{R-d/2}} + \frac{2\pi\epsilon_0\epsilon x}{\ln \frac{R+d/2}{R-d/2}}$$

$$\approx \frac{2\pi\epsilon_0}{d/R} [1-x + \epsilon x]$$

$$F_e = \left| \frac{dU}{dx} \right| = \frac{1}{2} V^2 \frac{dC_{eq}}{dx} = \frac{\pi\epsilon_0(\epsilon-1)RV^2}{d}$$

(Constant) //

$$\ln \frac{R+d/2}{R-d/2} = \ln \left(\frac{R-d/2}{R-d/2} + \frac{d}{R-d/2} \right)$$

$$= \ln \left(1 + \frac{d}{R-d/2} \right)$$

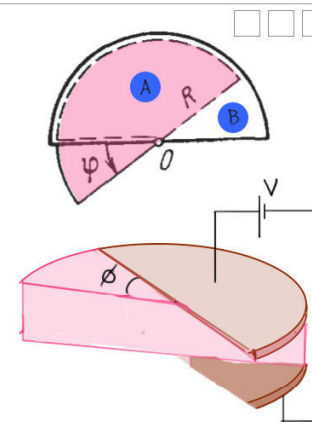
$$\approx \ln \left(1 + \frac{d}{R} \right)$$

$$\approx \frac{d}{R}$$

(Neglecting higher powers of d/R)

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3.146. A capacitor consists of two stationary plates shaped as a semi-circle of radius R and a movable plate made of dielectric with permittivity ϵ and capable of rotating about an axis O between the stationary plates (Fig. 3.34). The thickness of the movable plate is equal to d which is practically the separation between the stationary plates. A potential difference V is applied to the capacitor. Find the magnitude of the moment of forces relative to the axis O acting on the movable plate in the position shown in the figure.



$$C_{eq} = C_A + C_B$$

$$= \frac{(\pi - \phi)}{2\pi} \cdot \pi R^2 \frac{\epsilon \epsilon_0}{d} + \frac{\phi}{2\pi} \cdot \pi R^2 \frac{\epsilon_0}{d}$$

$$= \frac{\epsilon_0 R^2}{2d} (\epsilon(\pi - \phi) + \phi)$$

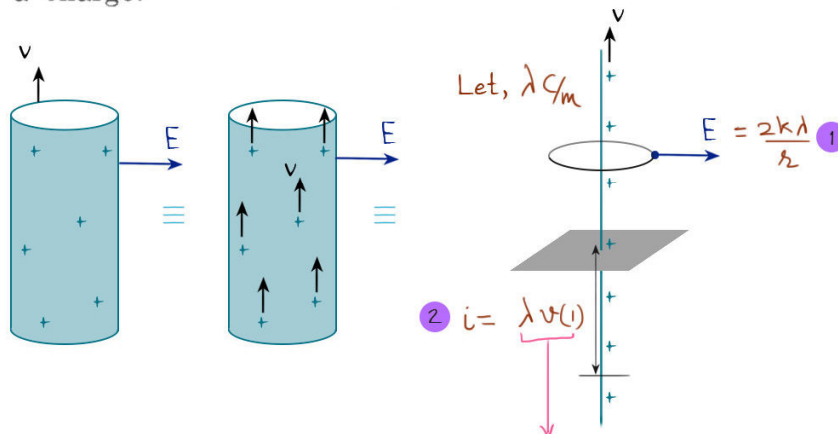
$$U = \frac{1}{2} C_{eq} V^2$$

$$\tau = \left| \frac{dU}{d\phi} \right| = \frac{1}{2} V^2 \cdot \frac{dC_{eq}}{d\phi}$$

$$= \frac{V^2}{2} \frac{\epsilon_0 R^2}{2d} (\epsilon - 1) //$$

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3.147. A long cylinder with uniformly charged surface and cross-sectional radius $a = 1.0$ cm moves with a constant velocity $v = 10$ m/s along its axis. An electric field strength at the surface of the cylinder is equal to $E = 0.9$ kV/cm. Find the resulting convection current, that is, the current caused by mechanical transfer of a charge.



$$\text{Let, } \lambda \text{ C/m}$$

$$E = \frac{2k\lambda}{r} \quad (1)$$

From 1,2:

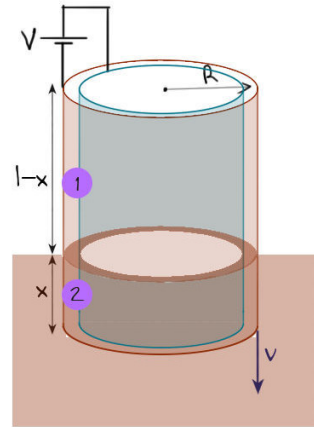
$$E = \frac{2k}{r} \left(\frac{i}{v} \right)$$

$$\Rightarrow i = \frac{Ev r}{2k} //$$

charge crossing a plane per second
i.e current

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3.148. An air cylindrical capacitor with a dc voltage $V = 200$ V applied across it is being submerged vertically into a vessel filled with water at a velocity $v = 5.0$ mm/s. The electrodes of the capacitor are separated by a distance $d = 2.0$ mm, the mean curvature radius of the electrodes is equal to $r = 50$ mm. Find the current flowing in this case along lead wires, if $d \ll r$.



From 3.145, for this capacitor,

$$C_{eq} = \frac{2\pi r \epsilon_0 (l-x + \epsilon x)}{d}$$

$$q = C_{eq} V$$

$$i = \frac{dq}{dt} = V \frac{dC_{eq}}{dt} = V \cdot \frac{2\pi r \epsilon_0 (\epsilon - 1)}{d} \frac{dx}{dt} //$$

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3.149. At the temperature 0°C the electric resistance of conductor 2 is η times that of conductor 1. Their temperature coefficients of resistance are equal to α_2 and α_1 respectively. Find the temperature coefficient of resistance of a circuit segment consisting of these two conductors when they are connected (a) in series; (b) in parallel.

Given,

	1	2
0°C	R	ηR
T	$R(1 + \alpha_1 \Delta T)$	$\eta R(1 + \alpha_2 \Delta T)$

(a)

$$R_f = R_i (1 + \alpha_{eq} \Delta T)$$

$$0^\circ\text{C} \quad R_i = R + \eta R$$

$$T \quad R_f = R(1 + \alpha_1 \Delta T) + \eta R(1 + \alpha_2 \Delta T)$$

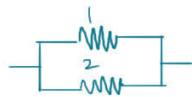
$$= (R + \eta R) + R(\alpha_1 + \eta \alpha_2) \Delta T$$

$$= (R + \eta R) \left[1 + \frac{R}{R + \eta R} (\alpha_1 + \eta \alpha_2) \Delta T \right] = R_i \left[1 + \left(\frac{\alpha_1 + \eta \alpha_2}{\eta + 1} \right) \Delta T \right] //$$

$$\alpha_{eq}$$

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B



$$R_f = R_i (1 + \alpha_{eq} \Delta T)$$

o.c $R_i = \frac{R \eta R}{(\eta + 1) R}$

T $R_f = \frac{R(1 + \alpha_1 \Delta T) \eta R(1 + \alpha_2 \Delta T)}{R(1 + \alpha_1 \Delta T) + \eta R(1 + \alpha_2 \Delta T)}$

$$= \frac{\eta R}{\eta + 1} \left[\frac{(\eta + 1) (1 + \alpha_1 \Delta T) (1 + \alpha_2 \Delta T)}{(\eta + 1) + (\alpha_1 + \eta \alpha_2) \Delta T} \right]$$

$$= R_i \left[(1 + (\alpha_1 + \alpha_2) \Delta T) \left[1 + \frac{(\alpha_1 + \eta \alpha_2) \Delta T}{(\eta + 1)} \right]^{-1} \right]$$

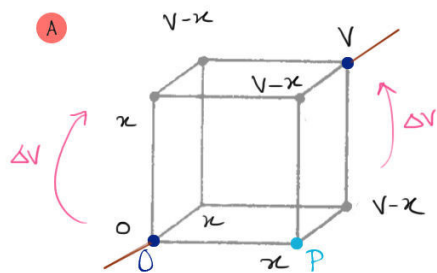
$$= R_i \left[\left[1 + (\alpha_1 + \alpha_2) \Delta T \right] \left[1 - \left(\frac{\alpha_1 + \eta \alpha_2}{\eta + 1} \right) \Delta T \right] \approx R_i \left(1 + \left[(\alpha_1 + \alpha_2) - \frac{(\alpha_1 + \eta \alpha_2)}{\eta + 1} \right] \Delta T \right) \right]$$

Compare

$$\alpha_{eq} = \frac{\eta \alpha_1 + \alpha_2}{\eta + 1} //$$

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A



Total current leaving at P = 0,

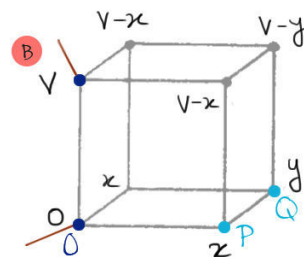
$$P: \frac{x - 0}{R} + \frac{x - (V - x)}{R} + \frac{x - (V - x)}{R} = 0$$

$$\Rightarrow x = 2V/5$$

Total current arriving at O = i,

$$i = \frac{V}{R_{eq}} = \frac{3x}{R} = \frac{6V}{5R} \Rightarrow R_{eq} = \frac{5R}{6} //$$

B



$$P: x + [x - (V - x)] + (x - y) = 0$$

$$Q: (y - x) + (y - x) + [y - (V - y)] = 0$$

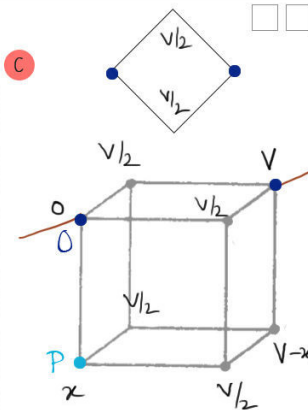
$$\text{Eliminating } y \Rightarrow x = \frac{5V}{14}$$

Total current arriving at O = i,

$$i = \frac{V}{R_{eq}} = \frac{2x}{R} + \frac{V}{R} = \frac{12V}{7R}$$

$$\Rightarrow R_{eq} = \frac{7R}{12} //$$

C



$$P: x + 2(x - \frac{V}{2}) = 0$$

$$\Rightarrow x = V/3$$

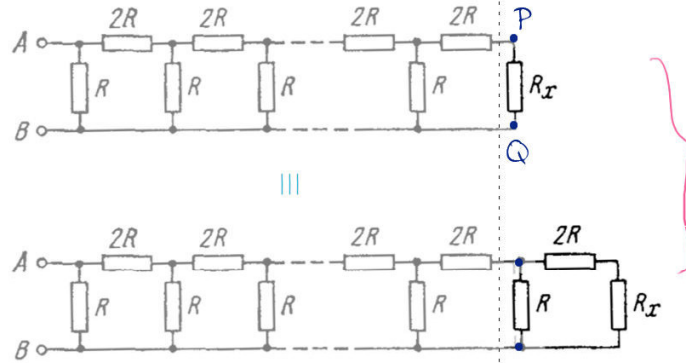
Total i arriving at O = i,

$$i = \frac{V}{R_{eq}} = \frac{V}{2R} + \frac{V}{2R} + \frac{x}{R} = \frac{4V}{3R}$$

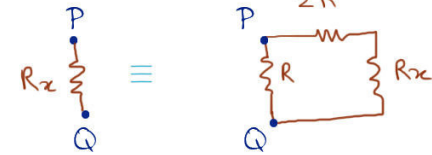
$$\Rightarrow R_{eq} = \frac{3R}{4} //$$

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3.151. At what value of the resistance R_x in the circuit shown in Fig. 3.36 will the total resistance between points A and B be independent of the number of cells?



• Total resistance is independent of number of cells



$$R_x = \frac{R(2R + R_x)}{R + (2R + R_x)}$$

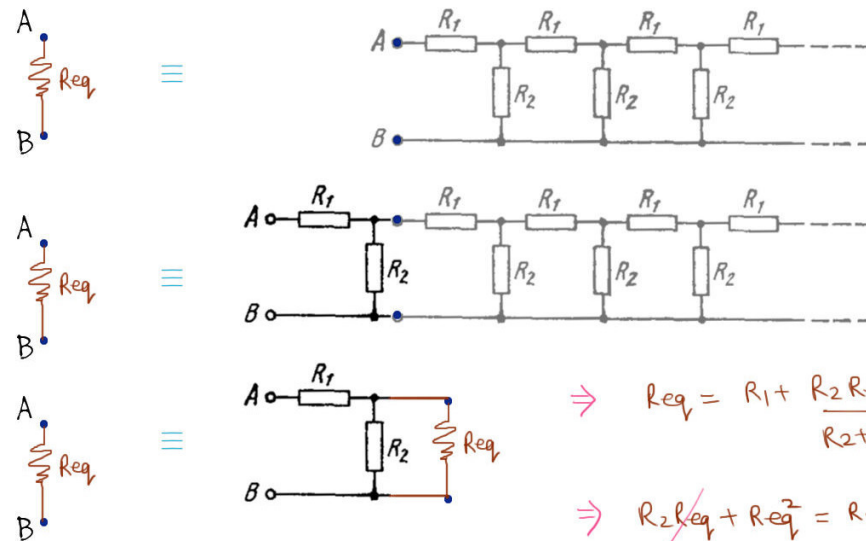
$$\Rightarrow 3RR_x + R_x^2 = 2R^2 + RR_x$$

$$\Rightarrow R_x^2 + 2RR_x - 2R^2 = 0$$

$$\Rightarrow R_x = (\sqrt{3} - 1)R //$$

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3.152. Fig. 3.37 shows an infinite circuit formed by the repetition of the same link, consisting of resistance $R_1 = 4.0 \Omega$ and $R_2 = 3.0 \Omega$. Find the resistance of this circuit between points A and B.



$$\Rightarrow R_{eq} = R_1 + \frac{R_2 R_{eq}}{R_2 + R_{eq}}$$

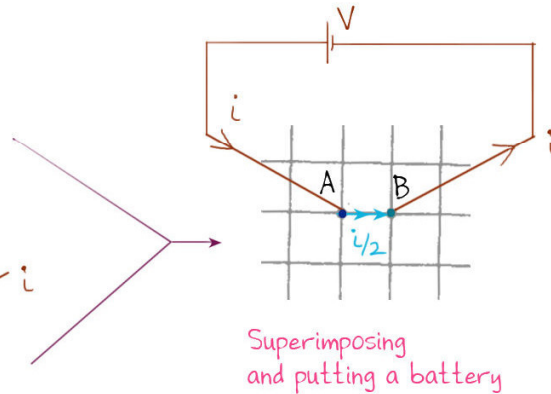
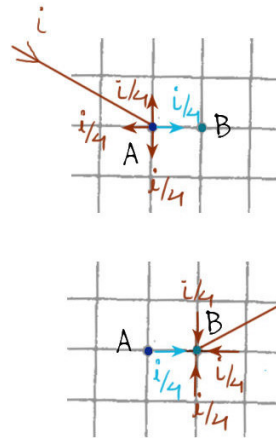
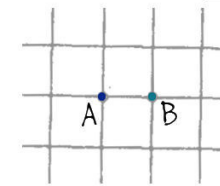
$$\Rightarrow R_2 R_{eq} + R_{eq}^2 = R_1 R_2 + R_1 R_{eq} + R_2 R_{eq}$$

$$\Rightarrow R_{eq} = \frac{R_1 + \sqrt{R_1^2 + 4R_1 R_2}}{2} //$$

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3.153. There is an infinite wire grid with square cells (Fig. 3.38). The resistance of each wire between neighbouring joint connections is equal to R_0 . Find the resistance R of the whole grid between points A and B .

Instruction. Make use of principles of symmetry and superposition.



Between A and B,

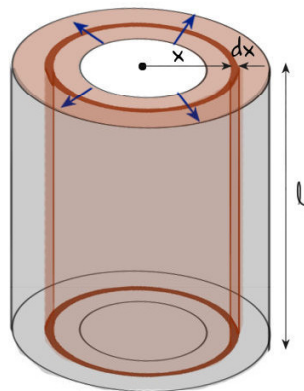
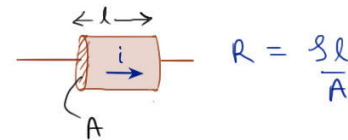
$$V = \frac{i}{2} R_0$$

By definition of resistance.
It's the ratio of V and i .

$$\therefore R_{eq} = \frac{V}{i} = \frac{R_0}{2}$$

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3.154. A homogeneous poorly conducting medium of resistivity ρ fills up the space between two thin coaxial ideally conducting cylinders. The radii of the cylinders are equal to a and b , with $a < b$, the length of each cylinder is l . Neglecting the edge effects, find the resistance of the medium between the cylinders.



For elemental cylindrical shell:

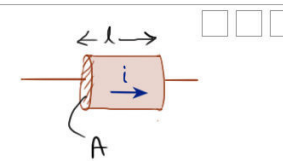
$$dR = \frac{\rho dx}{(2\pi x l)}$$

$$\Rightarrow R = \int dR = \frac{\rho}{2\pi l} \int_a^b \frac{dx}{x}$$

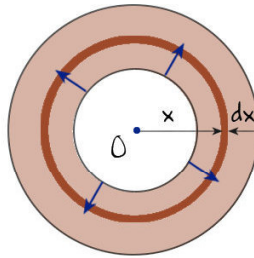
$$\Rightarrow R = \frac{\rho}{2\pi l} \ln \frac{b}{a}$$

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3.155. A metal ball of radius a is surrounded by a thin concentric metal shell of radius b . The space between these electrodes is filled up with a poorly conducting homogeneous medium of resistivity ρ . Find the resistance of the interelectrode gap. Analyse the obtained solution at $b \rightarrow \infty$.



$$R = \frac{\rho l}{A}$$



For elemental spherical shell:

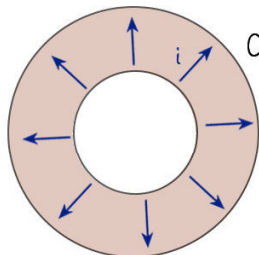
$$dR = \frac{\rho dx}{4\pi x^2}$$

$$\Rightarrow R = \int dR = \frac{\rho}{4\pi} \int_a^b \frac{dx}{x^2}$$

$$R = \frac{\rho}{4\pi} \left(\frac{1}{a} - \frac{1}{b} \right) \quad \xrightarrow{\text{As } b \rightarrow \infty} \quad R = \frac{\rho}{4\pi a}$$

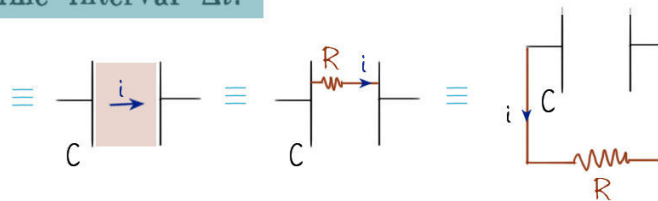
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3.156. The space between two conducting concentric spheres of radii a and b ($a < b$) is filled up with homogeneous poorly conducting medium. The capacitance of such a system equals C . Find the resistivity of the medium if the potential difference between the spheres, when they are disconnected from an external voltage, decreases η -fold during the time interval Δt .



$$R = \frac{\rho}{4\pi} \left(\frac{1}{a} - \frac{1}{b} \right) \quad (1)$$

(As derived in 3.155)



From 1 & 2,

$$\rho = 4\pi \frac{ab}{b-a} \cdot \frac{\Delta t}{C \ln \eta} //$$

For RC decay circuit,

$$\frac{q}{C} - iR = 0 \quad \rightarrow i = -\frac{dq}{dt}$$

$$\Rightarrow \frac{q}{C} = -\frac{dq}{dt} R$$

$$q \rightarrow CV$$

$$\Rightarrow V = -RC \frac{dV}{dt}$$

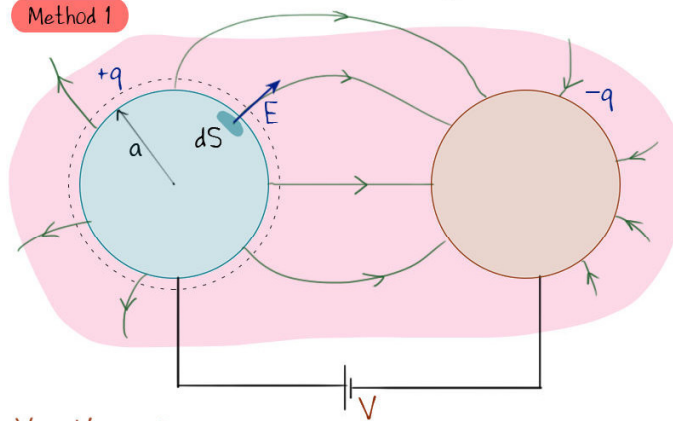
$$\Rightarrow \int_{V/n}^V \frac{dV}{V} = -\frac{1}{RC} \int_0^{\Delta t} dt$$

$$\Rightarrow \ln \eta = \frac{\Delta t}{RC} \quad (2)$$

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3.157. Two metal balls of the same radius a are located in a homogeneous poorly conducting medium with resistivity ρ . Find the resistance of the medium between the balls provided that the separation between them is much greater than the radius of the ball.

Method 1



$$V = V_+ - V_-$$

$$= \frac{kq}{a} - \frac{k(-q)}{a} = 2 \frac{kq}{a} \quad (1)$$

(Neglecting $\pm \frac{kq}{b}$)

Current leaving small element dS :

$$di = j dS = \frac{1}{\rho} \frac{E dS}{\epsilon_0}$$

As E just outside the conductor is perpendicular to surface.
 $= \vec{E} \cdot d\vec{S}$

∴ For complete surface:

$$i = \int di = \frac{1}{\rho} \oint \vec{E} \cdot d\vec{S}$$

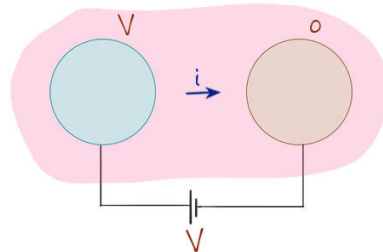
$$\Rightarrow i = \frac{q}{\rho \epsilon_0} \quad (2)$$

From 1,2:

$$i = \left(\frac{Va}{k} \right) \frac{1}{\rho \epsilon_0} = \frac{2\pi Va}{\rho} \Rightarrow R_{eq} = \frac{V}{i} = \frac{\rho}{2\pi a}$$

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Method 2

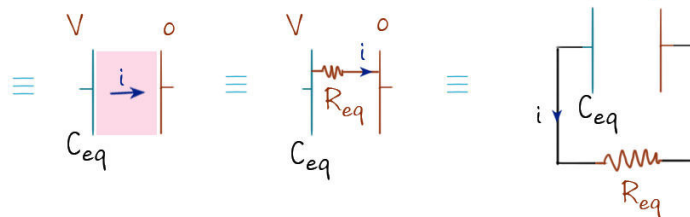


$$C = 2\pi \epsilon_0 a \quad (1)$$

(Derived in 3.110)

From 1,2:

$$R_{eq} = \frac{\rho \epsilon_0}{2\pi \epsilon_0 a} = \frac{\rho}{2\pi a}$$



∴ For equivalent circuit:

$$C_{eq} = \frac{A \epsilon_0}{d}$$

$$R_{eq} = \frac{\rho d}{A}$$

$$R_{eq} = \frac{\rho \epsilon_0}{C_{eq}} \quad (2)$$

A direct result for two conductor capacitor system with dielectric all around them.

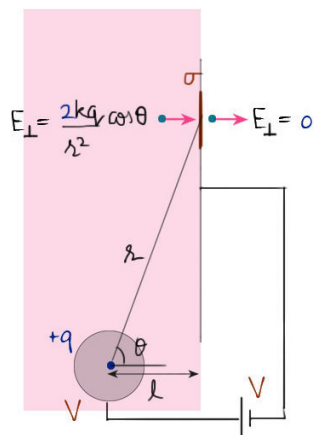
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3.158. A metal ball of radius a is located at a distance l from an infinite ideally conducting plane. The space around the ball is filled with a homogeneous poorly conducting medium with resistivity ρ . In the case of $a \ll l$ find:

Please refer 3.59

(a) the current density at the conducting plane as a function of distance r from the ball if the potential difference between the ball and the plane is equal to V ;

(b) the electric resistance of the medium between the ball and the plane.



$$A \quad j_r = \frac{1}{\rho} E = \frac{1}{\rho} \frac{2kq}{r^2} \cos\theta = \frac{1}{\rho} \frac{2kq}{r^2} \frac{l}{r} = \frac{1}{\rho} \frac{2kq}{r^3} l = \frac{1}{\rho} \frac{2kV a}{r^3} l = \frac{2Val}{\rho r^3} //$$

$$B \quad d\vec{i} = j dS = \frac{1}{\rho} E dS = \frac{1}{\rho} (\vec{E} \cdot d\vec{S})$$

$$\Rightarrow i = \int d\vec{i} = \frac{1}{\rho} \oint \vec{E} \cdot d\vec{S} = \frac{q}{\rho \epsilon_0} = \frac{Va}{k\epsilon_0}$$

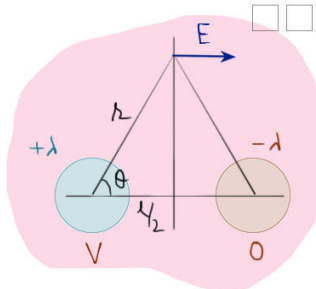
$$\Rightarrow R_{eq} = \frac{V}{i} = \frac{k\rho\epsilon_0}{a} = \frac{\rho}{4\pi a} //$$

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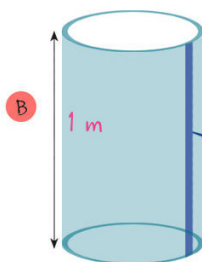
3.159. Two long parallel wires are located in a poorly conducting medium with resistivity ρ . The distance between the axes of the wires is equal to l , the cross-section radius of each wire equals a . In the case $a \ll l$ find:

(a) the current density at the point equally removed from the axes of the wires by a distance r if the potential difference between the wires is equal to V ;

(b) the electric resistance of the medium per unit length of the wires.



$$A \quad j_r = \frac{1}{\rho} E = \frac{1}{\rho} \frac{2 \cdot 2k\lambda}{r^2} \cos\theta = \frac{2k\lambda}{\rho r^2} \frac{l}{r} = \frac{2k\lambda}{\rho r^3} l = \frac{2k\lambda}{\rho r^3} \frac{V l}{4k \ln \frac{b}{a}} = \frac{V l}{2\rho r^3 \ln \frac{b}{a}} //$$

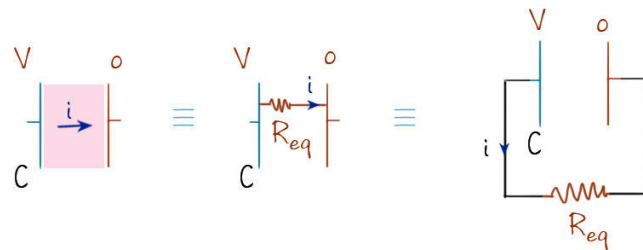


$$B \quad d\vec{i} = j dS = \frac{1}{\rho} E dS = \frac{1}{\rho} \vec{E} \cdot d\vec{S}$$

$$\Rightarrow i = \int d\vec{i} = \frac{1}{\rho} \oint \vec{E} \cdot d\vec{S} = \frac{\lambda(1)}{\rho \epsilon_0} = \frac{V}{4k \ln \frac{b}{a} \rho \epsilon_0} \Rightarrow R_{eq} = \frac{V}{i} = 4k \rho \epsilon_0 \ln \frac{b}{a} = \frac{\rho}{\pi} \ln \frac{b}{a} //$$

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3.160. The gap between the plates of a parallel-plate capacitor is filled with glass of resistivity $\rho = 100 \text{ G}\Omega \cdot \text{m}$. The capacitance of the capacitor equals $C = 4.0 \text{ nF}$. Find the leakage current of the capacitor when a voltage $V = 2.0 \text{ kV}$ is applied to it.

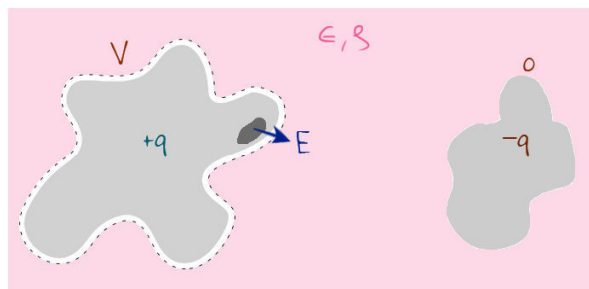


$$C = \frac{A\epsilon\epsilon_0}{d}$$

$$i = \frac{V}{R_{eq}} = \frac{V}{\frac{d}{\epsilon\epsilon_0 A}} = \frac{CV}{\epsilon\epsilon_0}$$

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3.161. Two conductors of arbitrary shape are embedded into an infinite homogeneous poorly conducting medium with resistivity ρ and permittivity ϵ . Find the value of a product RC for this system, where R is the resistance of the medium between the conductors, and C is the mutual capacitance of the wires in the presence of the medium.



$$di = j ds = \frac{1}{\rho} E ds = \frac{\vec{E} \cdot \vec{ds}}{\rho}$$

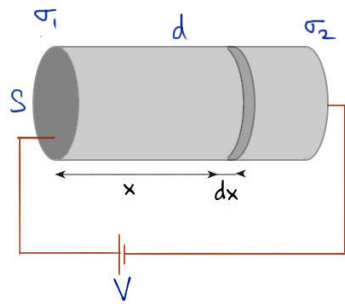
$$\Rightarrow i = \int di = \frac{\oint \vec{E} \cdot d\vec{s}}{\rho} = \frac{q}{\epsilon\epsilon_0 \rho} \quad \left(q = CV \right)$$

$$\Rightarrow R_{eq} = \frac{V}{i} = \frac{\epsilon\epsilon_0 \rho}{C}$$

$$\Rightarrow RC = \rho\epsilon\epsilon_0 //$$

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3.163. The gap between the plates of a parallel-plate capacitor is filled up with an inhomogeneous poorly conducting medium whose conductivity varies linearly in the direction perpendicular to the plates from $\sigma_1 = 1.0 \text{ pS/m}$ to $\sigma_2 = 2.0 \text{ pS/m}$. Each plate has an area $S = 230 \text{ cm}^2$, and the separation between the plates is $d = 2.0 \text{ mm}$. Find the current flowing through the capacitor due to a voltage $V = 300 \text{ V}$.



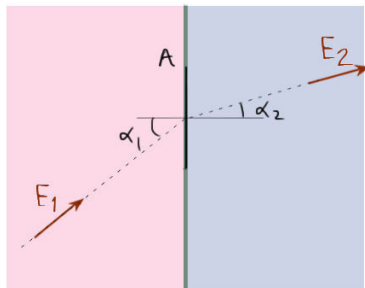
$$dR = \frac{\rho dx}{S} = \frac{dx}{\sigma_x S}$$

$$R = \int dR = \frac{1}{S} \int_0^d \frac{dx}{\left[\sigma_1 + \frac{(\sigma_2 - \sigma_1)x}{d} \right]} = \frac{d}{S(\sigma_2 - \sigma_1)} \ln \frac{\sigma_2}{\sigma_1}$$

$$\Rightarrow i = \frac{V}{R} = \frac{VS(\sigma_2 - \sigma_1)}{d \ln \sigma_2 / \sigma_1}$$

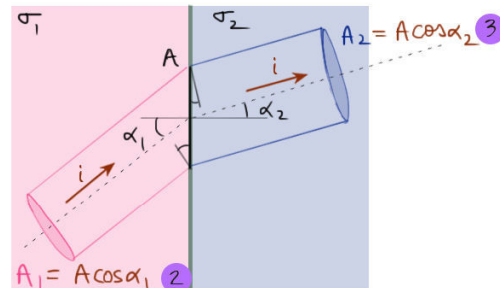
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3.164. Demonstrate that the law of refraction of direct current lines at the boundary between two conducting media has the form $\tan \alpha_2 / \tan \alpha_1 = \sigma_2 / \sigma_1$, where σ_1 and σ_2 are the conductivities of the media, α_2 and α_1 are the angles between the current lines and the normal of the boundary surface.



$$E_{\parallel} = E_{\parallel}$$

$$\Rightarrow E_1 \sin \alpha_1 = E_2 \sin \alpha_2 \quad (1)$$



$$i = i$$

$$j_1 A_1 = j_2 A_2$$

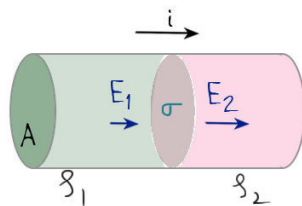
$$(\sigma_1 E_1) A_1 = (\sigma_2 E_2) A_2 \quad (4)$$

From 1,2,3,4:

$$\frac{\sigma_2}{\sigma_1} = \frac{E_1 A_1}{E_2 A_2} = \frac{\sin \alpha_2}{\sin \alpha_1} \cdot \frac{\cos \alpha_1}{\cos \alpha_2} = \frac{\tan \alpha_2}{\tan \alpha_1}$$

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3.165. Two cylindrical conductors with equal cross-sections and different resistivities ρ_1 and ρ_2 are put end to end. Find the charge at the boundary of the conductors if a current I flows from conductor 1 to conductor 2.



$$i = j_1 A = \frac{E_1}{\rho_1} A$$

$$i = j_2 A = \frac{E_2}{\rho_2} A$$

And,

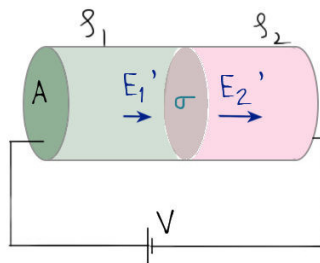
$$E_2 - E_1 = \frac{\sigma}{\epsilon_0}$$

$$\Rightarrow \frac{\rho_2 i}{A} - \frac{\rho_1 i}{A} = \frac{\sigma}{\epsilon_0}$$

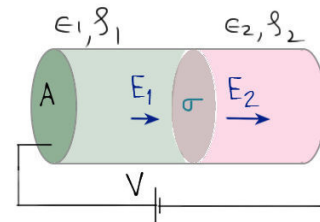
$$\Rightarrow q = \sigma A = \epsilon_0 (\rho_2 - \rho_1) i //$$

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3.166. The gap between the plates of a parallel-plate capacitor is filled up with two dielectric layers 1 and 2 with thicknesses d_1 and d_2 , permittivities ϵ_1 and ϵ_2 , and resistivities ρ_1 and ρ_2 . A dc voltage V is applied to the capacitor, with electric field directed from layer 1 to layer 2. Find σ , the surface density of extraneous charges at the boundary between the dielectric layers, and the condition under which $\sigma = 0$.



Without dielectric, there is ONLY extraneous charge on the boundary layer. Which is given by: $E_2' - E_1' = \frac{\sigma}{\epsilon_0}$ (1)



After filling dielectric:

$$E_1 = \frac{E_1'}{\epsilon_1}, E_2 = \frac{E_2'}{\epsilon_2} \quad (2)$$

$$j_1 = j_2 \Rightarrow \frac{E_1}{\rho_1} = \frac{E_2}{\rho_2} \quad (3)$$

$$E_1 d_1 + E_2 d_2 = V \quad (4)$$

Solving the equations:
From 1,2:

$$E_2 \epsilon_2 - E_1 \epsilon_1 = \frac{\sigma}{\epsilon_0} \quad (A)$$

$$\frac{(4)}{(A)} \Rightarrow \frac{d_1 + \frac{E_2 d_2}{E_1}}{\frac{E_2}{E_1} \epsilon_2 - \epsilon_1} = \frac{V \epsilon_0}{\sigma} \quad (B)$$

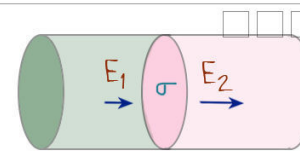
From 3,B:

$$\frac{d_1 + \frac{\rho_2 d_2}{\rho_1}}{\frac{\rho_2}{\rho_1} \epsilon_2 - \epsilon_1} = \frac{V \epsilon_0}{\sigma}$$

$$\Rightarrow \sigma = \frac{V \epsilon_0 (\rho_2 \epsilon_2 - \rho_1 \epsilon_1)}{d_1 \rho_1 + d_2 \rho_2}$$

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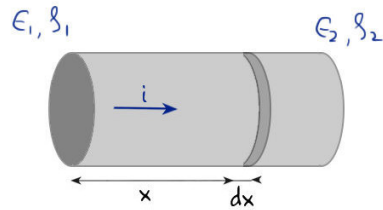
3.167. An inhomogeneous poorly conducting medium fills up the space between plates 1 and 2 of a parallel-plate capacitor. Its permittivity and resistivity vary from values ϵ_1, ρ_1 at plate 1 to values ϵ_2, ρ_2 at plate 2. A dc voltage is applied to the capacitor through which a steady current I flows from plate 1 to plate 2. Find the total extraneous charge in the given medium.



$$\epsilon_2 \epsilon_2 - \epsilon_1 \epsilon_1 = \frac{\sigma}{\epsilon_0} \quad (1)$$

(Derived in 3.166)

Calculating extraneous charge on an element



From 1,

$$d(\epsilon \epsilon) = \frac{\sigma}{\epsilon_0} = \frac{dq}{A \epsilon_0}$$

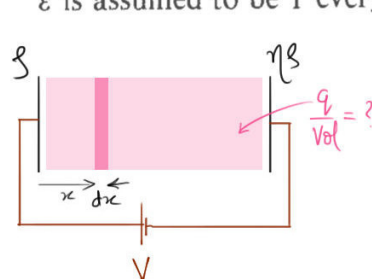
$$\Rightarrow \int_{\epsilon_1 \epsilon_1}^{\epsilon_2 \epsilon_2} d(\epsilon \epsilon) = \frac{1}{A \epsilon_0} \int_0^q dq \Rightarrow q = A \epsilon_0 (\epsilon_2 \epsilon_2 - \epsilon_1 \epsilon_1)$$

$$i = j A = \frac{E_2 A}{\rho_2} = \frac{E_1 A}{\rho_1}$$

$$\Rightarrow q = \epsilon_0 i (\rho_2 \epsilon_2 - \rho_1 \epsilon_1)$$

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3.168. The space between the plates of a parallel-plate capacitor is filled up with inhomogeneous poorly conducting medium whose resistivity varies linearly in the direction perpendicular to the plates. The ratio of the maximum value of resistivity to the minimum one is equal to η . The gap width equals d . Find the volume density of the charge in the gap if a voltage V is applied to the capacitor. ϵ is assumed to be 1 everywhere.



$$j = \sigma E$$

$$\Rightarrow \frac{i}{S} = \frac{1}{\rho} E$$

$$\Rightarrow \frac{V}{\rho S} = \frac{E}{S} \Rightarrow E = \frac{\rho V}{RS}$$

$$E_x \quad E_{x+dx}$$

$$E_{x+dx} - E_x = \frac{\sigma}{\epsilon_0} = \frac{dq}{S \epsilon_0}$$

$$\Rightarrow dE = \frac{dq}{S \epsilon_0}$$

$$\Rightarrow q = (E_2 - E_1) S \epsilon_0$$

$$= \frac{V}{RS} (\rho_2 - \rho_1) S \epsilon_0$$

$$q = \frac{V (\rho_2 - \rho_1) \epsilon_0}{R}$$

$$dR = \frac{\rho_x dx}{S}$$

$$R = \int_0^d dR = \int_0^d \left[\rho + (\eta \rho - \rho) \frac{x}{d} \right] \frac{dx}{S}$$

$$= \frac{\rho}{S} \left[x + (\eta - 1) \frac{x^2}{2d} \right]_0^d$$

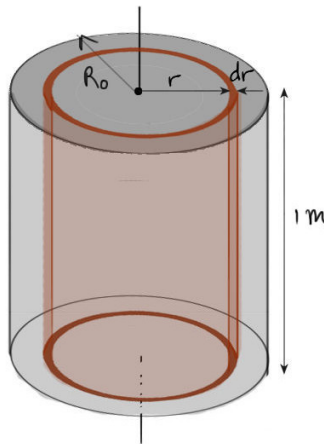
$$R = \frac{d \rho (\eta + 1)}{2S}$$

$$\frac{V \epsilon_0 \rho (\eta - 1) S}{d \rho (\eta + 1)} \Rightarrow \frac{q}{Sd} = \frac{2 \epsilon_0 (\eta - 1) V}{d^2 (\eta + 1)}$$

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3.169. A long round conductor of cross-sectional area S is made of material whose resistivity depends only on a distance r from the axis of the conductor as $\rho = \alpha/r^2$, where α is a constant. Find:

- (a) the resistance per unit length of such a conductor;
 (b) the electric field strength in the conductor due to which a current I flows through it.



A Resistance of elemental cylindrical shell:

$$dR = \frac{\rho \cdot (1 \text{ m})}{2\pi r dr} = \frac{\alpha}{2\pi r^3 dr}$$

All elemental cylindrical shell shaped resistances are in parallel.

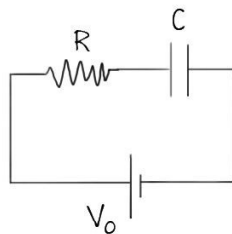
$$\therefore \frac{1}{R_{eq}} = \int \frac{1}{dR} = \int_0^{R_0} \frac{2\pi r^3 dr}{\alpha} = \frac{2\pi R_0^4}{\alpha \cdot 2} = \frac{(\pi R_0^2)^2}{2\alpha\pi} = \frac{S^2}{2\alpha\pi}$$

$$\Rightarrow R_{eq} = \frac{2\pi\alpha}{S^2}$$

B $E d = V = I R_{eq} \Rightarrow E = \frac{I R_{eq}}{d} = \frac{I}{\frac{S}{4\pi r}} \left(\frac{2\pi\alpha}{S^2} \right) = \frac{2\pi\alpha I}{S^2}$

Youtube / Arihant Senior

3.170. A capacitor with capacitance $C = 400 \text{ pF}$ is connected via a resistance $R = 650 \Omega$ to a source of constant voltage V_0 . How soon will the voltage developed across the capacitor reach a value $V = 0.90 V_0$?



For RC current growth circuit,

$$q = q_0 (1 - e^{-t/RC})$$

$$CV = CV_0 (1 - e^{-t/RC})$$

$$0.9V_0 = V_0 (1 - e^{-t/RC})$$

$$\Rightarrow 0.1 = e^{-t/RC}$$

$$\Rightarrow t = -RC \ln(0.1) = RC \ln 10 //$$

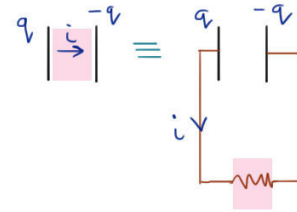
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3.171. A capacitor filled with dielectric of permittivity $\epsilon = 2.1$ loses half the charge acquired during a time interval $\tau = 3.0$ min. Assuming the charge to leak only through the dielectric filler, calculate its resistivity.

For a current decay RC circuit:

$$q = q_0 e^{-t/RC}$$

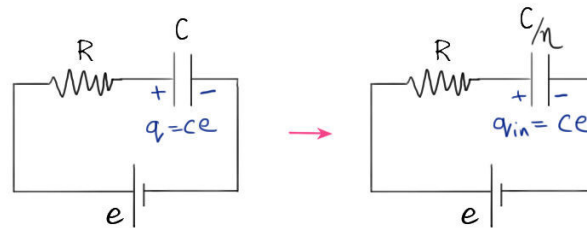
Annotations: $q_0/2$ (pointing to q), τ (pointing to t), $\frac{A\epsilon\epsilon_0}{d}$ (pointing to C), $\frac{\rho d}{A}$ (pointing to R)



$$\Rightarrow \ln 2 = \frac{\tau}{\frac{\rho d}{A} \cdot \frac{A\epsilon\epsilon_0}{d}} \Rightarrow \rho = \frac{\tau}{\epsilon\epsilon_0 \ln 2}$$

Youtube / Arihant Senior

3.172. A circuit consists of a source of a constant emf $\mathcal{E}$ and a resistance R and a capacitor with capacitance C connected in series. The internal resistance of the source is negligible. At a moment $t = 0$ the capacitance of the capacitor is abruptly decreased η -fold. Find the current flowing through the circuit as a function of time t .



For a RC circuit:

$$q = q_{ss} (1 - e^{-t/RC}) + q_{in} e^{-t/RC}$$

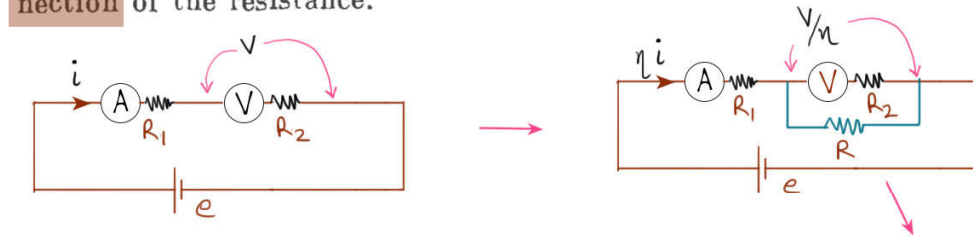
Annotations: q_{ss} is Charge on a plate. q_{in} is Charge at steady state on same plate. q_{in} is Initial charge on same plate.

$$q = \frac{ce}{\eta} (1 - e^{-\frac{\eta t}{RC}}) + ce e^{-\frac{\eta t}{RC}}$$

$$i = \frac{dq}{dt} = \frac{ce}{\eta} \cdot \frac{\eta}{RC} e^{-\frac{\eta t}{RC}} - ce \frac{\eta}{RC} e^{-\frac{\eta t}{RC}} = \frac{(1-\eta)ce}{R} e^{-\frac{\eta t}{RC}}$$

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3.173. An ammeter and a voltmeter are connected in series to a battery with an emf $\mathcal{E} = 6.0 \text{ V}$. When a certain resistance is connected in parallel with the voltmeter, the readings of the latter decrease $\eta = 2.0$ times, whereas the readings of the ammeter increase the same number of times. Find the voltmeter readings after the connection of the resistance.

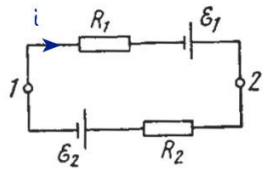


$$i = \frac{e}{R_1 + R_2}, \quad V = i R_2 = \frac{e R_2}{R_1 + R_2} \quad (1) \quad , \quad \frac{V}{\eta} = e - \eta i R_1 = e - \frac{\eta e}{R_1 + R_2} R_1 \quad (2)$$

Solving:

$$\frac{R_2}{R_1} \rightarrow t \quad \begin{aligned} (1) &\Rightarrow \eta = \frac{e t}{1+t} \\ (2) &\Rightarrow \eta = \frac{e - \frac{\eta e}{1+t} R_1}{e - \frac{\eta e}{1+t} R_1} \Rightarrow \eta = \frac{t}{1+t-\eta} \Rightarrow \eta^2 - \eta t - \eta + t = 0 \\ &\Rightarrow \eta = t, 1 \end{aligned} \quad \xrightarrow{(1)} \quad \frac{V}{\eta} = \frac{1}{\eta} \frac{e t}{1+t} = \frac{e}{1+\eta}$$

3.174. Find a potential difference $\varphi_1 - \varphi_2$ between points 1 and 2 of the circuit shown in Fig. 3.39 if $R_1 = 10 \, \Omega$, $R_2 = 20 \, \Omega$, $\mathcal{E}_1 = 5.0 \text{ V}$, and $\mathcal{E}_2 = 2.0 \text{ V}$. The internal resistances of the current sources are negligible.



$$i = \frac{e_1 - e_2}{R_1 + R_2}$$

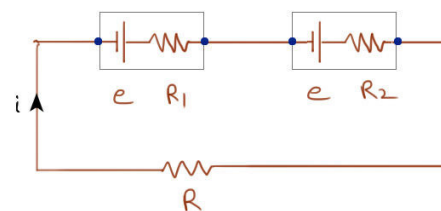
Using KVL from 1 to 2:

$$V_1 - i R_1 + e_1 = V_2$$

$$\Rightarrow V_2 - V_1 = e_1 - i R_1 = e_1 - \frac{e_1 - e_2}{R_1 + R_2} R_1$$

$$V_2 - V_1 = \frac{e_1 R_2 + e_2 R_1}{R_1 + R_2} //$$

3.175. Two sources of current of equal emf are connected in series and have different internal resistances R_1 and R_2 . ($R_2 > R_1$). Find the external resistance R at which the potential difference across the terminals of one of the sources (which one in particular?) becomes equal to zero.



Since same i flows through both batteries, potential drop across battery with higher internal resistance will be more. And hence total potential difference across that battery can become zero.

$$i = \frac{2e}{R_1 + R_2 + R}, \quad e - iR_2 = 0 \quad (\because R_2 > R_1)$$

eliminating i

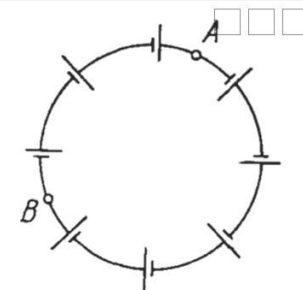
$$\Rightarrow \frac{e}{R_2} = \frac{2e}{R_1 + R_2 + R}$$

$$\Rightarrow R = R_2 - R_1 //$$

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3.176. N sources of current with different emf's are connected as shown in Fig. 3.40. The emf's of the sources are proportional to their internal resistances, i.e. $\mathcal{E} = \alpha R$, where α is an assigned constant. The lead wire resistance is negligible. Find:

- the current in the circuit;
- the potential difference between points A and B dividing the circuit in n and $N - n$ links.



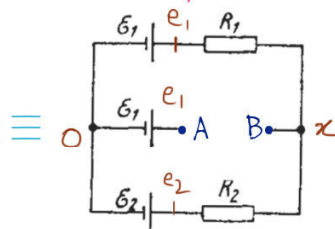
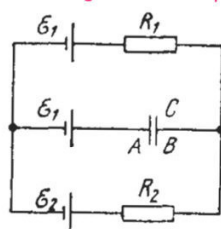
$$\text{A } i = \frac{\sum e}{\sum R} = \frac{\sum \alpha R}{\sum R} = \alpha //$$

$$\begin{aligned} \text{B } V_A - V_B &= \sum_1^n (e - iR) \\ &= \sum [(\alpha R) - \alpha R] \\ &= \sum 0 \\ &= 0 // \end{aligned}$$

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3.177. In the circuit shown in Fig. 3.41 the sources have emf's $\mathcal{E}_1 = 1.0 \text{ V}$ and $\mathcal{E}_2 = 2.5 \text{ V}$ and the resistances have the values $R_1 = 10 \Omega$ and $R_2 = 20 \Omega$. The internal resistances of the sources are negligible. Find a potential difference $\varphi_A - \varphi_B$ between the plates A and B of the capacitor C.

At steady state, capacitor will act as open circuit.



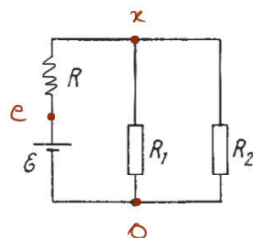
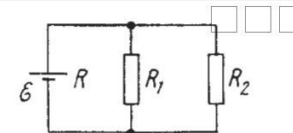
$$\frac{x - \mathcal{E}_1}{R_1} + \frac{x - \mathcal{E}_2}{R_2} = 0$$

$$\Rightarrow x = \frac{\mathcal{E}_1 R_2 + \mathcal{E}_2 R_1}{R_1 + R_2}$$

$$\Rightarrow V_A - V_B = \mathcal{E}_1 - x = \frac{(\mathcal{E}_1 - \mathcal{E}_2) R_1}{R_1 + R_2} //$$

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3.178. In the circuit shown in Fig. 3.42 the emf of the source is equal to $\mathcal{E} = 5.0 \text{ V}$ and the resistances are equal to $R_1 = 4.0 \Omega$ and $R_2 = 6.0 \Omega$. The internal resistance of the source equals $R = 0.10 \Omega$. Find the currents flowing through the resistances R_1 and R_2 .



$$\frac{x - \mathcal{E}}{R} + \frac{x}{R_1} + \frac{x}{R_2} = 0$$

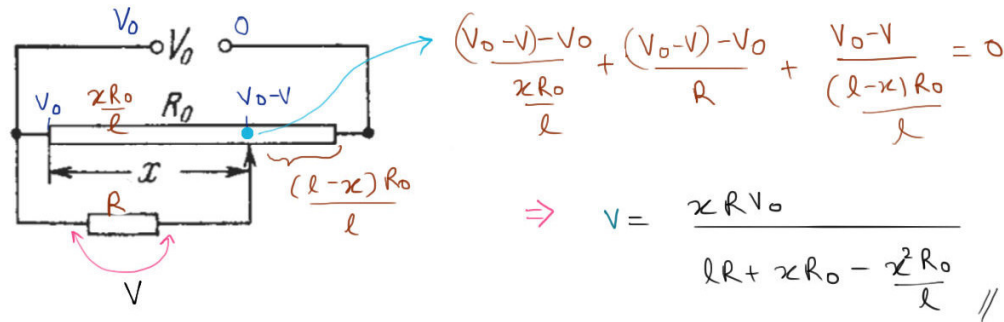
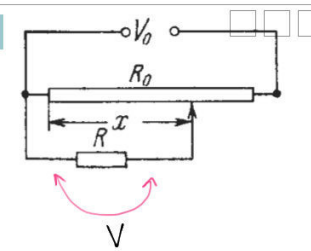
$$\Rightarrow x = \mathcal{E} \left(\frac{R_1 R_2}{\mathcal{E} R_1 R_2} \right) \Rightarrow$$

$$i_1 = \frac{x}{R_1} = \frac{\mathcal{E} R_2}{\mathcal{E} R_1 R_2}$$

$$i_2 = \frac{x}{R_2} = \frac{\mathcal{E} R_1}{\mathcal{E} R_1 R_2}$$

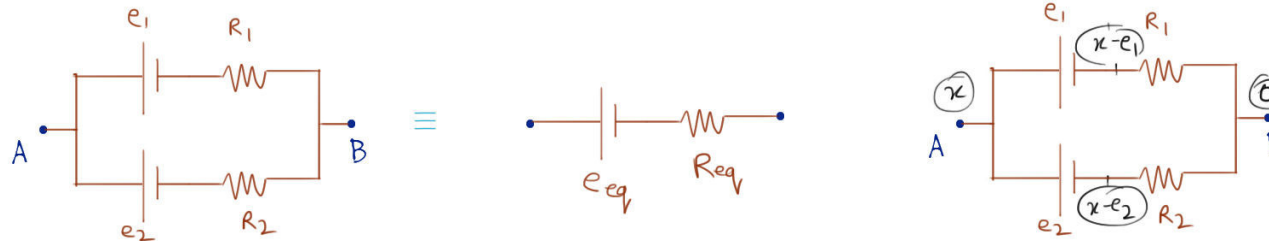
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3.179. Fig. 3.43 illustrates a potentiometric circuit by means of which we can vary a voltage V applied to a certain device possessing a resistance R . The potentiometer has a length l and a resistance R_0 , and voltage V_0 is applied to its terminals. Find the voltage V fed to the device as a function of distance x . Analyse separately the case $R \gg R_0$.



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3.180. Find the emf and the internal resistance of a source which is equivalent to two batteries connected in parallel whose emf's are equal to $\mathcal{E}_1$ and $\mathcal{E}_2$ and internal resistances to R_1 and R_2 .



Point B: $\frac{x - e_1}{R_1} + \frac{x - e_2}{R_2} = 0$

$$\Rightarrow x = e_{eq} = \frac{\frac{e_1}{R_1} + \frac{e_2}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}}$$

R_{eq} calculation is independent of batteries.

$$\therefore R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

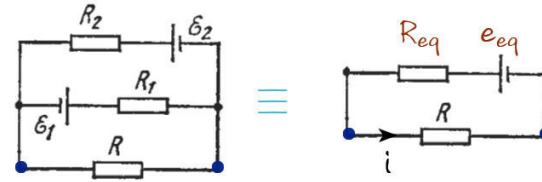
We can use these results of equivalent emf and resistance to calculate current in one resistor in the circuit. Its called Thevenins analysis. We will do it in next few problems.

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3.181. Find the magnitude and direction of the current flowing through the resistance R in the circuit shown in Fig. 3.44 if the emf's of the sources are equal to $\mathcal{E}_1 = 1.5 \text{ V}$ and $\mathcal{E}_2 = 3.7 \text{ V}$ and the resistances are equal to $R_1 = 10 \Omega$, $R_2 = 20 \Omega$, $R = 5.0 \Omega$. The internal resistances of the sources are negligible.

By using the concept of equivalent emf and resistance (derived in 3.180), also known as Thevenins analysis,

We find the current in one resistance easily.



$$e_{eq} = \frac{\frac{\mathcal{E}_2}{R_2} - \frac{\mathcal{E}_1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 \mathcal{E}_2 - R_2 \mathcal{E}_1}{R_1 + R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$i = \frac{e_{eq}}{R + R_{eq}} = \frac{(R_1 \mathcal{E}_2 - R_2 \mathcal{E}_1)}{\Sigma R R_1} //$$

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3.182. In the circuit shown in Fig. 3.45 the sources have emf's $\mathcal{E}_1 = 1.5 \text{ V}$, $\mathcal{E}_2 = 2.0 \text{ V}$, $\mathcal{E}_3 = 2.5 \text{ V}$, and the resistances are equal to $R_1 = 10 \Omega$, $R_2 = 20 \Omega$, $R_3 = 30 \Omega$. The internal resistances of the sources are negligible. Find:

(a) the current flowing through the resistance R_1 ;

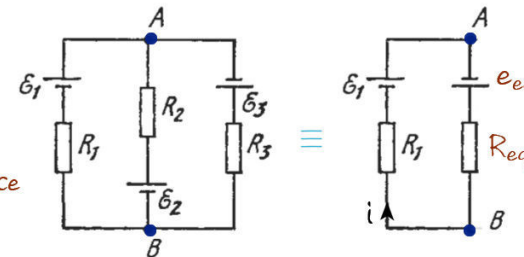
(b) a potential difference $\varphi_A - \varphi_B$ between the points A and B.

By using the concept of equivalent emf and resistance (derived in 3.180), also known as Thevenins analysis,

We find the current in one resistance easily.

$$e_{eq} = \frac{\frac{\mathcal{E}_3}{R_3} - \frac{\mathcal{E}_2}{R_2}}{\frac{1}{R_3} + \frac{1}{R_2}} = \frac{\mathcal{E}_3 R_2 - \mathcal{E}_2 R_3}{R_2 + R_3}$$

$$R_{eq} = \frac{R_2 R_3}{R_2 + R_3}$$



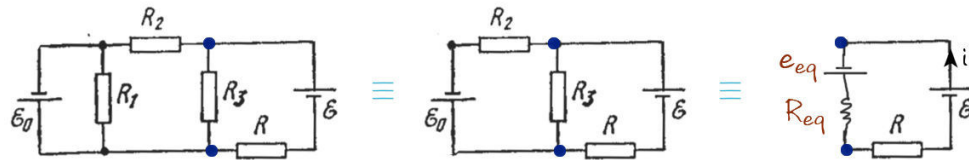
$$\text{A } i = \frac{\mathcal{E}_1 + e_{eq}}{R_1 + R_{eq}} = \frac{(\mathcal{E}_1 + \mathcal{E}_3) R_2 + (\mathcal{E}_1 - \mathcal{E}_2) R_3}{\Sigma R_1 R_2}$$

$$\text{B } V_A - \mathcal{E}_1 + i R_1 = V_B$$

$$\Rightarrow V_A - V_B = \mathcal{E}_1 - i R_1$$

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3.183. Find the current flowing through the resistance R in the circuit shown in Fig. 3.46. The internal resistances of the batteries are negligible.



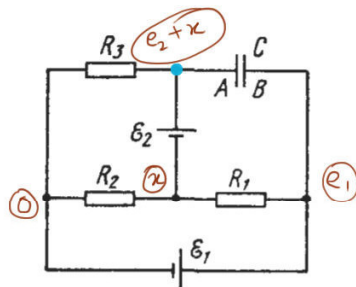
$$e_{eq} = \frac{E_0}{\frac{1}{R_2} + \frac{1}{R_3}} = \frac{E_0 R_2 R_3}{R_2 + R_3}$$

$$R_{eq} = \frac{R_2 R_3}{R_2 + R_3}$$

$$i = \frac{e + e_{eq}}{R + R_{eq}} = \frac{e + \frac{E_0 R_2 R_3}{R_2 + R_3}}{R + \frac{R_2 R_3}{R_2 + R_3}} = \frac{e(R_2 + R_3) + E_0 R_2 R_3}{R(R_2 + R_3) + R_2 R_3}$$

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3.184. Find a potential difference $\varphi_A - \varphi_B$ between the plates of a capacitor C in the circuit shown in Fig. 3.47 if the sources have emf's $\mathcal{E}_1 = 4.0$ V and $\mathcal{E}_2 = 1.0$ V and the resistances are equal to $R_1 = 10 \Omega$, $R_2 = 20 \Omega$, and $R_3 = 30 \Omega$. The internal resistances of the sources are negligible.



$$\frac{e_2 + x - 0}{R_3} + \frac{x - 0}{R_2} + \frac{x - e_1}{R_1} = 0$$

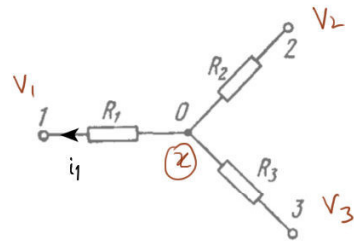
$$\Rightarrow x = \frac{\frac{e_1}{R_1} - \frac{e_2}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{(R_3 e_1 - R_1 e_2) R_2}{\Sigma R_1 R_2}$$

$$\Rightarrow V_A - V_B = (e_2 + x) - e_1 = \frac{-R_1 R_2 e_1 + R_2 R_3 e_2 + R_3 R_1 (e_2 - e_1)}{\Sigma R_1 R_2}$$

$$= \frac{R_3 e_2 (R_2 + R_1) - R_1 e_1 (R_2 + R_3)}{\Sigma R_1 R_2}$$

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3.185. Find the current flowing through the resistance R_1 of the circuit shown in Fig. 3.48 if the resistances are equal to $R_1 = 10 \Omega$, $R_2 = 20 \Omega$, and $R_3 = 30 \Omega$, and the potentials of points 1, 2, and 3 are equal to $\varphi_1 = 10 \text{ V}$, $\varphi_2 = 6 \text{ V}$, and $\varphi_3 = 5 \text{ V}$



$$\frac{x - V_1}{R_1} + \frac{x - V_2}{R_2} + \frac{x - V_3}{R_3} = 0$$

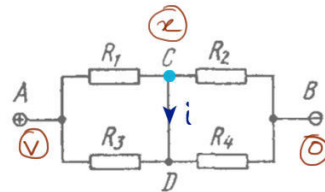
$$\Rightarrow x = \frac{\frac{V_1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

We need, $i_1 = \frac{x - V_1}{R_1}$

$$= \frac{R_2 R_3 V_1 + R_1 R_3 V_2 + R_1 R_2 V_3 - R_1 R_2 V_1 - R_2 R_3 V_1 - R_3 R_1 V_1}{R_1 R_2 R_3 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)} = \frac{R_3 (V_2 - V_1) + R_2 (V_3 - V_1)}{\sum R_1 R_2}$$

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3.186. A constant voltage $V = 25 \text{ V}$ is maintained between points A and B of the circuit (Fig. 3.49). Find the magnitude and direction of the current flowing through the segment CD if the resistances are equal to $R_1 = 1.0 \Omega$, $R_2 = 2.0 \Omega$, $R_3 = 3.0 \Omega$, and $R_4 = 4.0 \Omega$.



$$x = \left(\frac{\frac{R_2 R_4}{R_2 + R_4}}{\frac{R_2 R_4}{R_2 + R_4} + \frac{R_1 R_3}{R_1 + R_3}} \right) V$$

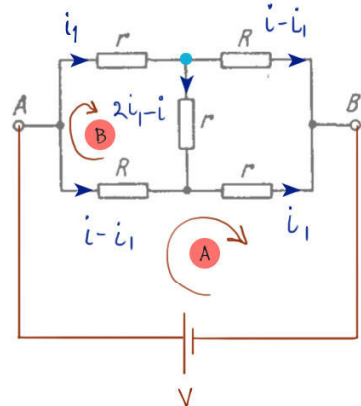
Current from C to D = Current coming to C from A and B

$$\therefore i = \frac{V - x}{R_1} + \frac{(-x)}{R_2}$$

$$= \frac{V}{R_1} - \frac{1}{R_1} \left(\frac{R_2 R_4}{R_2 + R_4} \right) \frac{V}{\left(\frac{R_2 R_4}{R_2 + R_4} \right) (R_1 + R_3) + R_1 R_3 (R_2 + R_4)} = \frac{25}{1} - \left(1 + \frac{1}{2} \right) \left(\frac{2 \cdot 4 \cdot 4 \cdot 25}{8 \cdot 4 + 3 \cdot 6} \right) = 25 - 3 \cdot \frac{800}{50} = 1 \text{ Amp}$$

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3.187. Find the resistance between points A and B of the circuit shown in Fig. 3.50.



$$\text{A} \quad V - i_1 r - (i - i_1) R = 0$$

$$\text{B} \quad -i_1 r - (2i_1 - i) R + (i - i_1) R = 0$$

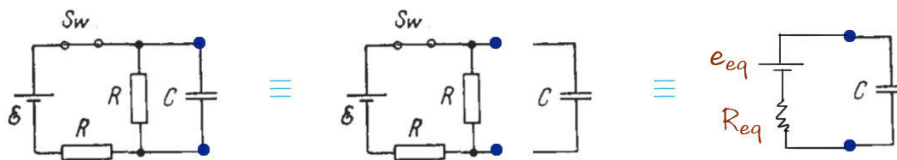
$$R_0 \rightarrow \frac{V}{i}, \quad \frac{i_1}{i} \rightarrow t$$

$$\Rightarrow \text{A} \rightarrow R_0 - R + (R - r)t = 0$$

$$\text{B} \rightarrow t = \frac{r + R}{3r + R} \Rightarrow R_0 = \frac{r(3R + r)}{3r + R} //$$

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3.188. Find how the voltage across the capacitor C varies with time t (Fig. 3.51) after the shorting of the switch Sw at the moment t = 0.



By using the concept of equivalent emf and resistance (derived in 3.180), also known as Thevenins analysis,

We find the parameters for equivalent RC circuit easily.

$$E_{eq} = \frac{\frac{E}{R} + 0}{\frac{1}{R} + \frac{1}{R}} = \frac{E}{2}$$

$$R_{eq} = R/2$$

For RC current growth circuit:

$$V = V_0 (1 - e^{-t/RC})$$

$$= \frac{E}{2} (1 - e^{-2t/RC}) //$$

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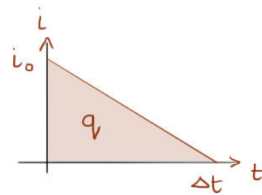
- 3.189. What amount of heat will be generated in a coil of resistance R due to a charge q passing through it if the current in the coil
- decreases down to zero uniformly during a time interval Δt ;
 - decreases down to zero halving its value every Δt seconds?

Approach:

1 Find i_t

2 $H = \int i_t^2 R dt$

A



Let initial current be i_0

$$\frac{1}{2} i_0 \Delta t = q \Rightarrow i_0 = \frac{2q}{\Delta t}$$

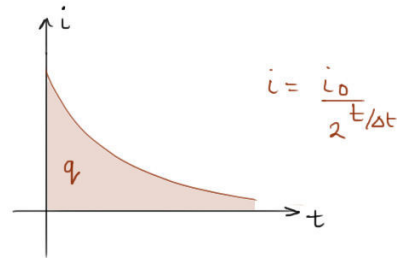
From graph:

$$i = i_0 - i_0 \frac{t}{\Delta t}$$

$$\begin{aligned} \therefore \text{Heat} &= \int i^2 R dt = \int_0^{\Delta t} i_0^2 \left(1 - \frac{t}{\Delta t}\right)^2 dt \\ &= \frac{4}{3} q^2 \frac{R}{\Delta t} // \end{aligned}$$

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B



$$i = \frac{i_0}{2^{t/\Delta t}}$$

Finding i_0 :

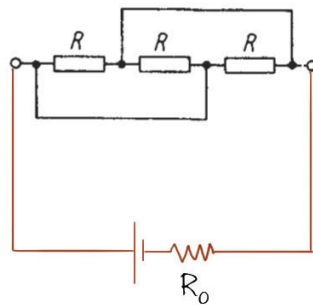
$$q = \int_0^{\infty} dq = \int_0^{\infty} i dt = i_0 \int_0^{\infty} 2^{-t/\Delta t} dt = \frac{i_0 \Delta t}{\ln 2} \Rightarrow i_0 = \frac{q \ln 2}{\Delta t}$$

Finding heat:

$$\text{Heat} = \int i^2 R dt = i_0^2 R \int_0^{\infty} 2^{-2t/\Delta t} dt = \frac{q^2 \ln 2}{\Delta t^2} R \left[\frac{\Delta t}{2 \ln 2} \right] = \frac{q^2 R \ln 2}{2 \Delta t} //$$

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3.190. A dc source with internal resistance R_0 is loaded with three identical resistances R interconnected as shown in Fig. 3.52. At what value of R will the thermal power generated in this circuit be the highest?



For thermal power to be maximum.

$$R_{\text{external}} = R_{\text{internal}}$$

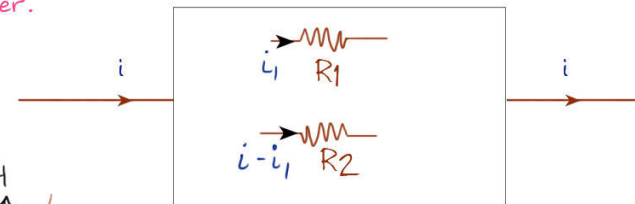
$$\frac{R}{3} = R_0$$

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3.191. Make sure that the current distribution over two resistances R_1 and R_2 connected in parallel corresponds to the minimum thermal power generated in this circuit.

Let say, we dont know how the resistors are connected in the box.
But we can conserve current and calculate total power.

Then we see when that power is minimum



$$\text{Heat} = i_1^2 R_1 + (i - i_1)^2 R_2$$

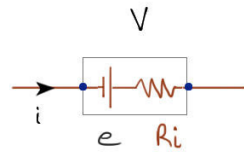
for minimum heat: $\frac{dH}{di_1} = 0 \Rightarrow 2i_1 R_1 + 2(i - i_1)(-1) R_2 = 0$

$$i_1 (R_1 + R_2) = i R_2$$

$$i_1 = \frac{i R_2}{R_1 + R_2} \Rightarrow R_1 \text{ \& } R_2 \text{ are in parallel}$$

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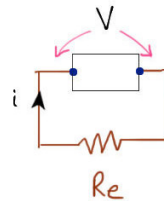
3.192. A storage battery with emf $\mathcal{E} = 2.6 \text{ V}$ loaded with an external resistance produces a current $I = 1.0 \text{ A}$. In this case the potential difference between the terminals of the storage battery equals $V = 2.0 \text{ V}$. Find the thermal power generated in the battery and the power developed in it by electric forces.



$$\mathcal{E} - iR_i = V$$

$$\Rightarrow R_i = \frac{\mathcal{E} - V}{i}$$

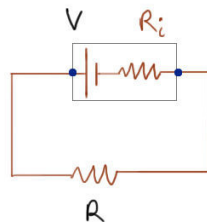
Power generated in battery: $i^2 R_i = i(\mathcal{E} - V)$



Power generated by battery externally: $i^2 R_e = Vi$

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3.193. A voltage V is applied to a dc electric motor. The armature winding resistance is equal to R . At what value of current flowing through the winding will the useful power of the motor be the highest? What is it equal to? What is the motor efficiency in this case?



Power of motor: $P = i^2 R$

$$= \frac{V^2}{(R + R_i)^2} R$$

At $P_{\max}$, $\frac{dP}{dR} = 0 \Rightarrow (R + R_i)^2 - R \cdot 2(R + R_i) = 0$

$$\Rightarrow R = R_i$$

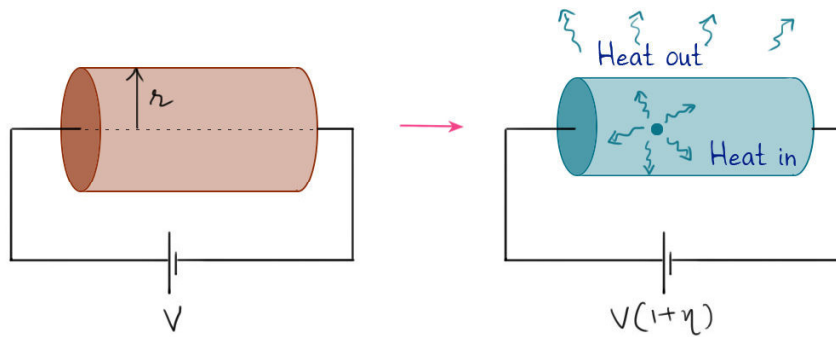
Power consumed by motor

Sol: $\therefore i = \frac{V}{2R}$ and, Motor efficiency = $\frac{i^2 R}{i^2 (R + R)} = \frac{1}{2}$

Power consumed by whole system

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3.194. How much (in per cent) has a filament diameter decreased due to evaporation if the maintenance of the previous temperature required an increase of the voltage by $\eta = 1.0\%$? The amount of heat transferred from the filament into surrounding space is assumed to be proportional to the filament surface area.



$$\frac{dV}{V} = \frac{\eta V}{V} = \eta$$

$$\Rightarrow (2V dV) r + V^2 dr = 0$$

$$\Rightarrow \frac{dr}{r} = -2 \frac{dV}{V} = -2\eta$$

If T is constant,

Heat in = Heat out

$$\Rightarrow \frac{V^2}{R} = C_1 \times 2\pi r l$$

$$\frac{8l}{\pi r^2}$$

$$\Rightarrow V^2 r^2 = C_2 r$$

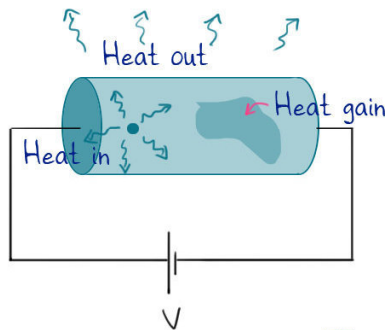
$$\Rightarrow V^2 r^2 \propto r$$

$$\Rightarrow V^2 r = \text{constant}$$

differentiate

Youtube / Arihant Senior

3.195. A conductor has a temperature-independent resistance R and a total heat capacity C . At the moment $t = 0$ it is connected to a dc voltage V . Find the time dependence of a conductor's temperature T assuming the thermal power dissipated into surrounding space to vary as $q = k(T - T_0)$, where k is a constant, T_0 is the environmental temperature (equal to the conductor's temperature at the initial moment).



Lets say the temperature of conductor rises by dT in time dt . Then,

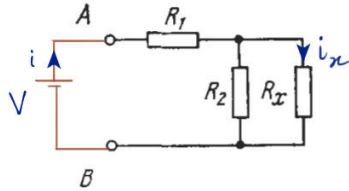
Heat gain = Heat in - Heat out

$$C dT = \frac{V^2}{R} dt - k(T - T_0) dt$$

$$\Rightarrow \int_{T_0}^T \frac{C}{\frac{V^2}{R} - k(T - T_0)} dT = \int_0^t dt \Rightarrow T = T_0 + \frac{V^2}{kR} (1 - e^{-kt/C}) //$$

Youtube / Arihant Senior

3.196. A circuit shown in Fig. 3.53 has resistances $R_1 = 20 \Omega$ and $R_2 = 30 \Omega$. At what value of the resistance R_x will the thermal power generated in it be practically independent of small variations of that resistance? The voltage between the points A and B is supposed to be constant in this case.



Let the potential across AB be V.

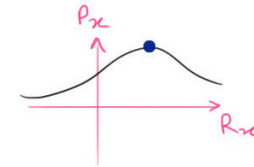
$$i_x = \frac{R_2}{R_2 + R_x} i = \frac{R_2}{(R_2 + R_x)} \cdot \frac{V}{R_1 + \frac{R_2 R_x}{R_2 + R_x}} = \frac{R_2 V}{R_1 R_2 + (R_1 + R_2) R_x}$$

$$\therefore P_x = i_x^2 R_x = (R_2 V)^2 \frac{R_x}{[R_1 R_2 + (R_1 + R_2) R_x]^2}$$

$$\text{For, } \frac{dP_x}{dR_x} = 0 \Rightarrow \left[\frac{d}{dR_x} \right] (1 - R_x) \cdot \frac{1}{[R_1 + R_2]} = 0$$

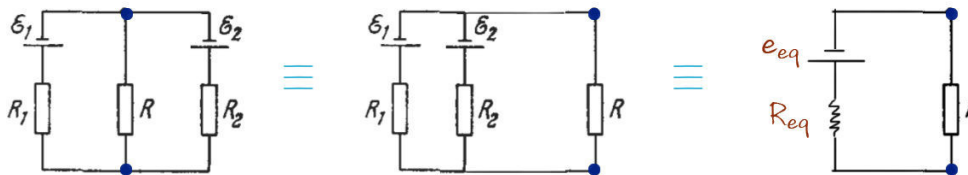
$$\Rightarrow R_1 R_2 + (R_1 + R_2) R_x - 2 R_x (R_1 + R_2) = 0$$

$$\Rightarrow R_x = \frac{R_1 R_2}{R_1 + R_2} //$$



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3.197. In a circuit shown in Fig. 3.54 resistances R_1 and R_2 are known, as well as emf's $\mathcal{E}_1$ and $\mathcal{E}_2$. The internal resistances of the sources are negligible. At what value of the resistance R will the thermal power generated in it be the highest? What is it equal to?



By using the concept of equivalent emf and resistance (derived in 3.180), also known as Thevenins analysis,

$$e_{eq} = \frac{e_1}{\frac{1}{R_1} + \frac{1}{R_2}}, \quad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

R for maximum power = Internal resistance of battery = $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

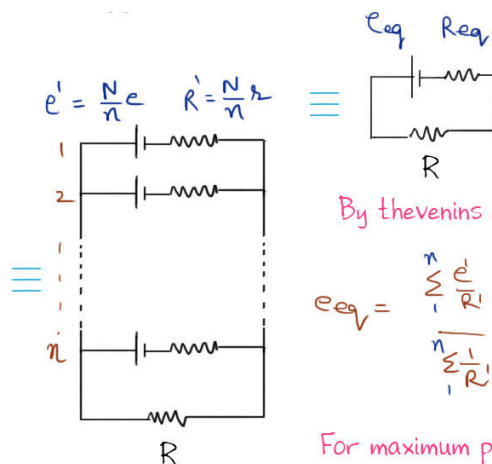
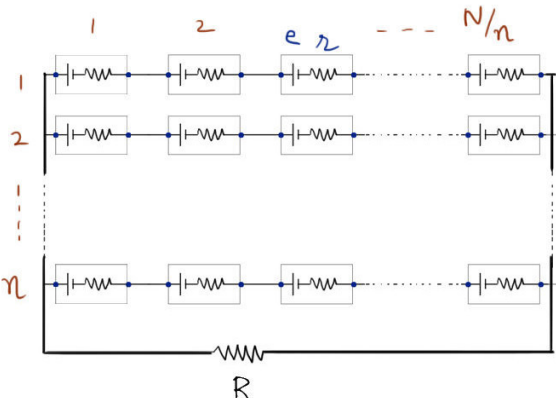
$$\text{Corresponding power in } R = i^2 R = \left(\frac{e_{eq}}{2 R_{eq}} \right)^2 \cdot R_{eq}$$

$$= \frac{(R_2 e_1 + R_1 e_2)^2}{4 (R_1 + R_2)^2} \cdot \frac{R_1 R_2}{R_1 + R_2} = \frac{(R_2 e_1 + R_1 e_2)^2}{4 (R_1 + R_2)^2} \cdot \frac{R_1 R_2}{R_1 + R_2} //$$

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3.198. A series-parallel combination battery consisting of a large number $N = 300$ of identical cells, each with an internal resistance $r = 0.3 \Omega$, is loaded with an external resistance $R = 10 \Omega$. Find the number n of parallel groups consisting of an equal number of cells connected in series, at which the external resistance generates the highest thermal power.

Total cells: N



By thevenins analysis,

$$e_{eq} = \frac{\sum e}{\sum \frac{1}{R'}} = \frac{e' \sum \frac{1}{R'}}{\sum \frac{1}{R'}} = \frac{Ne}{n}$$

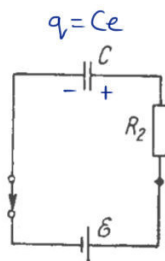
For maximum power in R

$$R = R_{eq} = \frac{R'}{n} = \frac{Nr}{n}$$

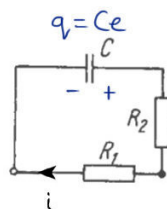
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3.199. A capacitor of capacitance $C = 5.00 \mu\text{F}$ is connected to a source of constant emf $\mathcal{E} = 200 \text{ V}$ (Fig. 3.55). Then the switch Sw was thrown over from contact 1 to contact 2. Find the amount of heat generated in a resistance $R_1 = 500 \Omega$ if $R_2 = 330 \Omega$.

Initially



Immediately after switching:



Initial energy stored in capacitor = $\frac{1}{2} C \mathcal{E}^2$

Power in a resistor = $i^2 R$

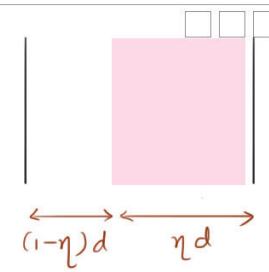
(Since current is same in both resistors) $P \propto R$

$$\Rightarrow E_1 = \left(\frac{R_1}{R_1 + R_2} \right) E$$

$$= \frac{R_1}{(R_1 + R_2)^2} \cdot \frac{1}{2} C \mathcal{E}^2 //$$

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3.200. Between the plates of a parallel-plate capacitor there is a metallic plate whose thickness takes up $\eta = 0.60$ of the capacitor gap. When that plate is absent the capacitor has a capacity $C = 20$ nF. The capacitor is connected to a dc voltage source $V = 100$ V. The metallic plate is slowly extracted from the gap. Find:
 (a) the energy increment of the capacitor;
 (b) the mechanical work performed in the process of plate extraction.



A
$$\Delta U = \frac{1}{2} C V^2 - \frac{1}{2} C_{eq} V^2$$

$$= \frac{1}{2} C V^2 \left(1 - \frac{1}{1-\eta} \right) = \frac{1}{2} C V^2 \left(\frac{-\eta}{1-\eta} \right)$$

$$C_{eq} = \frac{A \epsilon_0}{(1-\eta)d + \frac{\eta d}{\epsilon}}$$

$$= \frac{A \epsilon_0}{d} \left(\frac{1}{1-\eta + \frac{\eta}{\epsilon}} \right) = \frac{C}{1-\eta}$$

(For conductor)

B $W_{ext} + W_{battery} = \Delta U$
 $\Rightarrow W_{ext} = \Delta U - W_{battery}$

$$= \frac{1}{2} V^2 (C - C_{eq}) - (C - C_{eq}) V^2$$

$$= -\frac{1}{2} V^2 (C - C_{eq}) = -\Delta U = \frac{1}{2} C V^2 \frac{\eta}{1-\eta}$$

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3.201. A glass plate totally fills up the gap between the electrodes of a parallel-plate capacitor whose capacitance in the absence of that glass plate is equal to $C = 20$ nF. The capacitor is connected to a dc voltage source $V = 100$ V. The plate is slowly, and without friction, extracted from the gap. Find the capacitor energy increment and the mechanical work performed in the process of plate extraction.



A
$$\Delta U = \frac{1}{2} C V^2 - \frac{1}{2} C_{eq} V^2 = \frac{1}{2} C V^2 - \frac{1}{2} C \epsilon V^2$$

$$= -\frac{1}{2} C V^2 (\epsilon - 1)$$

$$C_{eq} = \frac{A \epsilon \epsilon_0}{d} = C \epsilon$$

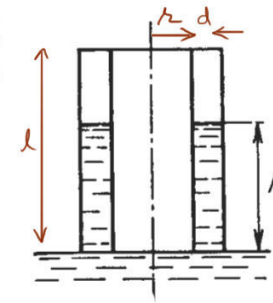
B $W_{ext} + W_{battery} = \Delta U$
 $\Rightarrow W_{ext} = \Delta U - W_{battery}$

$$= \frac{1}{2} V^2 (C - C_{eq}) - (C - C_{eq}) V^2$$

$$= -\frac{1}{2} V^2 (C - C_{eq}) = -\Delta U = \frac{1}{2} C V^2 (\epsilon - 1)$$

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3.202. A cylindrical capacitor connected to a dc voltage source V touches the surface of water with its end (Fig. 3.56). The separation d between the capacitor electrodes is substantially less than their mean radius. Find a height h to which the water level in the gap will rise. The capillary effects are to be neglected.



$$\begin{aligned}
 W_{\text{battery}} &= \Delta U \\
 &= \Delta q V = (C_f V - C_i V) V = (C_f - C_i) V^2 \\
 &= \Delta U_{\text{capacitor}} + \Delta U_{\text{gravitational}} = mgh = \rho (2\pi r h d) g h \\
 V^2 (C_f - C_i) &= \frac{1}{2} V^2 (C_f - C_i) + 2\pi g \rho r d h^2 \\
 \Rightarrow \frac{1}{2} V^2 \cdot \frac{2\pi \epsilon_0 (\epsilon - 1) l}{d} &= 2\pi g \rho r d h^2 \\
 \Rightarrow h &= \frac{V^2 \epsilon_0 (\epsilon - 1)}{2 g d^2}
 \end{aligned}$$

For force method, refer:
3.143 - 3.146

$$C = \frac{A \epsilon_0}{d} \approx \frac{(2\pi r l) \epsilon_0}{d} = \frac{2\pi r \epsilon_0 l}{d}$$

After water fills up:

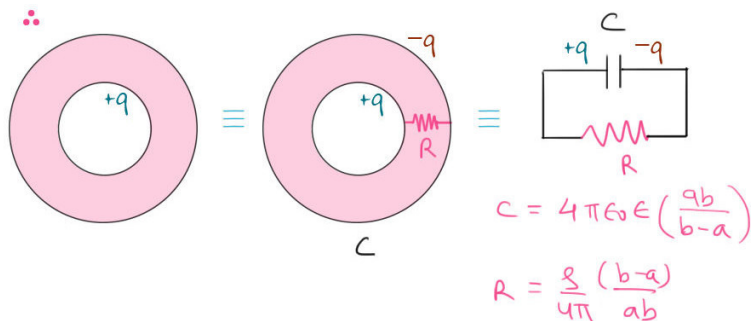
$$\begin{aligned}
 C_{eq} &= C_1 + C_2 \\
 &= \frac{2\pi r \epsilon_0 ((l-h) + \epsilon h)}{d}
 \end{aligned}$$

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3.203. The radii of spherical capacitor electrodes are equal to a and b , with $a < b$. The interelectrode space is filled with homogeneous substance of permittivity ϵ and resistivity ρ . Initially the capacitor is not charged. At the moment $t = 0$ the internal electrode gets a charge q_0 . Find:

- the time variation of the charge on the internal electrode;
- the amount of heat generated during the spreading of the charge.

Discharge in a spherical capacitor from inner to outer shell is independent of outer charge



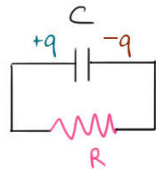
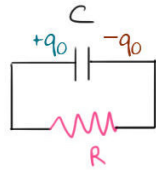
For the equivalent RC decay circuit:

$$\begin{aligned}
 \text{A } q &= q_0 e^{-t/RC} \\
 \text{B } & \text{Circuit diagram showing a capacitor with charges } +q \text{ and } -q \text{ discharging through a resistor } R. \\
 W_{\text{ext}} &= \Delta U + \text{Heat} \\
 \Rightarrow \text{Heat} &= -\Delta U = -\frac{1}{2C} (q_f^2 - q_i^2) \\
 &= \frac{1}{2C} q^2
 \end{aligned}$$

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3.204. The electrodes of a capacitor of capacitance $C = 2.00 \mu\text{F}$ carry opposite charges $q_0 = 1.00 \text{ mC}$. Then the electrodes are interconnected through a resistance $R = 5.0 \text{ M}\Omega$. Find:

- (a) the charge flowing through that resistance during a time interval $\tau = 2.00 \text{ s}$;
 (b) the amount of heat generated in the resistance during the same interval.



At time t

For the equivalent RC decay circuit:

A $q = q_0 e^{-t/RC}$

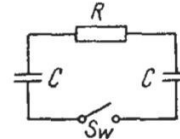
$\therefore$ Charge passed $= q_0 - q$
 $= q_0 (1 - e^{-t/RC})$

B $W_{\text{ext}} = \Delta U + \text{Heat}$
 $\Rightarrow \text{Heat} = -\Delta U$
 $= -\frac{1}{2C} (q_f^2 - q_i^2)$
 $= -\frac{1}{2C} (q^2 - q_0^2)$
 $= \frac{q_0^2}{2C} (1 - e^{-2t/RC})$

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3.205. In a circuit shown in Fig. 3.57 the capacitance of each capacitor is equal to C and the resistance, to R . One of the capacitors was connected to a voltage V_0 and then at the moment $t = 0$ was shorted by means of the switch Sw. Find:

- (a) a current I in the circuit as a function of time t ;
 (b) the amount of generated heat provided a dependence $I(t)$ is known.



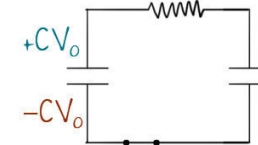
A $\frac{CV_0 - q}{C} - iR - \frac{dq}{dt} = 0$

$\Rightarrow \int_0^q \frac{dq}{CV_0 - 2q} = \int_0^t \frac{dt}{RC}$

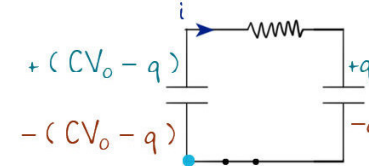
$\Rightarrow -\frac{1}{2} \ln \left(\frac{CV_0 - 2q}{CV_0} \right) = \frac{t}{RC}$
 $\Rightarrow q = \frac{CV_0}{2} (1 - e^{-2t/RC})$
 $\Rightarrow i = \frac{V_0}{R} e^{-2t/RC}$

B $\Delta H + \Delta U = 0$
 $\Rightarrow \Delta H = -\Delta U$
 $= -\frac{1}{2C} [2 \cdot q_f^2 - q_i^2]$
 $= \frac{CV_0^2}{4}$

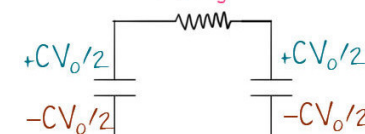
Initial instant



In-between

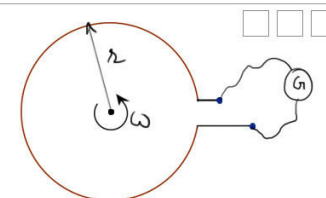


Finally



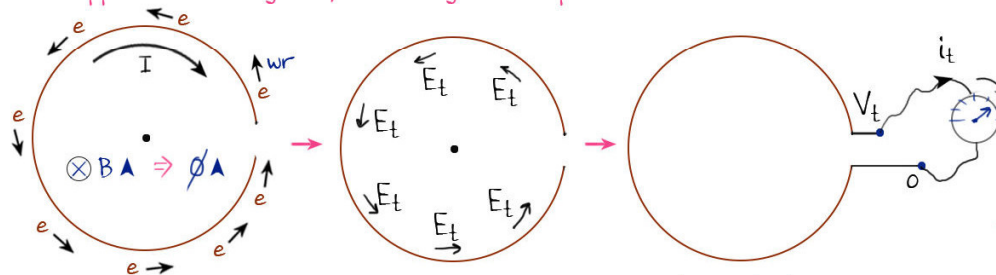
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3.206. A coil of radius $r = 25$ cm wound of a thin copper wire of length $l = 500$ m rotates with an angular velocity $\omega = 300$ rad/s about its axis. The coil is connected to a ballistic galvanometer by means of sliding contacts. The total resistance of the circuit is equal to $R = 21 \Omega$. Find the specific charge of current carriers in copper if a sudden stoppage of the coil makes a charge $q = 10$ nC flow through the galvanometer.



Resistance: R

On sudden stop, e 's will continue to move, generating current I . Induced field E_t will oppose this change in flux through the loop



From 1 and 2,

$$e \left(\frac{i_t R}{l} \right) dt = -m dv$$

$$\Rightarrow \frac{e}{m} \int_{q_1}^0 i_t dt = -\frac{l}{R} \int_{\omega}^0 d\omega$$

$$\Rightarrow \frac{e}{m} = \frac{l \omega}{q R}$$

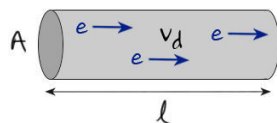
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Impulse due to induced electric field (due to TVMF) = Δp of electrons:

$$\therefore e E_t dt = -m dv \quad (1)$$

$$\text{For whole wire: } E_t l = V_t = i_t R \quad (2)$$

3.207. Find the total momentum of electrons in a straight wire of length $l = 1000$ m carrying a current $I = 70$ A.



$$\text{Total momentum of electrons, } p = \underbrace{n(Al)}_{\text{Total } e's} \underbrace{m v_d}_{\text{Momentum of one electron}} \quad (1)$$

And,

$$i = n e A v_d \quad (2)$$

$$\text{Eliminating } v_d \text{ from 1 and 2, } p = \frac{m i l}{e}$$

3.208. A copper wire carries a current of density $j = 1.0 \text{ A/mm}^2$. Assuming that one free electron corresponds to each copper atom, evaluate the distance which will be covered by an electron during its displacement $l = 10 \text{ mm}$ along the wire.

Assume the average velocity of electron to be $\langle v \rangle$ and electron density n .

$$\begin{aligned} \text{Distance traveled} &= \langle v \rangle t \\ &= \frac{l}{v_d} \\ &= \frac{\langle v \rangle l n e}{j} // \end{aligned}$$

$$\begin{aligned} i &= n e A v_d \\ \Rightarrow j &= \frac{i}{A} = n e v_d \\ \Rightarrow v_d &= \frac{j}{n e} \end{aligned}$$

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3.209. A straight copper wire of length $l = 1000 \text{ m}$ and cross-sectional area $S = 1.0 \text{ mm}^2$ carries a current $I = 4.5 \text{ A}$. Assuming that one free electron corresponds to each copper atom, find:

(a) the time it takes an electron to displace from one end of the wire to the other;

(b) the sum of electric forces acting on all free electrons in the given wire.

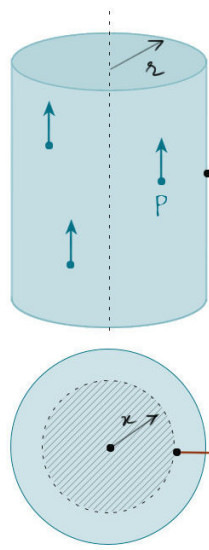
$$\begin{aligned} \text{A } t &= \frac{l}{v_d} = \frac{l n e A}{i} // \\ &\quad \downarrow \\ &\quad \frac{i}{n e A} \end{aligned}$$

$$\begin{aligned} \text{B } \text{Force on all electrons } F &= (n A l) e E \\ \Rightarrow f &= n e A l \frac{i S}{A} \\ &= n e i S l // \end{aligned}$$

$$\begin{aligned} j &= \frac{i}{A} = \frac{E}{S} \\ \Rightarrow F &= \frac{i S}{A} \end{aligned}$$

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3.210. A homogeneous proton beam accelerated by a potential difference $V = 600$ kV has a round cross-section of radius $r = 5.0$ mm. Find the electric field strength on the surface of the beam and the potential difference between the surface and the axis of the beam if the beam current is equal to $I = 50$ mA.



$\lambda = \frac{q}{l} = \frac{i dt}{v dt} = \frac{i}{v} = i \sqrt{\frac{m}{2eV}}$
 $\frac{1}{2}mv^2 = eV$

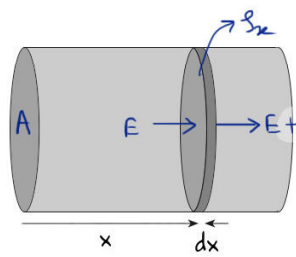
A $E = \frac{2k\lambda}{r} = \frac{2ki}{r} \sqrt{\frac{m}{2eV}}$

B $E_x = \frac{2k\lambda x}{x} = \frac{2k}{x} \lambda (\pi x^2) = \frac{2k\lambda x}{x^2}$
 $V = \int_0^r E_x dx = \frac{2k\lambda}{x^2} \int_0^r x dx = k\lambda = ki \sqrt{\frac{m}{2eV}}$

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3.211. Two large parallel plates are located in vacuum. One of them serves as a cathode, a source of electrons whose initial velocity is negligible. An electron flow directed toward the opposite plate produces a space charge causing the potential in the gap between the plates to vary as $\varphi = ax^{4/3}$, where a is a positive constant, and x is the distance from the cathode. Find:

- (a) the volume density of the space charge as a function of x ;
 (b) the current density.



$V = ax^{4/3}$
 $E_x = -\frac{dV}{dx} = -\frac{4}{3}ax^{1/3}$
 $dE_x = -\frac{4a}{9x^{2/3}}dx$ ①

Also,

$$dE_x = \frac{\sigma}{\epsilon_0} = \frac{\rho_x (A dx)}{A \epsilon_0}$$
 ②

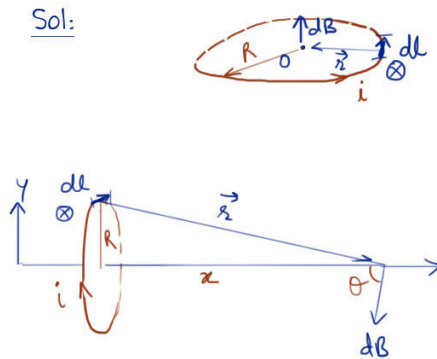
A From 1,2: $\frac{\rho_x}{\epsilon_0} = \frac{-4a}{9x^{2/3}}$
 $\Rightarrow \rho_x = -\frac{4a\epsilon_0}{9x^{2/3}}$

B $j = \frac{i}{A} = nevd$
 $e|V_x| = \frac{1}{2}mv^2$
 $= \frac{4\epsilon_0 a}{9x^{2/3}} \sqrt{\frac{2e(ax^{4/3})}{m}}$
 $= \frac{4\epsilon_0 a^{3/2}}{9} \sqrt{\frac{2e}{m}}$

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- 3.219. A current $I = 1.00$ A circulates in a round thin-wire loop of radius $R = 100$ mm. Find the magnetic induction
- at the centre of the loop;
 - at the point lying on the axis of the loop at a distance $x = 100$ mm from its centre.

Sol:



due to dl , $dB = \frac{\mu_0 i}{4\pi} \frac{dl \times \vec{r}}{r^3} = \frac{\mu_0 i}{4\pi R^2} dl$

$$B = \int dB = \frac{\mu_0 i}{4\pi R^2} \int_0^{2\pi R} dl = \frac{\mu_0 i}{2R}$$

due to dl , $dB = \frac{\mu_0 i}{4\pi} \frac{dl \times \vec{r}}{r^3} = \frac{\mu_0 i}{4\pi (x^2 + R^2)^{3/2}} dl$

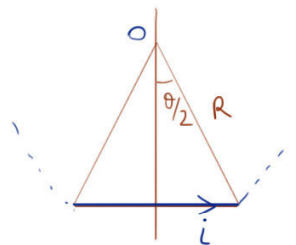
By symmetry net field will only be in axial direction

$$\therefore B = \int dB \cos \theta = \int_0^{2\pi R} \frac{\mu_0 i}{4\pi (x^2 + R^2)^{3/2}} \cdot \frac{R}{\sqrt{x^2 + R^2}} = \frac{\mu_0 i R^2}{2(x^2 + R^2)^{3/2}}$$

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- 3.220. A current I flows along a thin wire shaped as a regular polygon with n sides which can be inscribed into a circle of radius R . Find the magnetic induction at the centre of the polygon. Analyse the obtained expression at $n \rightarrow \infty$.

Sol:

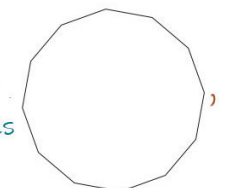


$$B_0 \text{ due to one side} = \frac{\mu_0 i}{4\pi (R \cos \frac{\theta}{2})} \left[\sin \frac{\theta}{2} + \sin \frac{\theta}{2} \right]$$

$$= \frac{\mu_0 i}{2\pi R} \tan \frac{\theta}{2} \rightarrow \theta = \frac{2\pi}{n}$$

$$B_0 \text{ due to } n \text{ sides} = n \frac{\mu_0 i}{2\pi R} \tan \frac{\pi}{n}$$

As $n \rightarrow \infty$, Loop approaches the circle

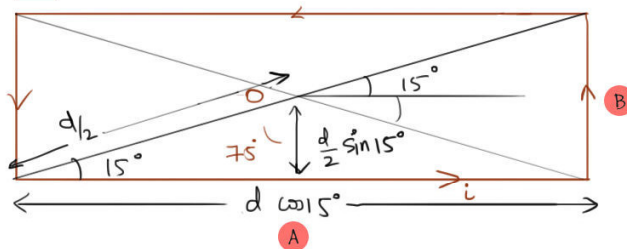


$$B \rightarrow \frac{\mu_0 i}{2R} \lim_{n \rightarrow \infty} \frac{\tan \frac{\pi}{n}}{\frac{\pi}{n}} = \frac{\mu_0 i}{2R} \rightarrow \text{Result of } B \text{ at center of a circular loop}$$

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3.221. Find the magnetic induction at the centre of a rectangular wire frame whose diagonal is equal to $d = 16$ cm and the angle between the diagonals is equal to $\phi = 30^\circ$; the current flowing in the frame equals $I = 5.0$ A.

Sol:



$$B_0 \text{ due to A} = \frac{\mu_0 i}{4\pi \frac{d}{2} \sin 15^\circ} (\sin 75^\circ + \sin 75^\circ)$$

$$= \frac{\mu_0 i}{\pi d} \tan 75^\circ$$

$$B_0 \text{ due to B} = \frac{\mu_0 i}{4\pi \frac{d}{2} \cos 15^\circ} (\sin 15^\circ + \sin 15^\circ)$$

$$= \frac{\mu_0 i}{\pi d} \tan 15^\circ$$

$$B_{\text{total}} = 2[B_A + B_B]$$

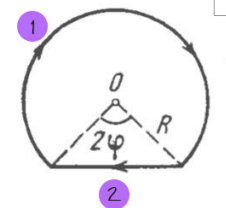
$$= 2 \frac{\mu_0 i}{\pi d} (\tan 15^\circ + \tan 75^\circ) = \frac{4\mu_0 i}{\pi d} (2 \cos 15^\circ \sin 15^\circ) = \frac{4\mu_0 i}{\pi d \sin 30^\circ}$$

General expression

$$= \frac{4\mu_0 i}{\pi d \sin \phi}$$

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3.222. A current $I = 5.0$ A flows along a thin wire shaped as shown in Fig. 3.59. The radius of a curved part of the wire is equal to $R = 120$ mm, the angle $2\phi = 90^\circ$. Find the magnetic induction of the field at the point O.



Sol:

$$B = B_1 + B_2$$

$$B = \left(\frac{2\pi - 2\phi}{2\pi} \right) \frac{\mu_0 i}{2R} + \frac{\mu_0 i}{4\pi (R \cos \phi)} (\sin \phi + \sin \phi)$$

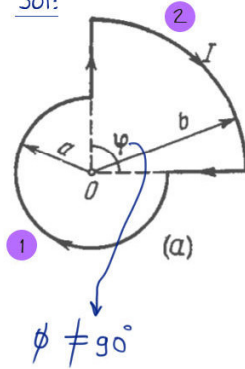
$$= \frac{\mu_0 i}{4\pi R} (2\pi - 2\phi + 2 \tan \phi) //$$

3.223. Find the magnetic induction of the field at the point O of a loop with current I , whose shape is illustrated

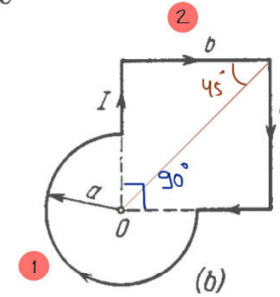
(a) in Fig. 3.60a, the radii a and b , as well as the angle ϕ are known;

(b) in Fig. 3.60b, the radius a and the side b are known.

Sol:



$$\begin{aligned} B &= B_1 + B_2 \\ &= \left(\frac{2\pi - \phi}{2\pi} \right) \frac{\mu_0 I}{2a} + \frac{\phi}{2\pi} \cdot \frac{\mu_0 I}{2b} \\ &= \frac{\mu_0 I}{4\pi} \left(\frac{2\pi - \phi}{a} + \frac{\phi}{b} \right) \end{aligned}$$

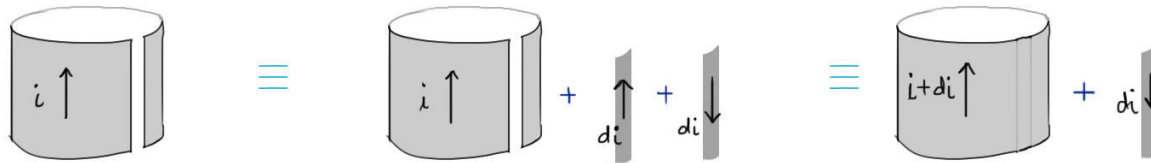


$$\begin{aligned} B &= B_1 + 2B_2 \\ &= \frac{3}{4} \frac{\mu_0 I}{2a} + 2 \left[\frac{\mu_0 I}{4\pi b} (\sin 0 + \sin 45^\circ) \right] \\ &= \frac{\mu_0 I}{4} \left(\frac{3}{2a} + \frac{\sqrt{2}}{\pi b} \right) \end{aligned}$$

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3.224. A current I flows along a lengthy thin-walled tube of radius R with longitudinal slit of width h . Find the induction of the magnetic field inside the tube under the condition $h \ll R$.

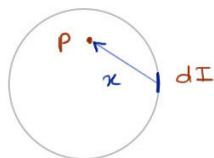
Principle of superposition: B remains unchanged if equal current is added and subtracted.



Field inside is 0 because of hollow wire.

Field inside is only due to this strip

Sol:

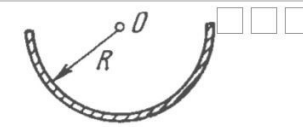


Field at any general point P:

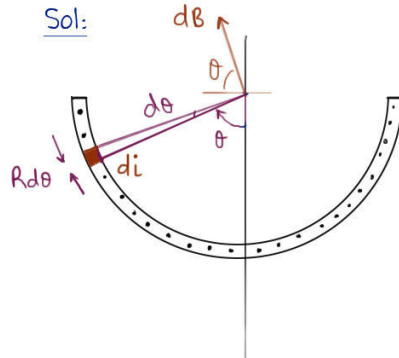
$$\begin{aligned} di &= \frac{h I}{(2\pi R - h)} \approx \frac{h I}{2\pi R} \\ \frac{\mu_0 di}{2\pi r} &= \frac{\mu_0}{2\pi r} \cdot \frac{h I}{2\pi R} = \frac{\mu_0 h I}{4\pi^2 r R} \end{aligned}$$

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3.225. A current I flows in a long straight wire with cross-section having the form of a thin half-ring of radius R (Fig. 3.61). Find the induction of the magnetic field at the point O .



Sol:



$$dB = \frac{\mu_0 di}{2\pi R}$$

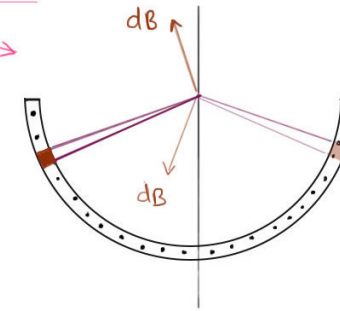
Net field will be towards X axis as Y components will cancel out due to symmetrical elements

$$\therefore B = \int dB \cos \theta$$

$$= \int \frac{\mu_0 di}{2\pi R} \cos \theta$$

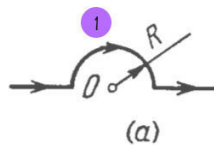
$$= \int \frac{\mu_0}{2\pi R} \cdot \frac{d\theta i}{\pi} \cos \theta$$

$$= \frac{\mu_0 i}{2\pi R^2} \int_{-\pi/2}^{\pi/2} \cos \theta d\theta = \frac{\mu_0 i}{\pi R^2}$$



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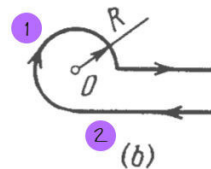
3.226. Find the magnetic induction of the field at the point O if a current-carrying wire has the shape shown in Fig. 3.62 a, b, c. The radius of the curved part of the wire is R , the linear parts are assumed to be very long.



Sol:

$$B = B_1$$

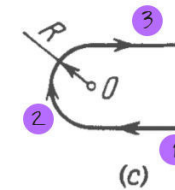
$$= \frac{1}{2} \left(\frac{\mu_0 i}{2R} \right)$$



$$B = B_1 + B_2$$

$$= \frac{3}{4} \left(\frac{\mu_0 i}{2R} \right) + \frac{\mu_0 i}{4\pi R}$$

$$= \frac{\mu_0 i}{4R} \left(\frac{3}{2} + \frac{1}{\pi} \right)$$



$$B = B_1 + B_2 + B_3$$

$$= B_2 + 2(B_1)$$

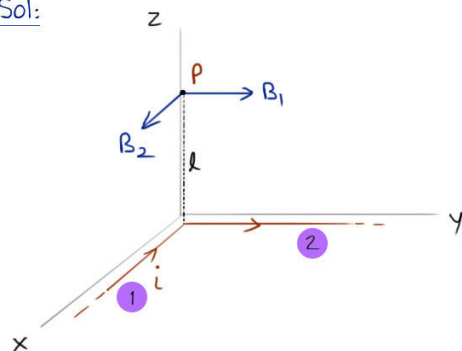
$$= \frac{1}{2} \frac{\mu_0 i}{2R} + 2 \left[\frac{\mu_0 i}{4\pi R} \right]$$

$$= \frac{\mu_0 i}{4R} \left(1 + \frac{2}{\pi} \right)$$

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3.227. A very long wire carrying a current $I = 5.0$ A is bent at right angles. Find the magnetic induction at a point lying on a perpendicular to the wire, drawn through the point of bending, at a distance $l = 35$ cm from it.

Sol:

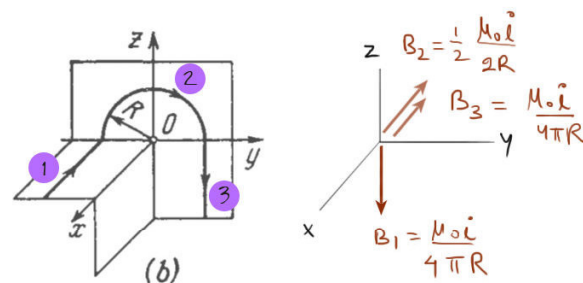
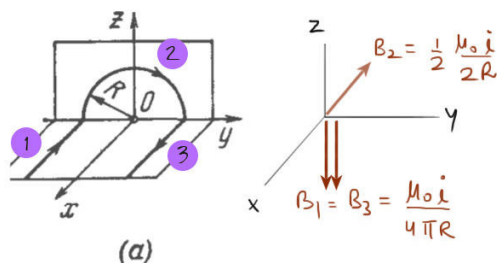


$$\begin{aligned} B &= B_1 + B_2 \\ &= B_1 \hat{j} + B_2 \hat{i} \\ &= \frac{\mu_0 I}{4\pi l} (\hat{j} + \hat{i}) \\ |B| &= \frac{\mu_0 I}{4\pi l} \sqrt{2} \end{aligned}$$

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3.228. Find the magnetic induction at the point O if the wire carrying a current $I = 8.0$ A has the shape shown in Fig. 3.63 a, b, c.

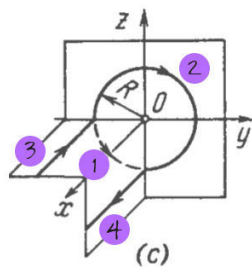
The radius of the curved part of the wire is $R = 100$ mm, the linear parts of the wire are very long.



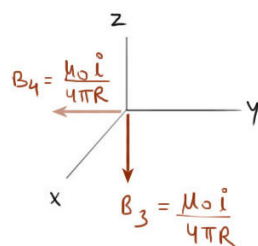
$$\begin{aligned} B &= \sqrt{B_2^2 + (B_1 + B_3)^2} \\ &= \sqrt{B_2^2 + (2B_1)^2} \\ &= \frac{\mu_0 I}{4R} \sqrt{1 + \frac{4}{\pi^2}} \end{aligned}$$

$$\begin{aligned} B &= \sqrt{B_1^2 + (B_2 + B_3)^2} \\ &= \frac{\mu_0 I}{4R} \sqrt{\frac{1}{\pi^2} + \left(1 + \frac{1}{\pi}\right)^2} \\ &= \frac{\mu_0 I}{4\pi R} \sqrt{2 + \pi^2 + 2\pi} \end{aligned}$$

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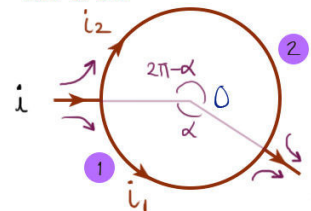
Net field will only be due to 3 and 4. Its zero due to combined effect of 1 and 2.



$$\therefore B = \sqrt{B_3^2 + B_4^2} = \frac{\mu_0 i}{4\pi R} \sqrt{2}$$

Proof

General result for field at center of circle when current splits into two arcs.



$$i_1 = \left(\frac{2\pi - \alpha}{2\pi}\right)i, \quad i_2 = \frac{\alpha}{2\pi}i$$

$$B_0 = B_1 - B_2 = \frac{\alpha}{2\pi} \frac{\mu_0 i_1}{2R} - \frac{2\pi - \alpha}{2\pi} \frac{\mu_0 i_2}{2R} = \frac{\mu_0}{4\pi R} (\alpha i_1 - (2\pi - \alpha) i_2)$$

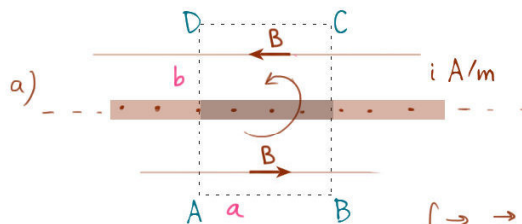
$$= \frac{\mu_0}{4\pi R} \left(\frac{\alpha(2\pi - \alpha) - (2\pi - \alpha)\alpha}{2\pi} \right) i = 0$$

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3.229. Find the magnitude and direction of the magnetic induction vector B

(a) of an infinite plane carrying a current of linear density i ; the vector i is the same at all points of the plane;

(b) of two parallel infinite planes carrying currents of linear densities i and $-i$; the vectors i and $-i$ are constant at all points of the corresponding planes.



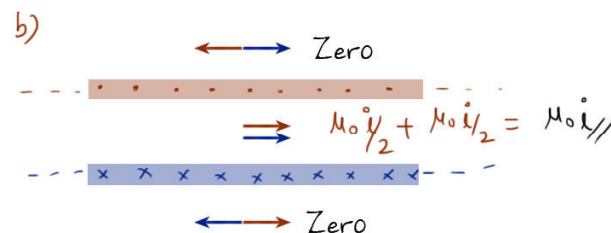
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{in}$$

ABCD

$$\oint \vec{B} \cdot d\vec{l} + \oint \vec{B} \cdot d\vec{l} + \oint \vec{B} \cdot d\vec{l} + \oint \vec{B} \cdot d\vec{l} = \mu_0 (i a)$$

AB BC CD DA

$$2Ba = \mu_0 i a \Rightarrow B = \mu_0 i / 2$$

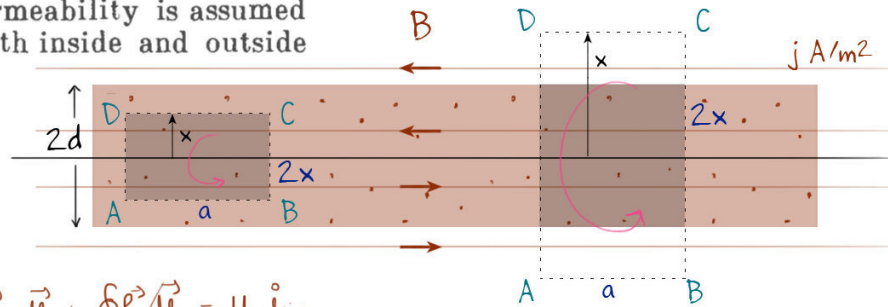


Using this result for both sheets

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3.230. A uniform current of density j flows inside an infinite plate of thickness $2d$ parallel to its surface. Find the magnetic induction induced by this current as a function of the distance x from the median plane of the plate. The magnetic permeability is assumed to be equal to unity both inside and outside the plate.

$$\oint_{ABCD} \vec{B} \cdot d\vec{l} = \mu_0 i_{in}$$



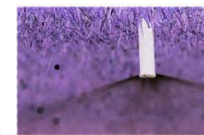
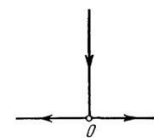
$$\oint_{AB} \vec{B} \cdot d\vec{l} + \oint_{BC} \vec{B} \cdot d\vec{l} + \oint_{CD} \vec{B} \cdot d\vec{l} + \oint_{DA} \vec{B} \cdot d\vec{l} = \mu_0 i_{in}$$

$$\text{Inside the plate } 2Ba = \mu_0 (j a \cdot 2x) \Rightarrow B_{in} = \mu_0 j x //$$

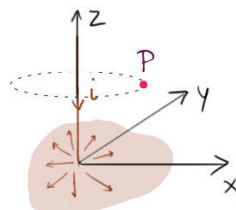
$$\text{Outside the plate } 2Ba = \mu_0 (j a \cdot 2d) \Rightarrow B_{out} = \mu_0 j d //$$

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3.231. A direct current I flows along a lengthy straight wire. From the point O (Fig. 3.64) the current spreads radially all over an infinite conducting plane perpendicular to the wire. Find the magnetic induction at all points of space.

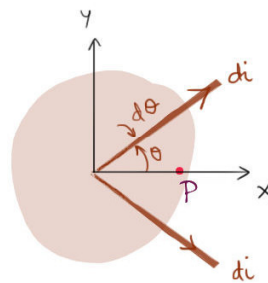


Lets take a point P above the X axis.



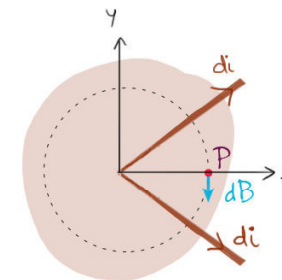
Front view

Due to straight part of current, B will be tangential to circular path.



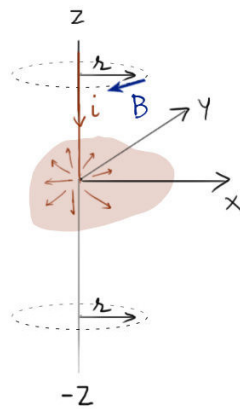
Top view

Due to flat part of current, we qualitatively find B due to two current elements symmetric to point P



- dB at P , due to these two elements is perpendicular to X axis in XY plane
- For all such pairs, B will be perpendicular to X axis in XY plane.
- Direction of B is tangential to a circular symmetrical path surrounding Z axis.

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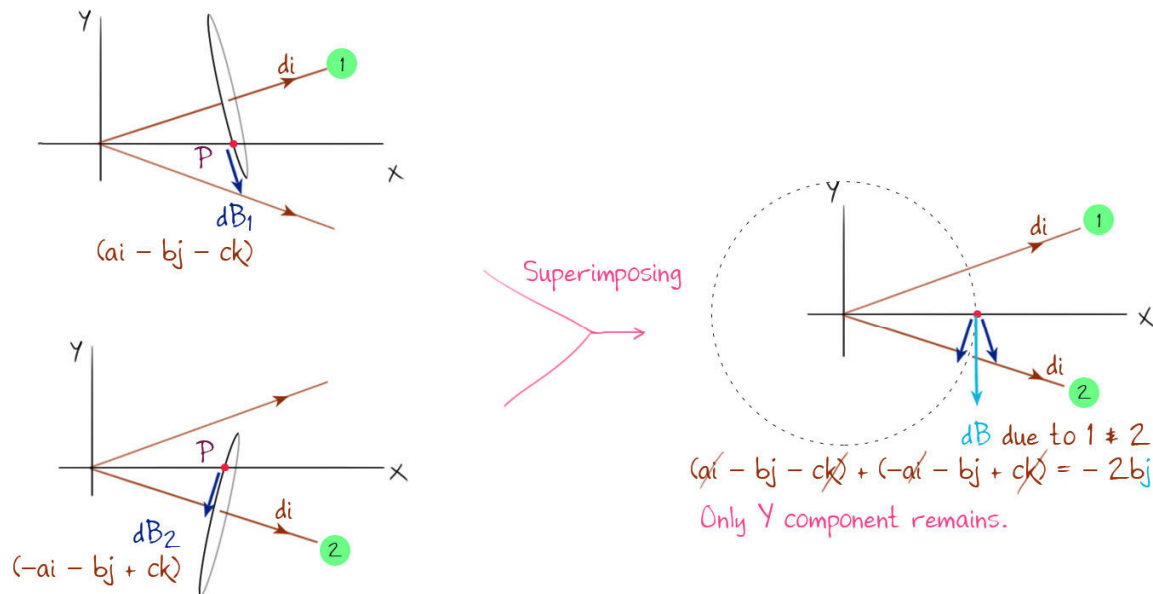
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{in}$$

Above XY plane $B(2\pi r) = \mu_0 i$
 $B = \frac{\mu_0 i}{2\pi r} //$

Below XY plane $B(2\pi r) = \mu_0 (0)$
 $B = 0 //$

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Visual proof that net field due to flat part of current is along tangent to circular path around Z axis. ☐ ☐ ☐



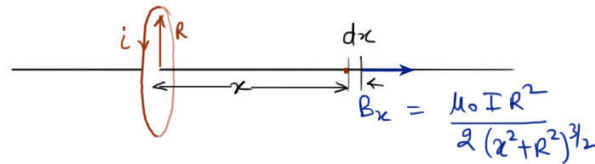
Magnitudes of dB_1 and dB_2 are same, as value of current and distance of P from both the elemental wires are same.

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3.232. A current I flows along a round loop. Find the integral

$\int \mathbf{B} \cdot d\mathbf{r}$ along the axis of the loop within the range from $-\infty$ to $+\infty$.

Explain the result obtained.



We need to find:

$$\int_{-\infty}^{\infty} B_x dx = \int_{-\infty}^{\infty} \frac{\mu_0 I R^2}{2 (x^2 + R^2)^{3/2}} dx$$

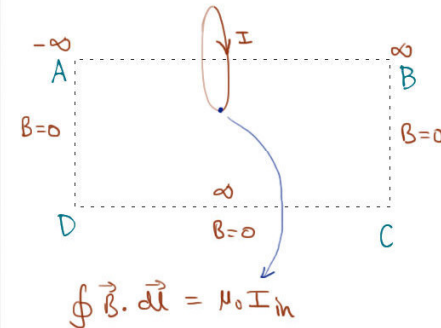
$$x \rightarrow R \tan \theta$$

$$dx \rightarrow R \sec^2 \theta d\theta$$

$$= \int \frac{\mu_0 I R^2 \sec^2 \theta}{2 R^3 \sec^3 \theta} d\theta$$

$$= \frac{\mu_0 I}{2} \int_{-\pi/2}^{+\pi/2} \cos \theta d\theta = \mu_0 I //$$

The result can also be obtained by Amperes law.
By considering a large loop

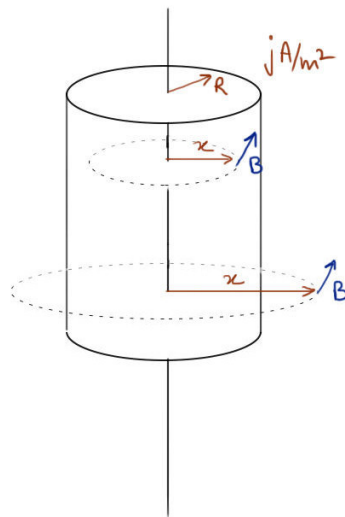


$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{in}$$

$$\Rightarrow \int_{-\infty}^{+\infty} B_x dx + 0 + 0 + 0 = \mu_0 I //$$

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3.233. A direct current of density j flows along a round uniform straight wire with cross-section radius R . Find the magnetic induction vector of this current at the point whose position relative to the axis of the wire is defined by a radius vector $\mathbf{r}$. The magnetic permeability is assumed to be equal to unity throughout all the space.



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{in}$$

Inside the conductor:

$$B (2\pi r) = \mu_0 (j \pi r^2)$$

$$B = \frac{\mu_0 j r}{2} = \frac{\mu_0}{2} (\vec{j} \times \vec{r})$$

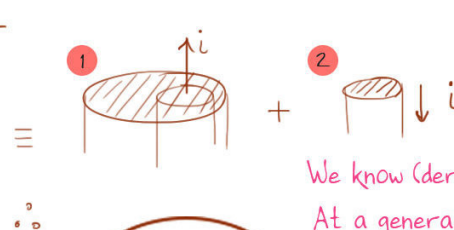
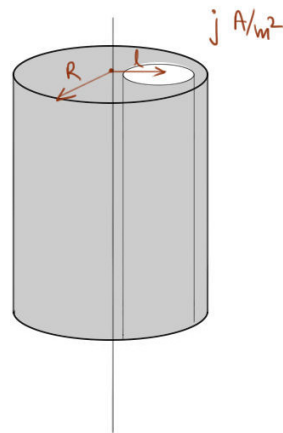
Outside the conductor:

$$B (2\pi r) = \mu_0 (j \pi R^2)$$

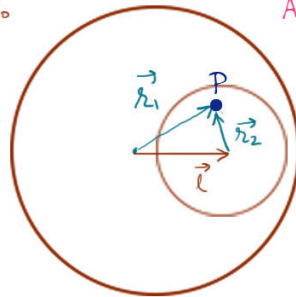
$$B = \frac{\mu_0 R^2 j}{r} = \frac{\mu_0 R^2}{r^2} (\vec{j} \times \vec{r})$$

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3.234. Inside a long straight uniform wire of round cross-section there is a long round cylindrical cavity whose axis is parallel to the axis of the wire and displaced from the latter by a distance l . A direct current of density j flows along the wire. Find the magnetic induction inside the cavity. Consider, in particular, the case $l = 0$.



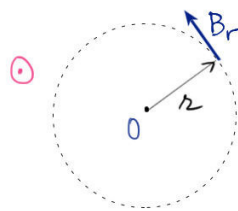
We know (derived in previous question), $B_{in} = \frac{\mu_0}{2} (\vec{j} \times \vec{r})$
At a general point P inside the cavity:



$$\begin{aligned} B &= B_1 + B_2 \\ &= \frac{\mu_0}{2} (\vec{j} \times \vec{r}_1) - \frac{\mu_0}{2} (\vec{j} \times \vec{r}_2) \\ &= \frac{\mu_0}{2} (\vec{j} \times (\vec{r}_1 - \vec{r}_2)) \\ &= \frac{\mu_0}{2} (\vec{j} \times \vec{l}) \end{aligned}$$

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3.235. Find the current density as a function of distance r from the axis of a radially symmetrical parallel stream of electrons if the magnetic induction inside the stream varies as $B = br^\alpha$, where b and α are positive constants.



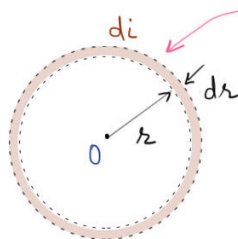
Its given: $B_r = br^\alpha$

From Amperes law: $(B_r) 2\pi r = \mu_0 i$

$$(br^\alpha) 2\pi r = \mu_0 i$$

differentiate

$$2\pi b(\alpha+1)r^\alpha dr = \mu_0 di \quad \text{--- (1)}$$



For a thin circular ring of width dr :

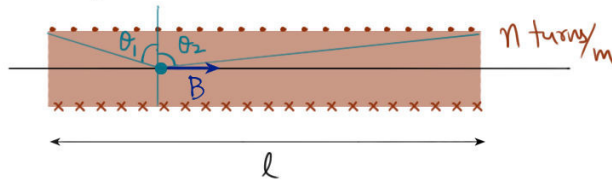
$$di = j_r dA$$

$$\Rightarrow di = j_r \cdot 2\pi r dr \quad \text{--- (2)}$$

$$\text{from (1) \& (2)} \quad j_r = \frac{2\pi b(\alpha+1)r^\alpha}{\mu_0 \cdot 2\pi r} = \frac{b(\alpha+1)}{\mu_0} r^{\alpha-1}$$

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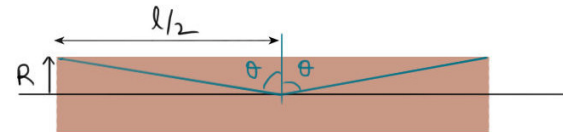
3.236. A single-layer coil (solenoid) has length l and cross-section radius R . A number of turns per unit length is equal to n . Find the magnetic induction at the centre of the coil when a current I flows through it.



We know for a general point inside a solenoid Field B at axis is:

$$B = \frac{\mu_0 n I}{2} (\sin \theta_1 + \sin \theta_2)$$

In our case:



$$B = \frac{\mu_0 n I}{2} (\sin \theta + \sin \theta)$$

$$= \frac{\mu_0 n I}{2} \cdot \frac{l/2}{\sqrt{(\frac{l}{2})^2 + R^2}} = \frac{\mu_0 n I}{\sqrt{1 + (\frac{2R}{l})^2}}$$

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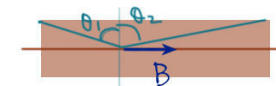
3.237. A very long straight solenoid has a cross-section radius R and n turns per unit length. A direct current I flows through the solenoid. Suppose that x is the distance from the end of the solenoid, measured along its axis. Find:

(a) the magnetic induction B on the axis as a function of x ; draw an approximate plot of B vs the ratio x/R ;

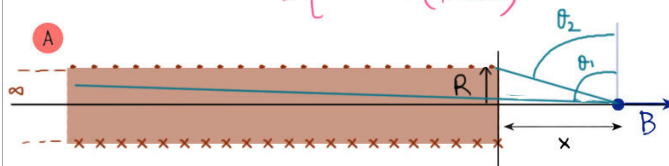
(b) the distance x_0 to the point on the axis at which the value of B differs by $\eta = 1\%$ from that in the middle section of the solenoid.

$\eta = 0.01$ (fraction)

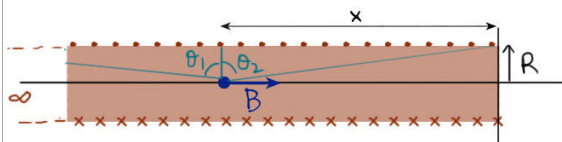
We have derived earlier for solenoid:



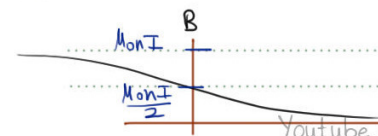
$$B = \frac{\mu_0 n I}{2} (\sin \theta_1 + \sin \theta_2)$$



$$\theta_1 = \pi/2, \theta_2 = -\tan^{-1} \frac{x}{R} \therefore B_{out} = \frac{\mu_0 n I}{2} \left(1 - \frac{|x|}{\sqrt{x^2 + R^2}}\right)$$



$$\theta_1 = \pi/2, \theta_2 = \tan^{-1} \frac{x}{R} \therefore B_{in} = \frac{\mu_0 n I}{2} \left(1 + \frac{|x|}{\sqrt{x^2 + R^2}}\right)$$



Youtube / Arihant Senior

(b) the distance x_0 to the point on the axis at which the value of B differs by $\eta = 1\%$ from that in the middle section of the solenoid.
 $\Rightarrow \eta = 0.01$

Ⓑ In middle of solenoid: $B_0 = \mu_0 n I$

Inside solenoid at distance x : $B = \frac{\mu_0 n I}{2} \left(1 + \frac{x}{\sqrt{x^2 + R^2}} \right)$

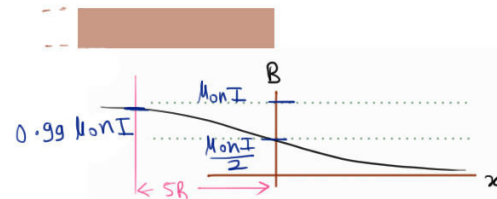
$$B_0 > B$$

$$\frac{B_0 - B}{B_0} = \eta$$

$$1 - \eta = \frac{B}{B_0} = \frac{1}{2} \left(1 + \frac{x}{\sqrt{x^2 + R^2}} \right) = \frac{1}{2} \left(1 + \frac{1}{\sqrt{1 + \left(\frac{R}{x}\right)^2}} \right)$$

Simplifying,

$$\frac{R}{x} = \sqrt{\frac{1}{(1-2\eta)^2} - 1} \Rightarrow x = \frac{R(1-2\eta)}{\sqrt{4\eta(1-\eta)}} \approx \frac{R(1-2\eta)}{2\sqrt{0.01}\sqrt{1-\eta}} = \frac{R}{2\sqrt{0.01}} = 5R$$



At end of solenoid field is 50% of its maximum value. But... Just $5R$ from the end, it reaches 99% of its maximum value.

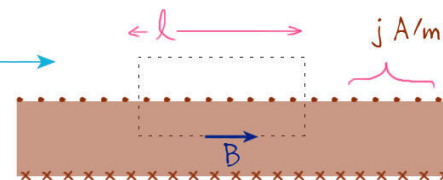
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3.238. A thin conducting strip of width $h = 2.0$ cm is tightly wound in the shape of a very long coil with cross-section radius $R = 2.5$ cm to make a single-layer straight solenoid. A direct current $I = 5.0$ A flows through the strip. Find the magnetic induction inside and outside the solenoid as a function of the distance r from its axis.

We know that,

Ⓐ For solenoid: $B_{in} = \mu_0 n i = \mu_0 j$ (Linear current density)

Proof

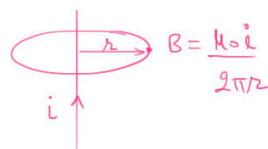


$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 i_{in}$$

$$B(l) = \mu_0 (j \cdot l)$$

$$B = \mu_0 j$$

Ⓑ Around a long wire:



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One round strip unwinded

$\sin \theta = \frac{h}{2\pi R}$ (1)
 $\cos \theta = \sqrt{1 - \left(\frac{h}{2\pi R}\right)^2}$ (2)

$j = \frac{i}{h}$ (3)
 $j \cos \theta$
 $j \sin \theta$
 $i_x = j \sin \theta (2\pi R)$ (5)

From 2, 3, 4
 $B_p = \mu_0 j \cos \theta$
 $B_{in} = \mu_0 \frac{i}{h} \sqrt{1 - \left(\frac{h}{2\pi R}\right)^2}$

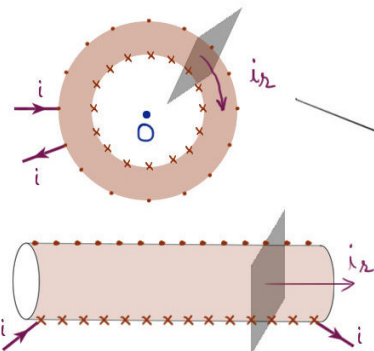
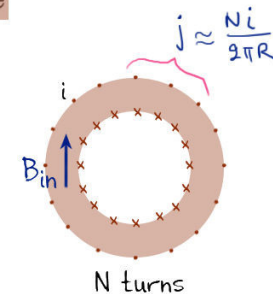
From 1, 3, 5
 $B_0 = \frac{\mu_0 i_x}{2\pi R} = \frac{\mu_0 j \frac{h}{2\pi R} \cdot 2\pi R}{2\pi R}$
 $B_{out} = \frac{\mu_0 i}{2\pi R}$

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3.239. $N = 2.5 \cdot 10^3$ wire turns are uniformly wound on a wooden toroidal core of very small cross-section. A current I flows through the wire. Find the ratio η of the magnetic induction inside the core to that at the centre of the toroid.

- A Just like solenoid, B inside a toroid is: $B_{in} = \mu_0 j = \mu_0 \left(\frac{Ni}{2\pi R}\right)$
 (Linear current density)

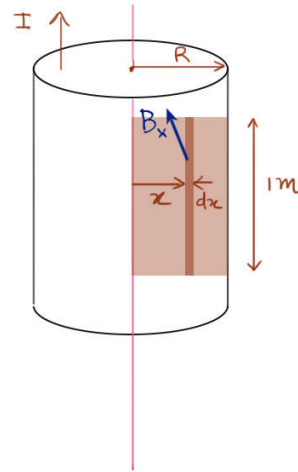
B $B = \frac{\mu_0 i}{2R}$



As across any plane cutting the solenoid or toroid, same amount of charge is crossing per unit time as is entering or leaving the whole circuit.

$$\frac{B_{in}}{B_0} = \frac{\mu_0 Ni / 2\pi R}{\mu_0 i / 2R} = \frac{N}{\pi}$$

3.240. A direct current $I = 10 \text{ A}$ flows in a long straight round conductor. Find the magnetic flux through a half of wire's cross-section per one metre of its length.



We know that B at distance x from center of a round conductor is: $B_x = \frac{\mu_0 I x}{2R^2}$ (Derived in problem 3.233)

$$\therefore B_x = \frac{\mu_0}{2} \left(\frac{I}{\pi R^2} \right) x$$

$d\phi$ through elemental strip:-

$$\begin{aligned} d\phi &= \vec{B}_x \cdot d\vec{S} = B_x (1) (dx) \\ &= \frac{\mu_0 I}{2\pi R^2} x dx \end{aligned}$$

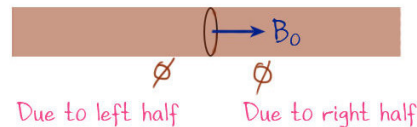
$$\phi = \int d\phi = \frac{\mu_0 I}{2\pi R^2} \int_0^R x dx = \frac{\mu_0 I}{2\pi R^2} \frac{R^2}{2} = \frac{\mu_0 I}{4\pi} //$$

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3.241. A very long straight solenoid carries a current I . The cross-sectional area of the solenoid is equal to S , the number of turns per unit length is equal to n . Find the flux of the vector $\mathbf{B}$ through the end plane of the solenoid.



For a long solenoid, flux in middle is twice of half solenoid as both halves are symmetrical.



Due to left half

Due to right half

Therefore,

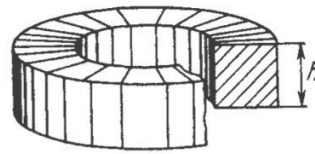
$$\phi_{\text{net}} = B_0 S = 2\phi$$

$$\Rightarrow (\mu_0 n I) S = 2\phi$$

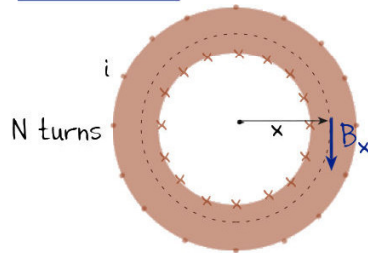
$$\Rightarrow \phi = \frac{\mu_0 n I S}{2} //$$

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3.242. Fig. 3.65 shows a toroidal solenoid whose cross-section is rectangular. Find the magnetic flux through this cross-section if the current through the winding equals $I = 1.7$ A, the total number of turns is $N = 1000$, the ratio of the outside diameter to the inside one is $\eta = 1.6$, and the height is equal to $h = 5.0$ cm.



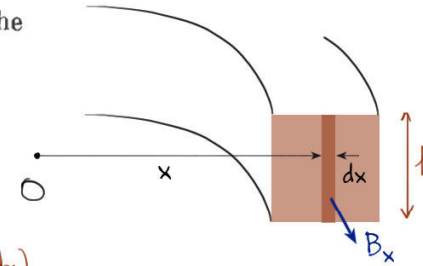
Field at a general point inside the toroid at distance x .



$$B_x (2\pi x) = \mu_0 i N = \mu_0 N i$$

$$\Rightarrow B_x = \frac{\mu_0 N i}{2\pi x}$$

Sol:



$$d\phi = B_x (h dx)$$

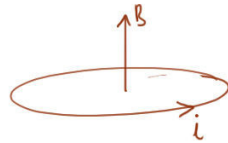
$$= \frac{\mu_0 N i}{2\pi x} h dx$$

$$\Rightarrow \phi = \int d\phi = \frac{\mu_0 N i h}{2\pi} \int_a^b \frac{dx}{x} = \frac{\mu_0 N i h}{2\pi} \ln \frac{b}{a}$$

$$= \frac{\mu_0 N i h}{2\pi} \ln \eta$$

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3.243. Find the magnetic moment of a thin round loop with current if the radius of the loop is equal to $R = 100$ mm and the magnetic induction at its centre is equal to $B = 6.0$ μ T.



$$\frac{\mu_0 i}{2R} = B$$

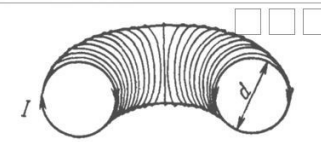
$$\Rightarrow i = \frac{2RB}{\mu_0}$$

$$\text{Magnetic moment } \mu = iA$$

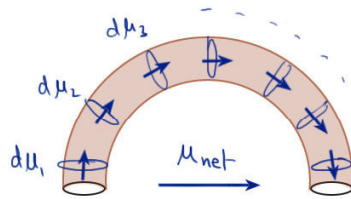
$$= \left(\frac{2RB}{\mu_0} \right) \cdot \pi R^2$$

$$= \frac{2\pi R^3 B}{\mu_0} //$$

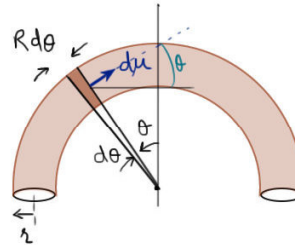
3.244. Calculate the magnetic moment of a thin wire with a current $I = 0.8$ A, wound tightly on half a toroid (Fig. 3.66). The diameter of the cross-section of the toroid is equal to $d = 5.0$ cm, the number of turns is $N = 500$.



Elemental magnetic moments due to elemental elements



Sol: Number of turns in elemental part: $dn = \frac{R d\theta}{\pi R} \cdot N$



$$d\mu = i A dn = i \pi r^2 \frac{R d\theta}{\pi R} \cdot N$$

$$= N i r^2 d\theta$$

$$\mu = \int d\mu \cos \theta \quad (\text{Because } \mu_{\text{net}} \text{ is in X direction})$$

$$= \int N i r^2 d\theta \cos \theta$$

$$= N i r^2 \int_{-\pi/2}^{\pi/2} \cos \theta d\theta = 2 N i r^2$$

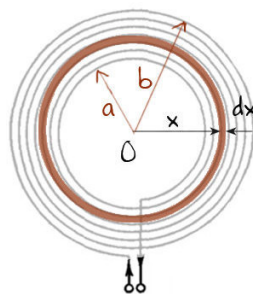
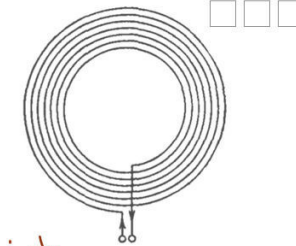
$$= \frac{N i d^2}{2}$$

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3.245. A thin insulated wire forms a plane spiral of $N = 100$ tight turns carrying a current $I = 8$ mA. The radii of inside and outside turns (Fig. 3.67) are equal to $a = 50$ mm and $b = 100$ mm.

Find:

- the magnetic induction at the centre of the spiral;
- the magnetic moment of the spiral with a given current.



Numer of turns in elemental ring: $dn = \frac{dx}{(b-a)} N$

∴ Current in ring: $di = i dn$

$$dB_0 = \frac{\mu_0 di}{2x} = \frac{\mu_0 (i dn)}{2x}$$

$$= \frac{\mu_0 i}{2x} \frac{dx}{(b-a)} N$$

$$B_0 = \int dB_0 = \frac{\mu_0 N i}{2(b-a)} \int_a^b \frac{dx}{x}$$

$$B_0 = \frac{\mu_0 N i}{2(b-a)} \ln \frac{b}{a} //$$

$$d\mu = di A = (i dn) A$$

$$= i \frac{dx}{(b-a)} N \cdot \pi x^2$$

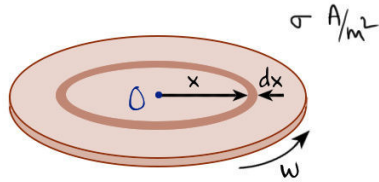
$$\mu = \int d\mu = \frac{\pi N i}{(b-a)} \int_a^b x^2 dx$$

$$\mu = \frac{\pi N i}{3} \frac{(b^3 - a^3)}{b-a}$$

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3.246. A non-conducting thin disc of radius R charged uniformly over one side with surface density σ rotates about its axis with an angular velocity ω . Find:

- (a) the magnetic induction at the centre of the disc;
(b) the magnetic moment of the disc.



$$di = \frac{dq}{dt} = \frac{\sigma dA}{T} = \frac{\sigma 2\pi x dx}{2\pi/\omega} = \sigma \omega x dx$$

$$dB = \frac{\mu_0 di}{2x} = \frac{\mu_0 \sigma \omega x dx}{2x}$$

$$B = \int dB = \frac{\mu_0 \sigma \omega}{2} \int_0^R dx = \frac{\mu_0 \sigma \omega R}{2} //$$

B Method 1

$$d\mu = diA = \sigma \omega x dx (\pi x^2) = \pi \sigma \omega x^3 dx$$

$$\mu = \int d\mu = \pi \sigma \omega \int_0^R x^3 dx = \pi \frac{\sigma \omega R^4}{4} //$$

B Method 2

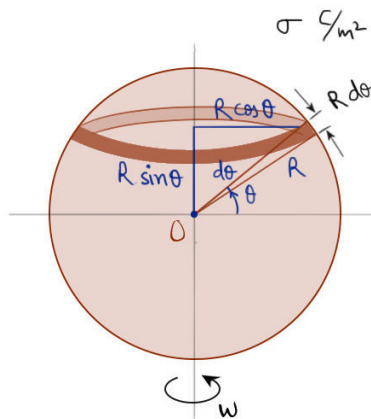
Using $\frac{\mu}{L} = \frac{q}{2m}$

$$\mu = \frac{Lq}{2m} = \frac{I\omega q}{2m} = \frac{\frac{1}{2} \pi R^2 \omega (\sigma \pi R^2)}{2m}$$

$$= \frac{\pi \sigma \omega R^4}{4}$$

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3.247. A non-conducting sphere of radius $R = 50$ mm charged uniformly with surface density $\sigma = 10.0 \mu\text{C}/\text{m}^2$ rotates with an angular velocity $\omega = 70$ rad/s about the axis passing through its centre. Find the magnetic induction at the centre of the sphere.



$$di = \frac{dq}{dt} = \frac{\sigma dA}{T} = \frac{\sigma 2\pi R \cos \theta \cdot R d\theta}{2\pi/\omega} = \sigma \omega R^2 \cos \theta d\theta$$

$$dB = \frac{\mu_0 di (R \cos \theta)^2}{2((R \sin \theta)^2 + (R \cos \theta)^2)^{3/2}} = \frac{\mu_0 \omega R^2}{2R} (\sigma \omega R^2 \cos \theta d\theta)$$

$$B_0 = \int dB = \frac{\mu_0 \sigma \omega R}{2} \int_{-\pi/2}^{\pi/2} \cos^3 \theta d\theta$$

$$= \frac{\mu_0 \sigma \omega R}{2} \cdot \frac{4}{3}$$

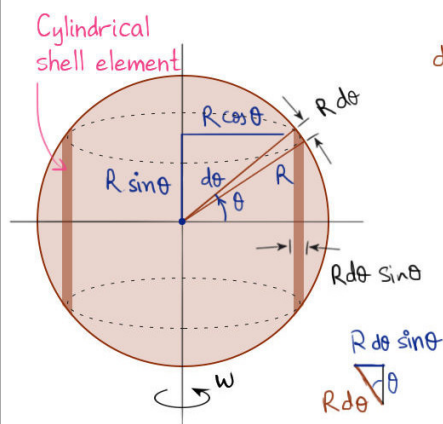
We know that field at axial point is:

$$B = \frac{\mu_0 i R^2}{2(x^2 + R^2)^{3/2}}$$

$$B_0 = \frac{2}{3} \mu_0 \sigma \omega R //$$

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3.248. A charge q is uniformly distributed over the volume of a uniform ball of mass m and radius R which rotates with an angular velocity ω about the axis passing through its centre. Find the respective magnetic moment and its ratio to the mechanical moment.



Ratio:

$$\frac{\mu}{L} = \frac{q\omega R^2}{\frac{2}{5}mR^2\omega} = \frac{q}{2m} \quad \text{B}$$

$$di = \frac{dq}{dt} = \frac{\delta dV}{T} = \left(\frac{q}{\frac{4}{3}\pi R^3} \right) \cdot \frac{(2\pi R \cos \theta \cdot R d\theta \sin \theta \cdot 2R \sin \theta)}{2\pi/\omega}$$

$$d\mu = diA = \left(\frac{3q\omega}{2\pi} \sin^2 \theta \cos \theta d\theta \right) \cdot (\pi R^2 \cos^2 \theta)$$

$$\mu = \int d\mu = \frac{3q\omega R^2}{2} \int_0^{\pi/2} \sin^2 \theta \cos^3 \theta d\theta$$

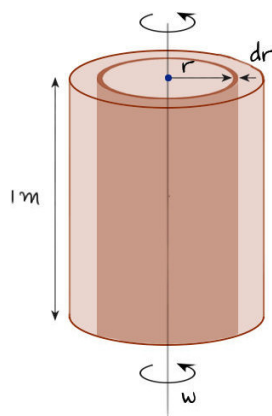
$$= \frac{3q\omega R^2}{2} \cdot \frac{2}{15}$$

$$\mu = \frac{q\omega R^2}{5} \quad \text{A}$$

$$\begin{aligned} & \rightarrow \int (\cos^3 \theta - \cos^5 \theta) d\theta \\ & = \left(\sin \theta - \frac{\sin^3 \theta}{3} \right) - \left(\sin \theta - \frac{\sin^5 \theta}{5} - \frac{2 \sin^3 \theta}{3} \right) \\ & = \frac{\sin^3 \theta}{3} - \frac{\sin^5 \theta}{5} \bigg|_0^{\pi/2} = \frac{2}{15} \end{aligned}$$

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3.249. A long dielectric cylinder of radius R is statically polarized so that at all its points the polarization is equal to $\mathbf{P} = \alpha \mathbf{r}$, where α is a positive constant, and $\mathbf{r}$ is the distance from the axis. The cylinder is set into rotation about its axis with an angular velocity ω . Find the magnetic induction $\mathbf{B}$ at the centre of the cylinder.



$$\rho = \alpha r \quad \text{C/m}^3$$

Small elemental cylindrical shell will act as a solenoid. With field at the axis because of it as: $dB = \mu_0 j_z$

Lets consider 1 m of cylinder.

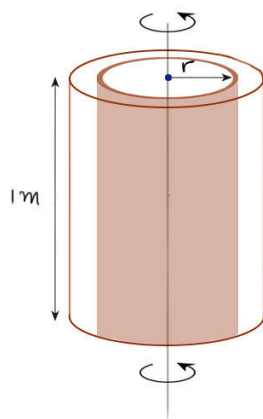
$$di = j_z \times (1\text{m}) = \frac{dq}{dt} = \frac{\delta dV}{T} = \frac{\alpha r (2\pi r dr)(1)}{2\pi/\omega} = \omega \alpha r^2 dr$$

Calculating field due to charge inside the dielectric:

$$dB = \mu_0 j_z = \mu_0 \omega \alpha r^2 dr$$

$$B_{in} = \int dB = \mu_0 \omega \alpha \int_0^R r^2 dr = \frac{\mu_0 \omega \alpha R^3}{3} \quad \text{1}$$

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Charge is polarized. That is, net charge is zero. So negative charge must be residing on the outermost layer of dielectric

For 1 m of cylinder

$$dq = \rho dv$$

$$= \rho \cdot 2\pi r dr \cdot 1$$

$$q = \int_0^R dq = \frac{2\pi \rho R^3}{3}$$

Therefore charge on outer surface = $-\frac{2\pi \rho R^3}{3}$

Calculating field due to charge on the surface:

$$B_{out} = \mu_0 j_R = \mu_0 i = \mu_0 \frac{q}{T} = \frac{\mu_0 \cdot (-\frac{2\pi \rho R^3}{3})}{\frac{2\pi}{\omega}}$$

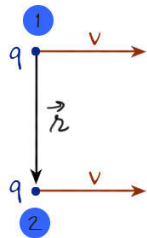
(As we took 1m wire)

$$B_{out} = -\frac{\mu_0 \omega \rho R^3}{3} \quad (2)$$

From (1) & (2), $B_{net} = B_{in} + B_{out} = 0$

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3.250. Two protons move parallel to each other with an equal velocity $v = 300$ km/s. Find the ratio of forces of magnetic and electrical interaction of the protons.



$$B_{21} = \frac{\mu_0}{4\pi} q \frac{(\vec{v} \times \vec{r})}{r^3} = \frac{\mu_0}{4\pi} \frac{qv}{r^2} \Rightarrow F_m = q(\vec{v} \times \vec{B}) = \frac{\mu_0}{4\pi} \frac{q^2 v^2}{r^2}$$

$$E_{21} = \frac{1}{4\pi \epsilon_0} \frac{q}{r^2} \Rightarrow F_E = qE = \frac{1}{4\pi \epsilon_0} \frac{q^2}{r^2}$$

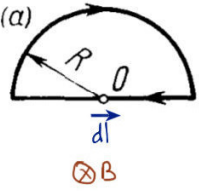
$$\therefore \frac{F_m}{F_E} = \frac{\frac{\mu_0}{4\pi} \frac{q^2 v^2}{r^2}}{\frac{1}{4\pi \epsilon_0} \frac{q^2}{r^2}} = \mu_0 \epsilon_0 v^2 = \left(\frac{v}{c}\right)^2 = 10^{-6} \quad \left(\text{As, } \mu_0 \epsilon_0 = \frac{1}{c^2}\right)$$

∴ Magnetic field force is much weaker than Electrostatic force at low speed of a charged beam.

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3.251. Find the magnitude and direction of a force vector acting on a unit length of a thin wire, carrying a current $I = 8.0$ A, at a point O , if the wire is bent as shown in

- (a) Fig. 3.68a, with curvature radius $R = 10$ cm;
 (b) Fig. 3.68b, the distance between the long parallel segments of the wire being equal to $l = 20$ cm.

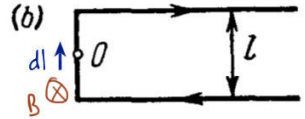
(a) 

$$\frac{dF}{dl} = ? \quad \frac{1}{2} \frac{\mu_0 I}{2R}$$

$$dF = i (d\vec{l} \times \vec{B})$$

$$dF = i dl \frac{\mu_0 I}{4R}$$

$$\frac{dF}{dl} = \frac{\mu_0 I^2}{4R}$$

(b) 

$$\frac{dF}{dl} = ? \quad 2 \left(\frac{\mu_0 I}{4\pi l} \right)$$

$$dF = i (d\vec{l} \times \vec{B})$$

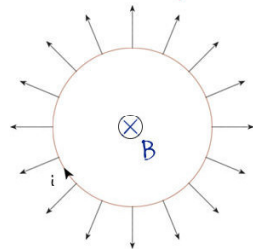
$$dF = i dl \frac{\mu_0 I}{\pi l}$$

$$\frac{dF}{dl} = \frac{\mu_0 I^2}{\pi l}$$

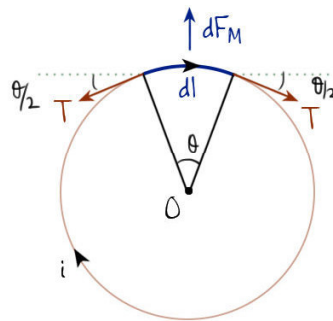
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3.252. A coil carrying a current $I = 10$ mA is placed in a uniform magnetic field so that its axis coincides with the field direction. The single-layer winding of the coil is made of copper wire with diameter $d = 0.10$ mm, radius of turns is equal to $R = 30$ mm. At what value of the induction of the external magnetic field can the coil winding be ruptured? Assuming breaking stress of copper as: σ_b

Cross section of solenoid



Magnetic field force is radially outwards on each element.
 Thus creating a tension in the wire.



Forces on a small element dl on the loop

As the loop is at rest

$$dF_m = 2T \sin \frac{\theta}{2} \approx T\theta$$

$$Bi dl = T\theta$$

$$Bi R \theta = T\theta \Rightarrow B = T/iR \quad (1)$$

It breaks if $\frac{T}{A} = \sigma_b$

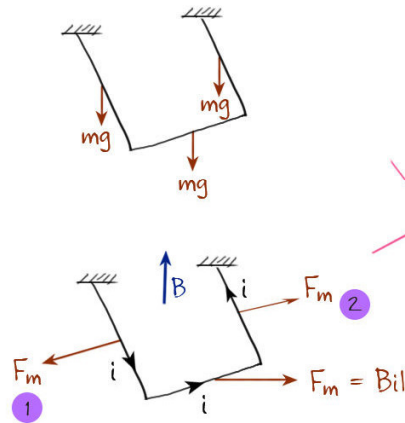
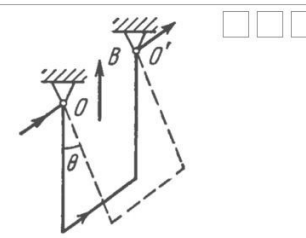
$$\text{i.e. } T = \sigma_b A = \sigma_b \pi \frac{d^2}{4} \quad (2)$$

① & ②

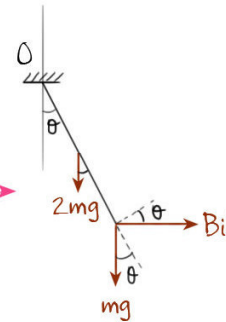
$$\Rightarrow B = \frac{\pi d^2 \sigma_b}{4iR}$$

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3.253. A copper wire with cross-sectional area $S = 2.5 \text{ mm}^2$ bent to make three sides of a square can turn about a horizontal axis OO' (Fig. 3.69). The wire is located in uniform vertical magnetic field. Find the magnetic induction if on passing a current $I = 16 \text{ A}$ through the wire the latter deflects by an angle $\theta = 20^\circ$.



① & ② cancel each other



Front view

Assume ℓ of wire.

Balancing torque about O:

$$\ell(mg \sin \theta) + \frac{\ell}{2}(2mg \sin \theta) = \ell(Bil \cos \theta)$$

$$2mg \sin \theta = Bil \cos \theta$$

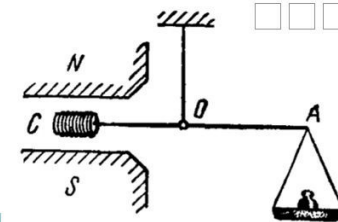
$$\Rightarrow B = \frac{2}{i} \left(\frac{m}{\ell} \right) g \tan \theta$$

$$B = \frac{2}{i} \rho A g \tan \theta$$

$$\begin{aligned} m &= \rho V \\ \Rightarrow m &= \rho A \ell \\ \Rightarrow \frac{m}{\ell} &= \rho A \end{aligned}$$

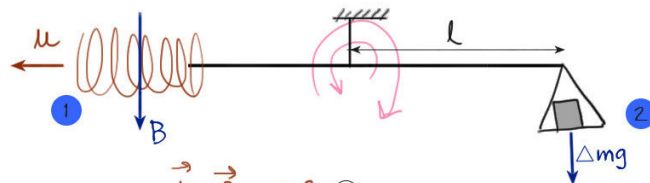
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3.254. A small coil C with $N = 200$ turns is mounted on one end of a balance beam and introduced between the poles of an electromagnet as shown in Fig. 3.70. The cross-sectional area of the coil is $S = 1.0 \text{ cm}^2$, the length of the arm OA of the balance beam is $l = 30 \text{ cm}$. When there is no current in the coil the balance is in equilibrium. On passing a current $I = 22 \text{ mA}$ through the coil the equilibrium is restored by putting the additional counterweight of mass $\Delta m = 60 \text{ mg}$ on the balance pan. Find the magnetic induction at the spot where the coil is located.



Initially: $\tau_1 + \tau_2 = 0$

Additional torque that appears:



$$\Delta \tau_1 = \vec{\mu} \times \vec{B} = \mu B \odot$$

$$\Delta \tau_2 = \Delta m g l \otimes = -\Delta m g l \odot$$

Finally: $\tau_1 + \Delta \tau_1 + \tau_2 + \Delta \tau_2 = 0$

$$\Delta \tau_1 + \Delta \tau_2 = 0$$

$$\mu B - \Delta m g l = 0$$

$$\Rightarrow B = \frac{\Delta m g l}{\mu} = \frac{\Delta m g l}{N i A}$$

Note: Distance of coil from the hinge is immaterial as torque in a current carrying loop is a couple torque, which means its value is same about all axes.

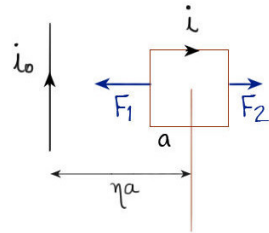
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3.255. A square frame carrying a current $I = 0.90$ A is located in the same plane as a long straight wire carrying a current $I_0 = 5.0$ A. The frame side has a length $a = 8.0$ cm. The axis of the frame passing through the midpoints of opposite sides is parallel to the wire and is separated from it by the distance which is $\eta = 1.5$ times greater than the side of the frame. Find:

(a) Ampere force acting on the frame;

(b) the mechanical work to be performed in order to turn the frame through 180° about its axis, with the currents maintained constant.

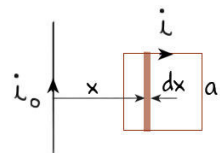
A



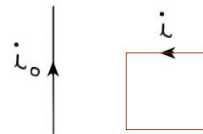
$$\begin{aligned} F_{\text{net}} &= F_1 - F_2 \\ &= B_1 i a - B_2 i a \\ &= i a \left(\frac{\mu_0 i_0}{2\pi(\eta a - \frac{a}{2})} - \frac{\mu_0 i_0}{2\pi(\eta a + \frac{a}{2})} \right) \\ &= \frac{\mu_0 i_0 i}{2\pi} \frac{4}{4\eta^2 - 1} = \frac{2\mu_0 i_0 i}{\pi(4\eta^2 - 1)} \end{aligned}$$

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B



flipped



Using, $W_{\text{ext}} = -i\Delta\phi$

$$\begin{aligned} &= -i(\phi_2 - \phi_1) \\ &= -i(-\phi_1 - \phi_1) \\ &= 2i\phi_1 \quad (1) \end{aligned}$$

(Except current direction, everything else is unchanged)
Hence, $\phi_1 = -\phi_2$

Finding ϕ_1 by dividing loop in vertical strips

$$d\phi_1 = \frac{\mu_0 i_0}{2\pi x} a dx = \frac{\mu_0 i_0 a}{2\pi} \frac{dx}{x}$$

$$\phi_1 = \int d\phi_1 = \frac{\mu_0 i_0 a}{2\pi} \int_{\eta a - \frac{a}{2}}^{\eta a + \frac{a}{2}} \frac{dx}{x} = \frac{\mu_0 i_0 a}{2\pi} \ln \frac{2\eta + 1}{2\eta - 1} \quad (2)$$

$$\text{From (1) \& (2), } W_{\text{ext}} = \frac{\mu_0 a i i_0}{\pi} \ln \left(\frac{2\eta + 1}{2\eta - 1} \right)$$

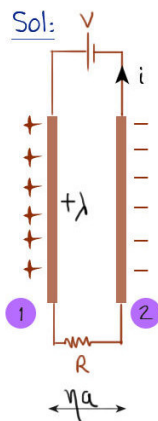
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3.256. Two long parallel wires of negligible resistance are connected at one end to a resistance R and at the other end to a dc voltage source. The distance between the axes of the wires is $\eta = 20$ times greater than the cross-sectional radius of each wire. At what value of resistance R does the resultant force of interaction between the wires turn into zero?

For two parallel cylindrical conductors, capacitance per unit length is: $C = \frac{\pi \epsilon_0}{\ln \eta}$

E due to charged long wire is given by: $E = \frac{2k\lambda}{r}$

Sol:



$$E = \frac{2k\lambda}{\eta a}$$

$$F_e = \lambda E = \frac{2k\lambda^2}{\eta a}$$

$$F_e = \frac{2}{\eta a} \cdot \frac{1}{4\pi\epsilon_0} \cdot \frac{\pi^2 \epsilon_0^2 V^2}{\ln^2 \eta} \quad (1)$$

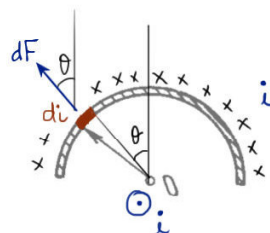
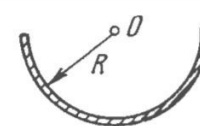
$$F_m = \frac{\mu_0 i^2}{2\pi (\eta a)} \quad (2)$$

$$F_e = F_m \Rightarrow \frac{2}{\eta a} \cdot \frac{1}{4\pi\epsilon_0} \cdot \frac{\pi^2 \epsilon_0^2 (iR)^2}{\ln^2 \eta} = \frac{\mu_0 i^2}{2\pi (\eta a)}$$

$$\Rightarrow R = \sqrt{\frac{\mu_0}{\epsilon_0} \frac{\ln \eta}{\pi}}$$

Note: All calculations are for per unit length of wire

3.257. A direct current I flows in a long straight conductor whose cross-section has the form of a thin half-ring of radius R . The same current flows in the opposite direction along a thin conductor located on the "axis" of the first conductor (point O in Fig. 3.61). Find the magnetic interaction force between the given conductors reduced to a unit of their length.



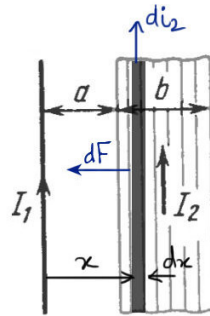
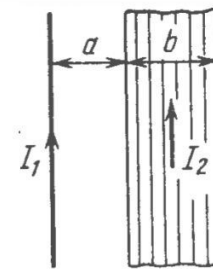
$$dF = \frac{\mu_0 i di}{2\pi R}$$

$$F = \int dF \cos \theta \quad (\text{As force components in X direction will cancel out})$$

$$= \int \frac{\mu_0 i}{2\pi R} \cdot \frac{i}{\pi} \cos \theta d\theta$$

$$= \frac{\mu_0 i^2}{2\pi^2 R} \int_{-\pi/2}^{\pi/2} \cos \theta d\theta = \frac{\mu_0 i^2}{\pi^2 R}$$

3.258. Two long thin parallel conductors of the shape shown in Fig. 3.71 carry direct currents I_1 and I_2 . The separation between the conductors is a , the width of the right-hand conductor is equal to b . With both conductors lying in one plane, find the magnetic interaction force between them reduced to a unit of their length.



$$dF = \frac{\mu_0 i_1 di_2}{2\pi x} \quad \text{with } di_2 = \frac{I_2 dx}{b}$$

$$\int dF = \frac{\mu_0 i_1 I_2}{2\pi b} \int \frac{dx}{x}$$

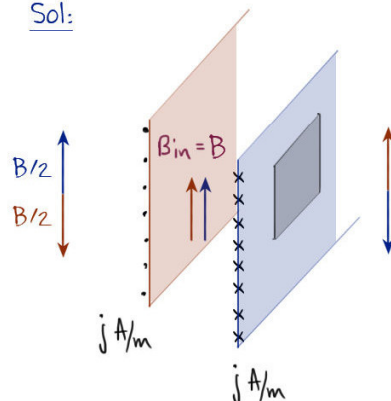
$$F = \int dF = \frac{\mu_0 i_1 I_2}{2\pi b} \int_a^{a+b} \frac{dx}{x} = \frac{\mu_0 i_1 I_2}{2\pi b} \ln \frac{a+b}{a}$$

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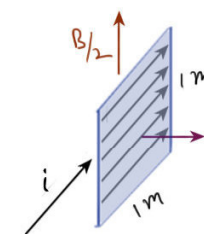
3.259. A system consists of two parallel planes carrying currents producing a uniform magnetic field of induction B between the planes. Outside this space there is no magnetic field. Find the magnetic force acting per unit area of each plane.

Magnetic field due to an infinite plane is: $\frac{\mu_0 j}{2}$, where j is linear current density.

Sol:



$$\frac{\mu_0 j}{2} + \frac{\mu_0 j}{2} = B \Rightarrow j = \frac{B}{\mu_0}$$



$$\begin{aligned} F &= \frac{B i l}{2} \\ &= \frac{B}{2} j (1) (1) \\ &= \frac{B j}{2} \\ &= \frac{B}{2} \left(\frac{B}{\mu_0} \right) \end{aligned}$$

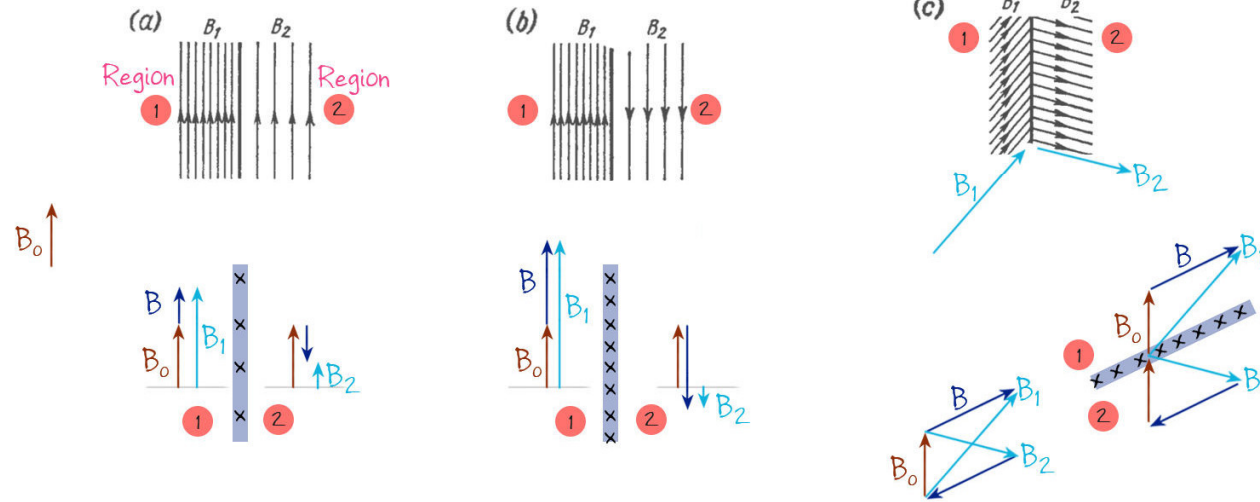
$$F = P = \frac{B^2}{\mu_0}$$

(As we calculated force per unit area)

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3.260. A conducting **current-carrying plane** is placed in an external uniform magnetic field. As a result, the magnetic induction becomes equal to B_1 on one side of the plane and to B_2 , on the other. Find the magnetic **force acting per unit area** of the plane in the cases illustrated in Fig. 3.72. **Determine the direction of the current** in the plane in each case.

Let the field due to plane be B and uniform magnetic field B_0 .



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A

$$B_0 + B = B_1$$

$$B_0 - B = B_2$$

$$B_0 = \frac{B_1 + B_2}{2} \quad (1)$$

$$B = \frac{B_1 - B_2}{2}$$

$$B = \frac{\mu_0 j}{2} \Rightarrow j = \frac{2B}{\mu_0} = \frac{B_1 - B_2}{\mu_0} \quad (2)$$

$$F = P = B_{\text{ext}} i l = B_0 (j i) (1) = B_0 j$$

$$\textcircled{1} \Delta \textcircled{2}: = \left(\frac{B_1 + B_2}{2} \right) \left(\frac{B_1 - B_2}{\mu_0} \right) = \frac{B_1^2 - B_2^2}{2\mu_0}$$

B

$$B_0 + B = B_1$$

$$B - B_0 = B_2$$

$$B_0 = \frac{B_1 - B_2}{2} \quad (1)$$

$$B = \frac{B_1 + B_2}{2}$$

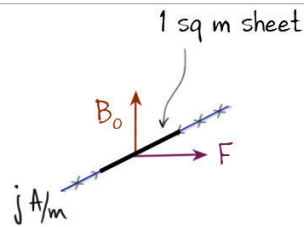
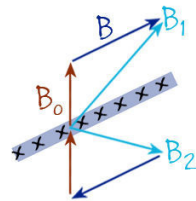
$$B = \frac{\mu_0 j}{2} \Rightarrow j = \frac{2B}{\mu_0} = \frac{B_1 + B_2}{\mu_0} \quad (2)$$

$$P = B_0 j, \textcircled{1} \Delta \textcircled{2}: = \left(\frac{B_1 - B_2}{2} \right) \left(\frac{B_1 + B_2}{\mu_0} \right) = \frac{B_1^2 - B_2^2}{2\mu_0}$$

Similar as above

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C



$$\begin{aligned} f &= B_{\text{ext}} i l \\ &= B_0 j^{(1)} (1) \\ &= B_0 j \end{aligned}$$

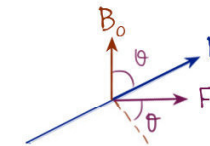
$$\text{Pressure} = \frac{F_{\perp}}{A}$$

$$= f_{\perp} = f \cos \theta$$

$$= B_0 j \cos \theta$$

$$= B_0 \cdot \frac{2B}{\mu_0} \cdot \frac{\vec{B} \cdot \vec{B}_0}{B B_0}$$

$$= \frac{2}{\mu_0} \frac{B_1^2 - B_2^2}{4} = \frac{B_1^2 - B_2^2}{2\mu_0}$$

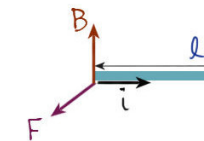
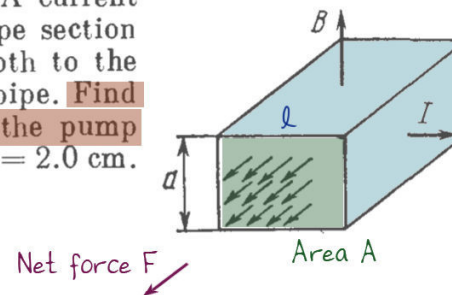


$$\left. \begin{aligned} \vec{B}_0 + \vec{B} &= \vec{B}_1 \\ \vec{B}_0 - \vec{B} &= \vec{B}_2 \end{aligned} \right\} \begin{aligned} \vec{B}_0 &= \frac{\vec{B}_1 + \vec{B}_2}{2} \\ \vec{B} &= \frac{\vec{B}_1 - \vec{B}_2}{2} \end{aligned}$$

$$B = \frac{\mu_0 j}{2} \Rightarrow j = \frac{2B}{\mu_0}$$

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3.261. In an **electromagnetic pump** designed for transferring molten metals a pipe section with metal is located in a uniform magnetic field of induction B (Fig. 3.73). A current I is made to flow across this pipe section in the direction perpendicular both to the vector B and to the axis of the pipe. Find the gauge pressure produced by the pump if $B = 0.10 \text{ T}$, $I = 100 \text{ A}$, and $a = 2.0 \text{ cm}$.



Pressure produced by the pump
= Force F (on all the liquid inside pump/cuboid) divided by cross sectional area A .

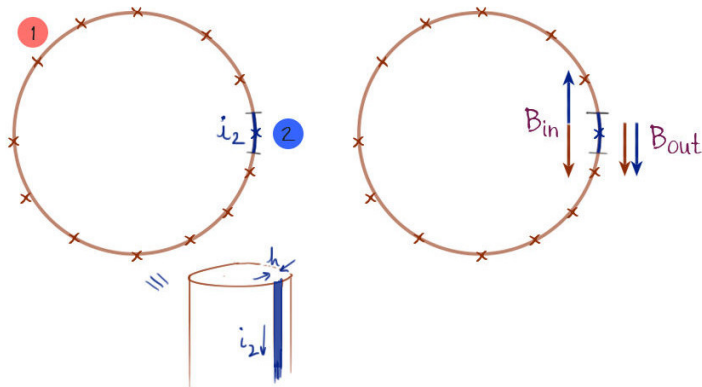
$$= \frac{F}{A} = \frac{B i l}{A} = \frac{B i k}{\cancel{A} a} = \frac{B i}{a}$$

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3.262. A current I flows in a long thin-walled cylinder of radius R . What pressure do the walls of the cylinder experience?

Let us divide the conductor in a small strip of width h ② and the rest of it ①

Therefore, total field anywhere is now given by superposition of magnetic fields due to 1 and 2.



We know that just inside the conductor, ☐☐☐

$$B_{in} = 0 \Rightarrow B_1 = B_2 \quad ①$$

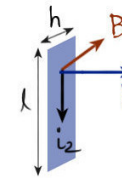
Also just outside the conductor,

$$B_{out} = B_1 + B_2 = \frac{\mu_0 I}{2\pi R} \quad ②$$

$$① \& ② \Rightarrow B_1 = \frac{B_{out}}{2} = \frac{1}{2} \frac{\mu_0 I}{2\pi R}$$

Force on strip of thickness h and length l :

$$F = B_{ext} i_2 l = B_1 i_2 l = \left(\frac{1}{2} \frac{\mu_0 I}{2\pi R} \right) \left(\frac{h}{2\pi R} \right) l$$



$$\text{Pressure on strip} = \frac{F}{A} = \frac{F}{lh}$$

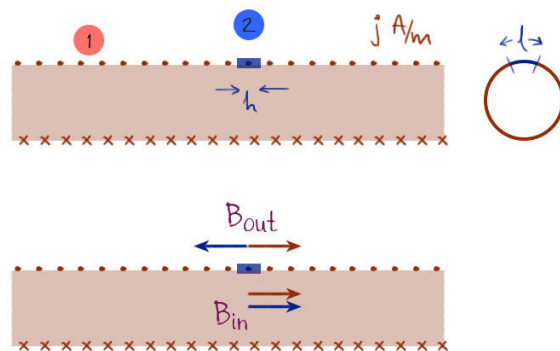
$$= \frac{\mu_0 I^2}{8\pi^2 R^2}$$

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3.263. What pressure does the lateral surface of a long straight solenoid with n turns per unit length experience when a current I flows through it?

Let us divide the solenoid in a small strip of thickness h and length l ② and the rest of it ①

Therefore, total field anywhere is now given by superposition of magnetic fields due to 1 and 2.



We know that just inside the solenoid, ☐☐☐

$$B_{in} = B_1 + B_2 = \mu_0 j \quad ①$$

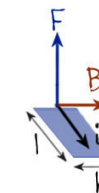
Also just outside the solenoid,

$$B_{out} = 0 \Rightarrow B_1 = B_2 \quad ②$$

$$① \& ② \Rightarrow B_1 = \frac{\mu_0 j}{2}$$

Force on conductor on the surface:

$$F = B_{ext} i_2 l = B_1 i_2 l = \frac{\mu_0 j}{2} (jh) l$$



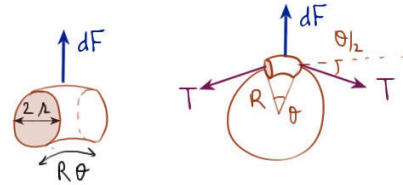
$$\text{Pressure on strip} = \frac{F}{A} = \frac{F}{lh}$$

$$= \frac{\mu_0 j^2}{2} = \frac{\mu_0 (nI)^2}{2}$$

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3.264. A current I flows in a long single-layer solenoid with cross-sectional radius R . The number of turns per unit length of the solenoid equals n . Find the limiting current at which the winding may rupture if the tensile strength of the wire is equal to F_{lim} .

From previous question 3.263 we know that pressure on a solenoid element is: $P = \frac{\mu_0 n^2 i^2}{2}$



Considering a small section of a wire.

$$dF = P \cdot A$$

$$= \frac{\mu_0 n^2 i^2}{2} (2R \cdot R\theta) \quad (1)$$

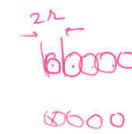
Also as the section is at rest,

$$dF = 2T \sin \frac{\theta}{2}$$

$$\approx T\theta \quad (2)$$

$$(1) \& (2) \Rightarrow T = \frac{\mu_0 n^2 i^2}{2} R$$

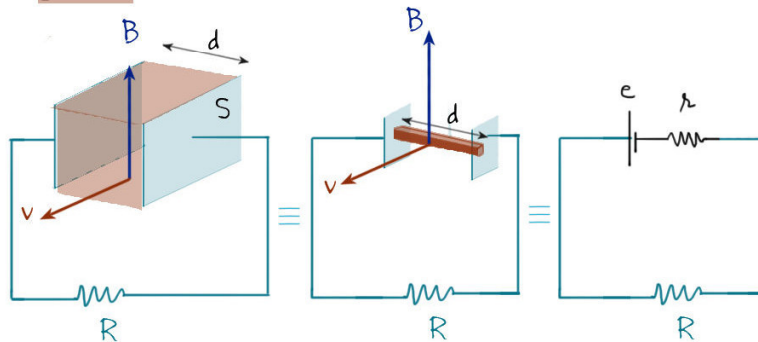
$$= \frac{\mu_0 n i^2 R}{2} \Rightarrow i = \frac{\sqrt{2F_{lim}}}{\sqrt{\mu_0 n R}}$$



$2R$ length $\rightarrow$ 1 wire
 1 m length $\rightarrow \frac{1}{2R}$ wires $= n$
 $\Rightarrow R = \frac{1}{2n}$

3.265. A parallel-plate capacitor with area of each plate equal to S and the separation between them to d is put into a stream of conducting liquid with resistivity ρ . The liquid moves parallel to the plates with a constant velocity v . The whole system is located in a uniform magnetic field of induction B , vector B being parallel to the plates and perpendicular to the stream direction. The capacitor plates are interconnected by means of an external resistance R .

A What amount of power is generated in that resistance? B At what value of R is the generated power the highest? C What is this highest power equal to?



emf induced across rod: $e = Bvd$

Equivalent battery's resistance: $r = \frac{\rho d}{S}$

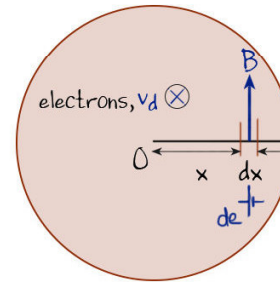
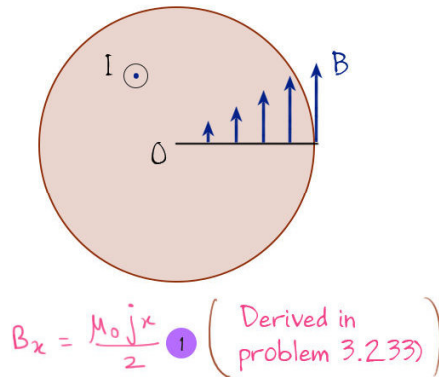
$$A \quad P_R = i^2 R = \left(\frac{e}{R+r} \right)^2 R = \frac{B^2 v^2 d^2 R}{(R + \rho d/S)^2}$$

For maximum power, external resistance must be equal to internal resistance of battery. Therefore,

$$B \quad R = \frac{\rho d}{S}$$

$$C \quad P_{R_{max}} = i^2 R = \left(\frac{e}{2R} \right)^2 R = \frac{B^2 v^2 d^2}{4R} = \frac{B^2 v^2 d S}{4\rho}$$

3.266. A straight round copper conductor of radius $R = 5.0$ mm carries a current $I = 50$ A. Find the potential difference between the axis of the conductor and its surface. The concentration of the conduction electrons in copper is equal to $n = 0.9 \cdot 10^{23} \text{ cm}^{-3}$.



We know that, $I = neAv_d$

$$\Rightarrow v_d = \frac{I}{ne(\pi R^2)}$$

Small motional emf de across dx due to electrons, traveling with v_d is given as:

$$de = d\vec{x} \cdot (\vec{v}_d \times \vec{B})$$

$$= v_d B dx$$

$$\textcircled{1} \& \textcircled{2} \Rightarrow = \frac{I}{ne(\pi R^2)} \frac{\mu_0 I x}{2} dx$$

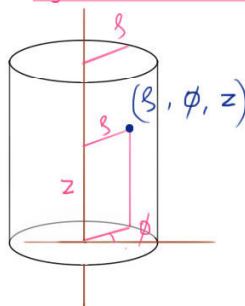
Total emf between axis and the surface: $\mathcal{E} = \int de = \frac{I}{ne(\pi R^2)} \frac{\mu_0 I}{2} \int_0^R x dx = \frac{\mu_0 I^2}{4\pi ne R^2}$

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3.269. A small current-carrying loop is located at a distance r from a long straight conductor with current I . The magnetic moment of the loop is equal to $\vec{p}_m$. Find the magnitude and direction of the force vector applied to the loop if the vector $\vec{p}_m$

- is parallel to the straight conductor;
- is oriented along the radius vector $\vec{r}$;
- coincides in direction with the magnetic field produced by the current I at the point where the loop is located.

Cylindrical coordinate system



$$\vec{\nabla} = \frac{\partial}{\partial s} \hat{s} + \frac{1}{s} \frac{\partial}{\partial \phi} \hat{\phi} + \frac{\partial}{\partial z} \hat{z} \quad \text{and} \quad \vec{B} = \frac{\mu_0 I}{2\pi s} \hat{\phi}$$

Sol: $\vec{F} = (\vec{\mu} \cdot \vec{\nabla}) \vec{B}$

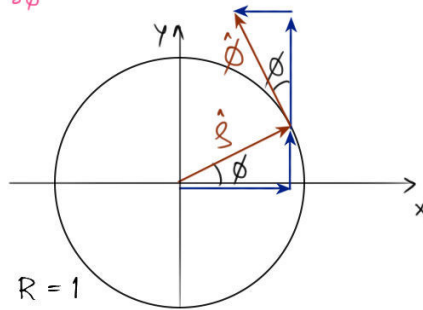
A $\vec{F} = \mu \frac{\partial \vec{B}}{\partial z} = 0$ (As $\vec{B}$ does not depend on z)

B $\vec{F} = \mu \frac{\partial \vec{B}}{\partial s} = \mu \frac{\mu_0 I}{2\pi} \frac{\partial}{\partial s} \left(\frac{\hat{\phi}}{s} \right) = \mu \frac{\mu_0 I}{2\pi} \left(-\frac{\hat{\phi}}{s^2} + \frac{1}{s} \frac{\partial \hat{\phi}}{\partial s} \right) \Rightarrow \vec{F} = -\frac{\mu \mu_0 I}{2\pi s^2} \hat{\phi}$

C $\vec{F} = \mu \frac{\partial \vec{B}}{\partial \phi} = \mu \frac{\mu_0 I}{2\pi} \frac{\partial}{\partial \phi} \left(\frac{\hat{\phi}}{s} \right) = \mu \frac{\mu_0 I}{2\pi} \left(\frac{1}{s} \frac{\partial \hat{\phi}}{\partial \phi} + \hat{\phi} \frac{\partial}{\partial \phi} \left(\frac{1}{s} \right) \right) \Rightarrow \vec{F} = -\frac{\mu \mu_0 I}{2\pi s^2} \hat{s}$

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Proof $\frac{\partial \hat{\phi}}{\partial \phi} = -\hat{s}$



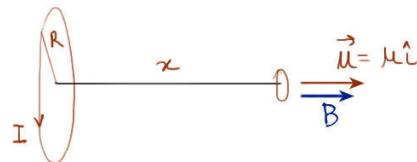
$$\hat{s} = \cos \phi \hat{i} + \sin \phi \hat{j}$$

$$\hat{\phi} = -\sin \phi \hat{i} + \cos \phi \hat{j}$$

$$\begin{aligned} \frac{\partial \hat{\phi}}{\partial \phi} &= -\cos \phi \hat{i} - \sin \phi \hat{j} & \frac{\partial \hat{s}}{\partial \phi} &= -\sin \phi \hat{i} + \cos \phi \hat{j} \\ \Rightarrow \frac{\partial \hat{\phi}}{\partial \phi} &= -\hat{s} & \Rightarrow \frac{\partial \hat{s}}{\partial \phi} &= \hat{\phi} \end{aligned}$$

Youtube / Arihant Senior

3.270. A small current-carrying coil having a magnetic moment $\vec{p}_m$ is located at the axis of a round loop of radius R with current I flowing through it. Find the magnitude of the vector force applied to the coil if its distance from the centre of the loop is equal to x and the vector $\vec{p}_m$ coincides in direction with the axis of the loop.



In cartesian coordinate system

$$\vec{\nabla} = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \quad \text{and} \quad \vec{B} = \frac{\mu_0 i R^2}{2(x^2 + R^2)^{3/2}} \hat{i}$$

$$\vec{F} = (\vec{\mu} \cdot \vec{\nabla}) \vec{B}$$

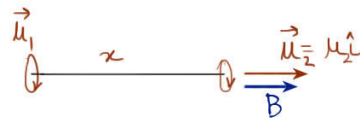
$$= \mu \frac{\partial \vec{B}}{\partial x}$$

$$= \frac{\mu \mu_0 i R^2}{2} \left[\frac{\partial}{\partial x} \frac{1}{(x^2 + R^2)^{3/2}} \right] \hat{i}$$

$$= \frac{\mu \mu_0 i R^2}{2} \left(-\frac{3}{2} \right) \frac{1}{(x^2 + R^2)^{5/2}} (2x) \hat{i} = -\frac{3 \mu \mu_0 i R^2 x}{2 (x^2 + R^2)^{5/2}} \hat{i}$$

Youtube / Arihant Senior

3.271. Find the interaction force of two coils with magnetic moments $p_{1m} = 4.0 \text{ mA} \cdot \text{m}^2$ and $p_{2m} = 6.0 \text{ mA} \cdot \text{m}^2$ and collinear axes if the separation between the coils is equal to $l = 20 \text{ cm}$ which exceeds considerably their linear dimensions.



In cartesian coordinate system

$$\vec{\nabla} = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \quad \text{and} \quad \vec{B} = \frac{\mu_0 \mu_1}{2\pi x^3} \hat{i}$$

$$\vec{F} = (\vec{\mu}_2 \cdot \vec{\nabla}) \vec{B}$$

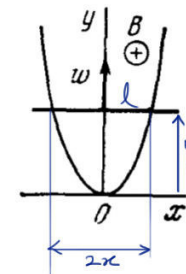
$$= \mu_2 \frac{\partial}{\partial x} \vec{B}$$

$$= \frac{\mu_2 \mu_0 \mu_1}{2\pi} \frac{\partial}{\partial x} \left(\frac{1}{x^3} \right)$$

$$= -3 \frac{\mu_0 \mu_1 \mu_2}{2\pi x^4} \xrightarrow{x=l} -3 \frac{\mu_0 \mu_1 \mu_2}{2\pi l^4}$$

Youtube / Arihant Senior

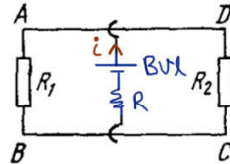
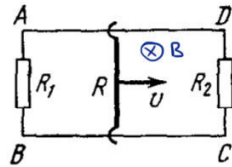
3.288. A wire bent as a parabola $y = ax^2$ is located in a uniform magnetic field of induction B , the vector $\vec{B}$ being perpendicular to the plane x, y . At the moment $t = 0$ a connector starts sliding translationwise from the parabola apex with a constant acceleration w (Fig. 3.78). Find the emf of electromagnetic induction in the loop thus formed as a function of y .



$$e = B v_y l \Rightarrow e = 2\sqrt{2} B y \sqrt{\frac{w}{a}}$$

$\swarrow \sqrt{2\omega y}$ $\searrow 2x$ $\searrow \sqrt{y/a}$

3.289. A rectangular loop with a sliding connector of length l is located in a uniform magnetic field perpendicular to the loop plane (Fig. 3.79). The magnetic induction is equal to B . The connector has an electric resistance R , the sides AB and CD have resistances R_1 and R_2 respectively. Neglecting the self-inductance of the loop, find the current flowing in the connector during its motion with a constant velocity v .



$$\Rightarrow i = \frac{Blv}{R + \frac{R_1 R_2}{R_1 + R_2}}$$

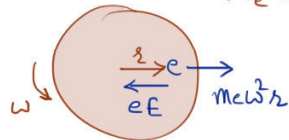
Youtube / Arihant Senior

3.290. A metal disc of radius $a = 25$ cm rotates with a constant angular velocity $\omega = 130$ rad/s about its axis. Find the potential difference between the centre and the rim of the disc if

- the external magnetic field is absent;
- the external uniform magnetic field of induction $B = 5.0$ mT is directed perpendicular to the disc.

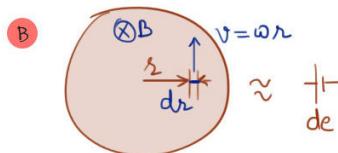
- (A) Electrons will drift towards the rim due to centrifugal force until electric field thus generated cancels it.

That is: $m_e \omega^2 r = eE$



$$\Rightarrow E = \frac{m_e \omega^2 r}{e}$$

$$\Rightarrow |dV| = \int E dr = \frac{m_e \omega^2}{e} \int_0^a r dr = \frac{m_e \omega^2 a^2}{2e} \quad \left\{ \text{very small magnitude} \right\}$$



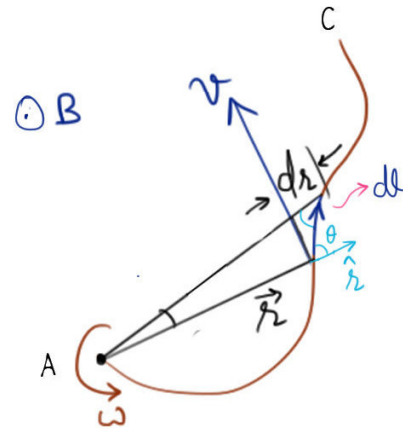
Let de be small motional emf induced due to length dr :

$$de = B v dr \Rightarrow e = \int de = B \omega \int_0^a r dr = \frac{B \omega a^2}{2}$$

Youtube / Arihant Senior

3.291. A thin wire AC shaped as a semi-circle of diameter $d = 20$ cm rotates with a constant angular velocity $\omega = 100$ rad/s in a uniform magnetic field of induction $B = 5.0$ mT, with $\omega \uparrow \uparrow B$. The rotation axis passes through the end A of the wire and is perpendicular to the diameter AC . Find the value of a line integral

$\int E dr$ along the wire from point A to point C . Generalize the obtained result.



$$\begin{aligned} \text{emf induced across } dl: de &= \vec{dl} \cdot (\vec{v} \times \vec{B}) \\ &= \vec{dl} \cdot [v B \hat{n}] \\ &= Bv (dl \cdot \hat{n}) \end{aligned}$$

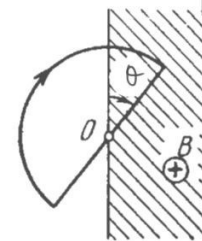
$$\begin{aligned} &= B\omega dl \cos \theta \\ &= B\omega dr \\ &= B\omega r dr \end{aligned}$$

$$e = \int de = B\omega \int_0^l r dr = \frac{B\omega l^2}{2} \quad (\text{general result})$$

(Therefore emf induced is independent of shape of wire)

Youtube / Arihant Senior

3.292. A wire loop enclosing a semi-circle of radius a is located on the boundary of a uniform magnetic field of induction B (Fig. 3.80). At the moment $t = 0$ the loop is set into rotation with a constant angular acceleration β about an axis O coinciding with a line of vector B on the boundary. Find the emf induced in the loop as a function of time t . Draw the approximate plot of this function. The arrow in the figure shows the emf direction taken to be positive.



$$\theta = \frac{1}{2} \beta t^2$$

$$e = -\frac{d\phi}{dt}$$

When, $2n\pi < \theta < (2n+1)\pi$, ϕ increases $\Rightarrow e$ -ve

When, $(2n-1)\pi < \theta < 2n\pi$, ϕ decreases $\Rightarrow e$ +ve

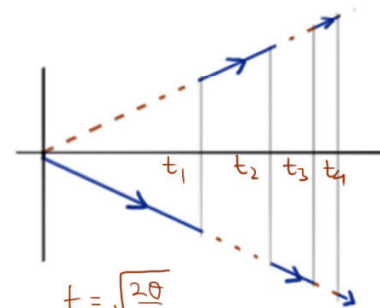
$$\text{At } \theta, \phi = B \cdot A = B \left(\frac{\theta}{2\pi} \right) \pi a^2$$

$$\text{emf } e = -\frac{d\phi}{dt} = -\frac{Ba^2}{2} \frac{d\theta}{dt} = -\frac{Ba^2}{2} (\beta t)$$

$$= (-1)^n Ba^2/2 (\beta t_n) //$$

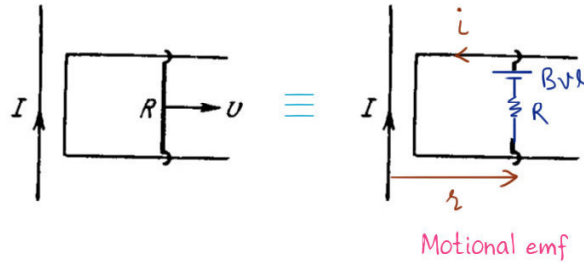
$$t = \sqrt{\frac{2\theta}{\beta}}$$

$$t_n = \sqrt{\frac{2(n\pi)}{\beta}}, n=1,2,3 \dots$$



Youtube / Arihant Senior

3.293. A long straight wire carrying a current I and a Π -shaped conductor with sliding connector are located in the same plane as shown in Fig. 3.81. The connector of length l and resistance R slides to the right with a constant velocity v . Find the current induced in the loop as a function of separation r between the connector and the straight wire. The resistance of the Π -shaped conductor and the self-inductance of the loop are assumed to be negligible.

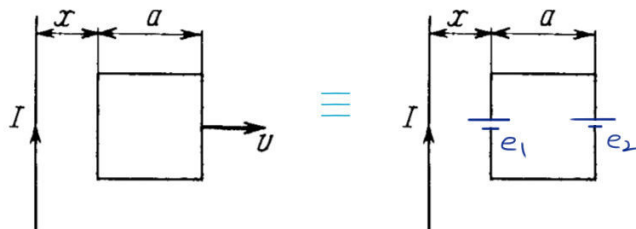


$$i = \frac{Bvl}{R} = \frac{\mu_0 I}{2\pi r} \cdot vl \cdot \frac{1}{R} = \left(\frac{\mu_0 Ivl}{2\pi R} \right) \frac{1}{r}$$

(Inversely proportional to r)

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3.294. A square frame with side a and a long straight wire carrying a current I are located in the same plane as shown in Fig. 3.82. The frame translates to the right with a constant velocity v . Find the emf induced in the frame as a function of distance x .



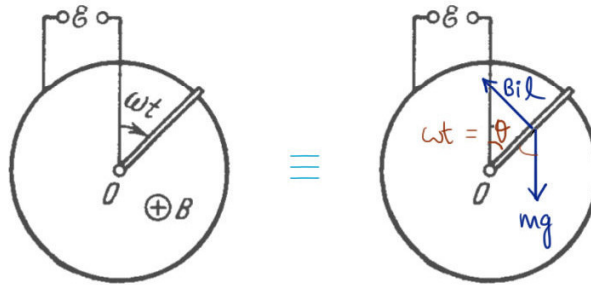
$$e_1 = B_1 va = \frac{\mu_0 I}{2\pi x} va$$

$$e_2 = B_2 va = \frac{\mu_0 I}{2\pi(x+a)} va$$

$$\therefore e_{\text{net}} = e_1 - e_2 = \frac{\mu_0 a^2 I v}{2\pi x(x+a)}$$

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3.295. A metal rod of mass m can rotate about a horizontal axis O , sliding along a circular conductor of radius a (Fig. 3.83). The arrangement is located in a uniform magnetic field of induction B directed perpendicular to the ring plane. The axis and the ring are connected to an emf source to form a circuit of resistance R . Neglecting the friction, circuit inductance, and ring resistance, find the law according to which the source emf must vary to make the rod rotate with a constant angular velocity ω .



$$Bil = B \left(\frac{e_{\text{ext}} + e_{\text{induced}}}{R} \right) l \quad (1)$$

$\frac{B\omega l^2}{2}$

τ_0 is balanced, as ω_{rod} is constant.

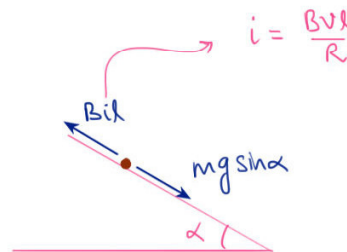
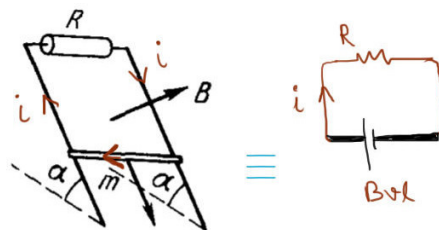
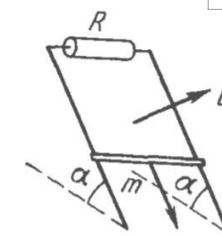
$$\Rightarrow mg \frac{l}{2} \sin \theta = Bil \frac{l}{2} \quad (2)$$

$$\text{From (1) \& (2)} \quad e_{\text{ext}} = \frac{1}{2Bl} \left[2mgR \sin \omega t - \omega^2 l^3 \right]$$

(Answer given in source is incorrect)

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3.296. A copper connector of mass m slides down two smooth copper bars, set at an angle α to the horizontal, due to gravity (Fig. 3.84). At the top the bars are interconnected through a resistance R . The separation between the bars is equal to l . The system is located in a uniform magnetic field of induction B , perpendicular to the plane in which the connector slides. The resistances of the bars, the connector and the sliding contacts, as well as the self-inductance of the loop, are assumed to be negligible. Find the steady-state velocity of the connector.



$$i = \frac{Bvl}{R}$$

At steady state,

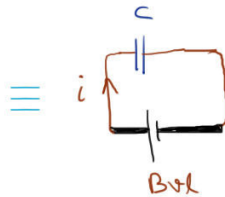
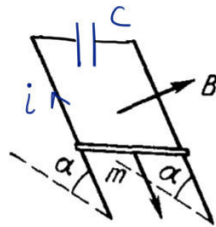
$$mg \sin \alpha = Bil$$

$$mg \sin \alpha = B \left(\frac{Bvl}{R} \right) l$$

$$v = \frac{mgR \sin \alpha}{B^2 l^2}$$

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3.297. The system differs from the one examined in the foregoing problem (Fig. 3.84) by a capacitor of capacitance C replacing the resistance R . Find the acceleration of the connector.

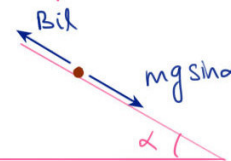


$$Bvl - \frac{q}{C} = 0$$

$$Bvl = \frac{q}{C}$$

differentiate w.r.t t

$$CB l \frac{dv}{dt} = i$$



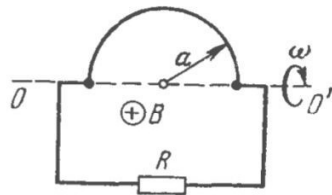
$$mg \sin \alpha - B il = ma$$

$$mg \sin \alpha - B(CB l a)l = ma$$

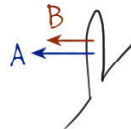
$$a = \frac{mg \sin \alpha}{m + CB^2 l^2}$$

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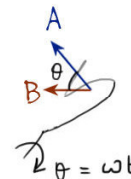
3.298. A wire shaped as a semi-circle of radius a rotates about an axis OO' with an angular velocity ω in a uniform magnetic field of induction B (Fig. 3.85). The rotation axis is perpendicular to the field direction. The total resistance of the circuit is equal to R . Neglecting the magnetic field of the induced current, find the mean amount of thermal power being generated in the loop during a rotation period.



Initially:



When turned by θ



$$\phi = \vec{B} \cdot \vec{A}$$

$$= BA \cos \theta$$

$$= BA \cos \omega t$$

$$|i| = \left| \frac{1}{R} \frac{d\phi}{dt} \right| = \frac{BA\omega \sin \omega t}{R}$$

Instantaneous power: $P = i^2 R$

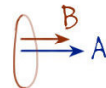
$$= \frac{B^2 A^2 \omega^2}{R} \sin^2 \omega t$$

$$\langle \sin^2 \omega t \rangle = \frac{1}{2} \text{ over a cycle} \Rightarrow P_{av} = \frac{B^2 A^2 \omega^2}{2R} = \frac{B^2 \left(\frac{\pi a^2}{2} \right)^2 \omega^2}{2R} = \frac{B^2 \pi^2 a^4 \omega^2}{8R}$$

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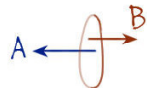
3.299. A small coil is introduced between the poles of an electro-magnet so that its axis coincides with the magnetic field direction. The cross-sectional area of the coil is equal to $S = 3.0 \text{ mm}^2$, the number of turns is $N = 60$. When the coil turns through 180° about its diameter, a ballistic galvanometer connected to the coil indicates a charge $q = 4.5 \mu\text{C}$ flowing through it. Find the magnetic induction magnitude between the poles provided the total resistance of the electric circuit equals $R = 40 \Omega$.

Initially



$$\phi = BA$$

Finally



$$\phi = -BA$$

$$\Rightarrow |\Delta\phi| = 2BA = 2B(NS)$$

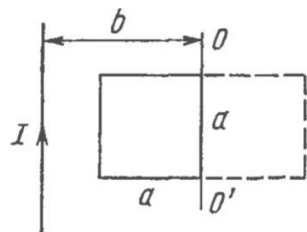
for N turns

$$\text{We know, } \Delta q = \frac{\Delta\phi}{R} = \frac{2BNS}{R}$$

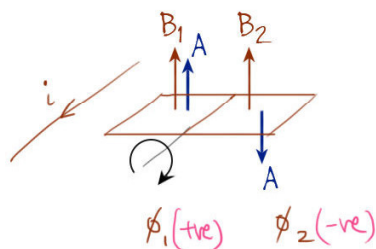
$$\Rightarrow B = \frac{\Delta q R}{2NS}$$

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3.300. A square wire frame with side a and a straight conductor carrying a constant current I are located in the same plane (Fig. 3.86). The inductance and the resistance of the frame are equal to L and R respectively. The frame was turned through 180° about the axis OO' separated from the current-carrying conductor by a distance b . Find the electric charge having flown through the frame.



Initial Final



$$\phi = \phi_{\text{self}} + \phi_{\text{ext}}$$

Due to self induction Due to external fields

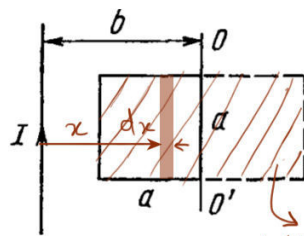
$$\Delta\phi = \Delta\phi_{\text{self}} + \Delta\phi_{\text{ext}}$$

$$= L(i_f - i_i) + \phi_2 - \phi_1$$

$$= \phi_2 - \phi_1$$

$$|\Delta\phi| = |\phi_2| + |\phi_1|$$

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$$|\phi_1| + |\phi_2|$$

$$|\Delta\phi| = |\phi_2| + |\phi_1|$$

$$= \int \vec{B} \cdot d\vec{A} = \int \frac{\mu_0 I}{2\pi x} a dx = \frac{\mu_0 I a}{2\pi} \int_a^b \frac{dx}{x} = \frac{\mu_0 I a}{2\pi} \ln \frac{b+a}{b-a}$$

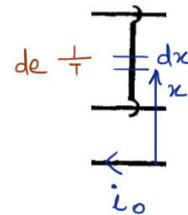
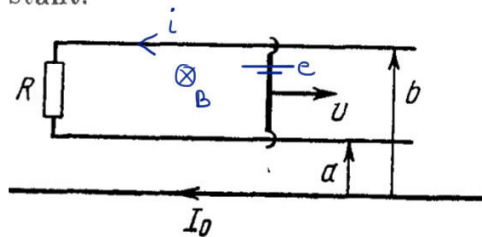
We know,

$$\Delta\phi = \frac{\Delta\phi}{R} = \frac{\mu_0 I a}{2\pi R} \ln \frac{b+a}{b-a}$$

Youtube / Arihant Senior

3.301. A long straight wire carries a current I_0 . At distances a and b from it there are two other wires, parallel to the former one, which are interconnected by a resistance R (Fig. 3.87). A connector slides without friction along the wires with a constant velocity v . Assuming the resistances of the wires, the connector, the sliding contacts, and the self-inductance of the frame to be negligible, find:

- the magnitude and the direction of the current induced in the connector;
- the force required to maintain the connector's velocity constant.

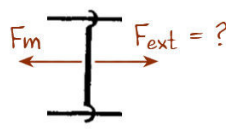


$$de = B_x v dx = \frac{\mu_0 I_0 v}{2\pi x} dx$$

$$e = \int de = \frac{\mu_0 I_0 v}{2\pi} \int_a^b \frac{dx}{x} = \frac{\mu_0 I_0 v}{2\pi} \ln \frac{b}{a}$$

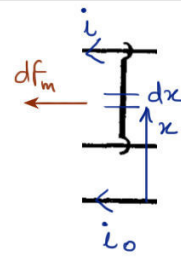
$$i = \frac{e}{R} = \frac{\mu_0 I_0 v}{2\pi R} \ln \frac{b}{a}$$

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Net force on rod is zero

$$F_m = F_{ext}$$



$$df_m = Bx i dx$$

$$= \frac{\mu_0 i_0}{2\pi R} i dx$$

$$\therefore F_{ext} = F_m = \int df_m = \frac{\mu_0 i_0}{2\pi R} i \int_a^b \frac{dx}{x}$$

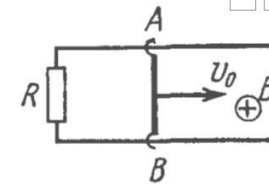
$$= \frac{\mu_0 i_0}{2\pi R} \left(\frac{\mu_0 i_0 v}{2\pi R} \ln \frac{b}{a} \right) \ln \frac{b}{a}$$

$$F_{ext} = \frac{v}{R} \left(\frac{\mu_0 i_0}{2\pi} \ln \frac{b}{a} \right)^2$$

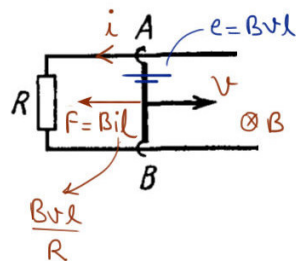
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3.302. A conducting rod AB of mass m slides without friction over two long conducting rails separated by a distance l (Fig. 3.88). At the left end the rails are interconnected by a resistance R . The system is located in a uniform magnetic field perpendicular to the plane of the loop. At the moment $t = 0$ the rod AB starts moving to the right with an initial velocity v_0 . Neglecting the resistances of the rails and the rod AB , as well as the self-inductance, find:

- (a) the distance covered by the rod until it comes to a standstill;
 (b) the amount of heat generated in the resistance R during this process.



After some time



$$F = Bil = B \left(\frac{Blv}{R} \right) l = \frac{B^2 l^2 v}{R}$$

$$\Rightarrow ma = - \frac{B^2 l^2 v}{R}$$

$$v \frac{dv}{dx} = - \frac{B^2 l^2}{mR} v$$

$$\int_{v_0}^0 dv = - \frac{B^2 l^2}{mR} \int_0^{x_0} dx$$

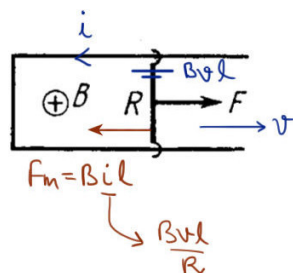
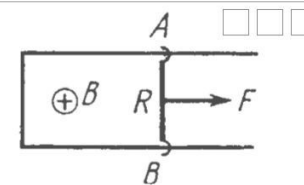
(A) $x_0 = \frac{mv_0 R}{B^2 l^2}$

Magnetic force doesn't do work. Therefore whole KE is converted into heat.

(B) $\therefore \Delta H = \frac{1}{2} m v_0^2$

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3.303. A connector AB can slide without friction along a Π -shaped conductor located in a horizontal plane (Fig. 3.89). The connector has a length l , mass m , and resistance R . The whole system is located in a uniform magnetic field of induction B directed vertically. At the moment $t = 0$ a constant horizontal force F starts acting on the connector shifting it translationwise to the right. Find how the velocity of the connector varies with time t . The inductance of the loop and the resistance of the Π -shaped conductor are assumed to be negligible.



$$F_{\text{net}} = ma = F - B i l$$

$$= F - B \left(\frac{B v l}{R} \right) l$$

$$m \frac{dv}{dt} = F - \frac{B^2 l^2 v}{R}$$

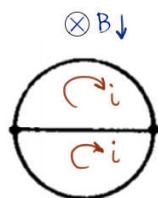
$$\frac{m dv}{F - \frac{B^2 l^2 v}{R}} = dt \Rightarrow \left. -\frac{mR}{B^2 l^2} \ln \left(F - \frac{B^2 l^2 v}{R} \right) \right|_0^v = t$$

$$\Rightarrow v = \frac{F m R}{B^2 l^2} \left(1 - e^{-\frac{B^2 l^2}{m R} t} \right)$$

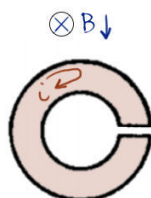
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3.304. Fig. 3.90 illustrates plane figures made of thin conductors which are located in a uniform magnetic field directed away from a reader beyond the plane of the drawing. The magnetic induction starts diminishing. Find how the currents induced in these loops are directed.

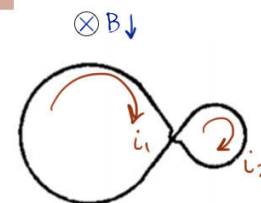
Directions of induced current, such that it increases the field B in the loop should be clockwise.



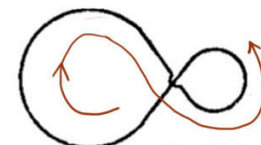
Areas of both loops are same so same current will flow.
Therefore current in connector would be zero



In both circular loops clockwise current will flow.
Current always flows in loop so zero in connector.

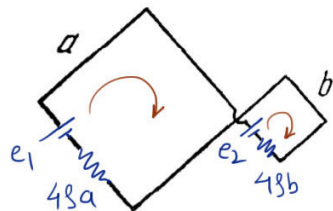
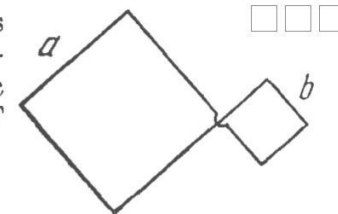


More current will flow in bigger loop because emf induced will be more.
Therefore final current in smaller loop will be reverse of induced current



Youtube/Arihant Senior

3.305. A plane loop shown in Fig. 3.91 is shaped as two squares with sides $a = 20$ cm and $b = 10$ cm and is introduced into a uniform magnetic field at right angles to the loop's plane. The magnetic induction varies with time as $B = B_0 \sin \omega t$, where $B_0 = 10$ mT and $\omega = 100$ s $^{-1}$. Find the amplitude of the current induced in the loop if its resistance per unit length is equal to $\rho = 50$ m Ω /m. The inductance of the loop is to be neglected.



$$\phi_1 = a^2 B_0 \sin \omega t$$

$$\phi_2 = b^2 B_0 \sin \omega t$$

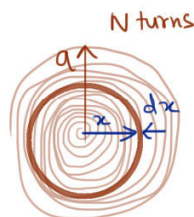
$$i = \frac{e_1 - e_2}{R_1 + R_2} = \frac{\frac{d\phi_1}{dt} - \frac{d\phi_2}{dt}}{R_1 + R_2} = \frac{(a^2 - b^2) B_0 \omega \cos \omega t}{48(a+b)}$$

$$= \frac{(a-b) B_0 \omega \cos \omega t}{48}$$

Amplitude

Youtube / Arihant Senior

3.306. A plane spiral with a great number N of turns wound tightly to one another is located in a uniform magnetic field perpendicular to the spiral's plane. The outside radius of the spiral's turns is equal to a . The magnetic induction varies with time as $B = B_0 \sin \omega t$, where B_0 and ω are constants. Find the amplitude of emf induced in the spiral.



At time t ,

$$d\phi \text{ in small ring} = B(\pi x^2) \underbrace{\frac{dx}{a} \cdot N}_{\text{Turns in small ring}}$$

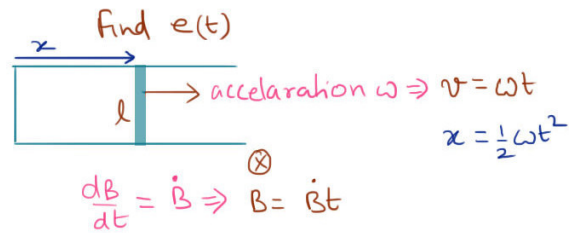
$$\Rightarrow \phi = \int d\phi = \frac{BN\pi}{a} \int_0^a x^2 dx = \frac{BN\pi a^2}{3}$$

$$e = \frac{d\phi}{dt} = \frac{n\pi a^2}{3} \frac{dB}{dt} = \frac{n\pi a^2 B_0 \omega \cos \omega t}{3}$$

Amplitude of e

Youtube / Arihant Senior

3.307. A Π -shaped conductor is located in a uniform magnetic field perpendicular to the plane of the conductor and varying with time at the rate $\dot{B} = 0.10 \text{ T/s}$. A conducting connector starts moving with an acceleration $w = 10 \text{ cm/s}^2$ along the parallel bars of the conductor. The length of the connector is equal to $l = 20 \text{ cm}$. Find the emf induced in the loop $t = 2.0 \text{ s}$ after the beginning of the motion, if at the moment $t = 0$ the loop area and the magnetic induction are equal to zero. The inductance of the loop is to be neglected.



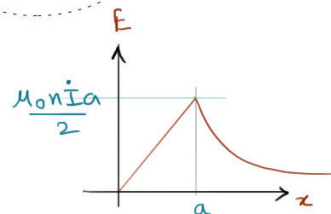
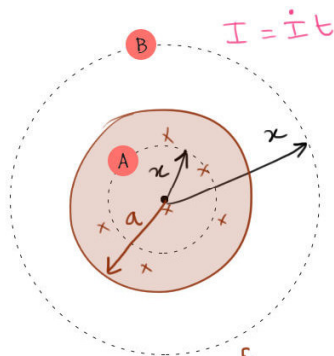
When its at distance x away:

$$\begin{aligned}\phi &= B \cdot A \\ &= B \cdot xl \\ &= (\dot{B}t) \left(\frac{1}{2}wt^2 \right) l \\ &= \frac{\dot{B}wl}{2} t^3\end{aligned}$$

$$e = \frac{d\phi}{dt} = \frac{3\dot{B}wl}{2} t^2 //$$

Youtube / Arihant Senior

3.308. In a long straight solenoid with cross-sectional radius a and number of turns per unit length n a current varies with a constant velocity $\dot{I} \text{ A/s}$. Find the magnitude of the eddy current field strength as a function of the distance r from the solenoid axis. Draw the approximate plot of this function.



A Inside the solenoid:

$$\begin{aligned}\phi &= BA \\ &= (\mu_0 n i) \pi x^2\end{aligned}$$

$$\begin{aligned}e &= \frac{d\phi}{dt} = \mu_0 n \pi x^2 \frac{di}{dt} \\ &= \mu_0 n \pi x^2 \dot{I} = E \cdot dl \\ &= E(2\pi x)\end{aligned}$$

$$\Rightarrow E_{in} = \frac{\mu_0 n \dot{I}}{2} x //$$

B Outside the solenoid:

$$\begin{aligned}\phi &= BA \\ &= (\mu_0 n i) \pi a^2\end{aligned}$$

$$\begin{aligned}e &= \frac{d\phi}{dt} = \mu_0 n \pi a^2 \frac{di}{dt} \\ &= \mu_0 n \pi a^2 \dot{I} = E \cdot dl \\ &= E(2\pi x)\end{aligned}$$

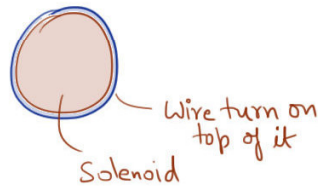
$$\Rightarrow E_{out} = \frac{\mu_0 n \dot{I} a^2}{2x} //$$

Youtube / Arihant Senior

3.309. A long straight solenoid of cross-sectional diameter $d = 5 \text{ cm}$ and with $n = 20$ turns per one cm of its length has a round turn of copper wire of cross-sectional area $S = 1.0 \text{ mm}^2$ tightly put on its winding. Find the current flowing in the turn if the current in the solenoid winding is increased with a constant velocity $\dot{I} = 100 \text{ A/s}$. The inductance of the turn is to be neglected.

We know that induced electric field on surface of a solenoid is given by:

$$E = \frac{r}{2} \frac{dB}{dt} = \frac{(d/2)}{2} \mu_0 n \frac{dI}{dt} = \frac{\mu_0 n d}{4} \dot{I}$$

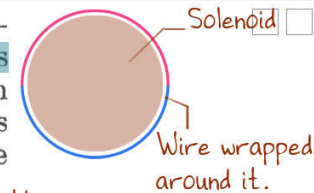


Therefore induced emf $\mathcal{E} = E \cdot \pi d = \pi \frac{\mu_0 n d^2}{4} \dot{I}$

$$i = \frac{\mathcal{E}}{R} = \frac{\mathcal{E}}{(\frac{\rho l}{S})} = \frac{\mathcal{E}}{(\frac{\rho \pi d}{S})} = \frac{\mu_0 n S d}{4 \rho} \dot{I} //$$

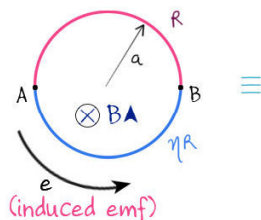
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3.310. A long solenoid of cross-sectional radius a has a thin insulated wire ring tightly put on its winding; one half of the ring has the resistance η times that of the other half. The magnetic induction produced by the solenoid varies with time as $B = bt$, where b is a constant. Find the magnitude of the electric field strength in the ring.

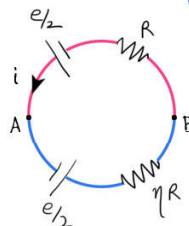


Sol:

He is asking for: 'conservative' electric field
Not induced electric field.



$$\begin{aligned} e &= \frac{d\Phi}{dt} \\ &= \frac{d}{dt} (\pi a^2 B) \\ e &= \pi a^2 b \quad (1) \end{aligned}$$



$$i = \frac{e}{R + \eta R}$$

$$i = \frac{e}{R(\eta + 1)} \quad (2)$$

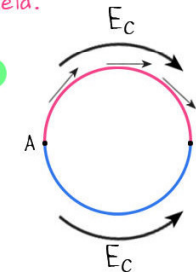
$$V_{AB} = \frac{e}{2} - iR$$

$$V_{AB} = -\frac{e}{2} + i\eta R$$

$$2V_{AB} = iR(\eta - 1) \quad (3)$$

Potential difference is only due to conservative field.

$$\therefore V_{AB} = E_c \pi a \quad (4)$$



From 1, 2, 3, 4:

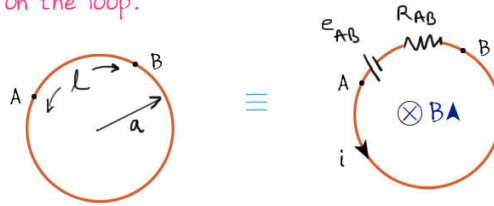
$$2 E_c \pi a = \frac{\pi a^2 b R (\eta - 1)}{R (\eta + 1)} \Rightarrow E_c = \frac{ab}{2} \frac{(\eta - 1)}{(\eta + 1)}$$

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1 Potential difference due to non-conservative field is zero.



For any two general points on the loop.



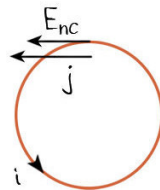
$$V_A - V_B = e_{AB} - iR_{AB}$$

$$= \left(\frac{L}{2\pi a}\right)e - \frac{e}{R} \cdot \left(\frac{L}{2\pi a}\right)R$$

$$\Rightarrow V_{AB} = 0 //$$

Induced emf e
Total Resistance R

2 Direction of E_c in a circular loop is tangential.



We know that direction of non conservative induced emf is tangential.

$$\text{Also: } \vec{j} = \sigma \vec{E}_{\text{total}}$$

$$\Rightarrow \vec{j} = \sigma (\vec{E}_{nc} + \vec{E}_c)$$

Tangential $\Rightarrow$ Must be tangential too. //

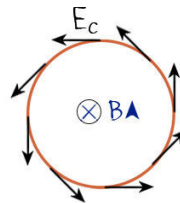
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3 E_c in a uniform conducting ring is zero.



Lets say its non-zero

By symmetry its direction will be in same sense



$\therefore$ over the whole loop:

$$\oint E_c \cdot d\vec{l} \neq 0$$

Which is not possible for a conservative field.

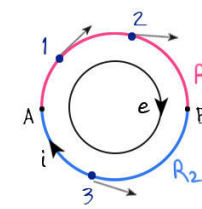
$$E_c = 0 //$$

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4 In a non-uniform conducting ring made of two halves: $E_{c1} = E_{c2} = E_{c3}$

E_{nc} : Non-conservative field due to induced emf.

E_c : Conservative electric field.



Same current flows.

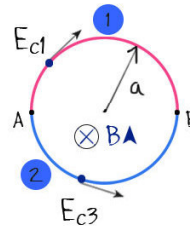
$$\therefore i_1 = i_2$$

$$\sigma E_1 = \sigma E_2$$

$$E_1 = E_2$$

$$E_{nc} + E_{c1} = E_{nc} + E_{c2}$$

$$\Rightarrow E_{c1} = E_{c2} //$$



Path 1

$$V_{AB} = E_{c1} (\pi a)$$

Path 2

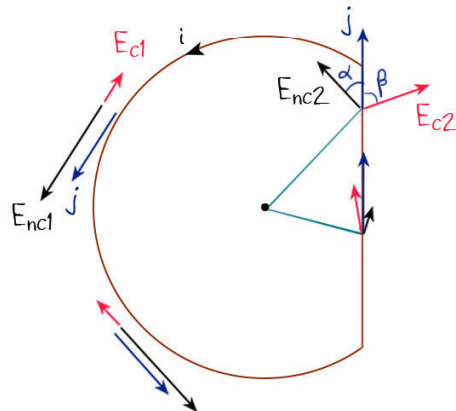
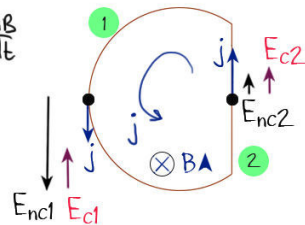
$$V_{AB} = E_{c3} (\pi a)$$

$$\Rightarrow E_{c1} = E_{c3} //$$

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5 E_c in a non-circular conducting loop.

$$E_{nc} = \frac{x}{2} \frac{dB}{dt}$$



A Direction of E_c in 1 and 2:

Take the midpoints of section 1 and 2. j has to be same. But E_{nc1} is larger than E_{nc2} . Therefore E_{c1} must retard current in 1, and E_{c2} must accelerate current in 2. Thus we get the desired directions.

$$\vec{j} = \sigma \vec{E}$$

$$\Rightarrow \textcircled{B} j = \sigma (E_{nc2} \cos \alpha + E_{c2} \cos \beta) \textcircled{1}$$

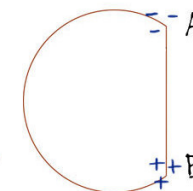
$$= \sigma (E_{nc1} - E_{c1})$$

and

$$E_{nc2} \sin \alpha = E_{c2} \sin \beta \textcircled{2}$$

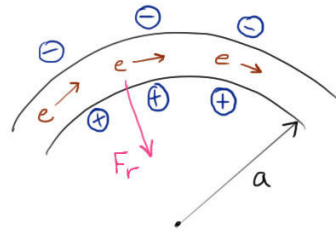
From 1 and 2 we can find the values of E_{c2} and β

ⓐ A is at lowest potential in loop
B is at highest.



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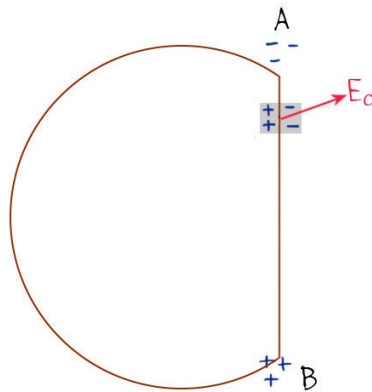
6 Polarization of charge in current carrying wire



It provides centripetal acceleration for electrons to turn in loop.

$$F_c = \frac{m_e v_d^2}{a}$$

F_r is very less as v_d is very small.



Charges will polarise so as to give the final magnitude and direction to E_c .

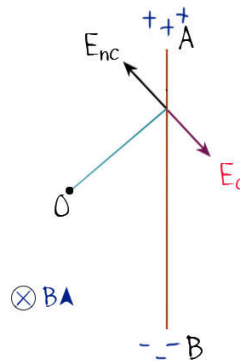
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7 E_c on a piece of wire kept in Time varying magnetic field



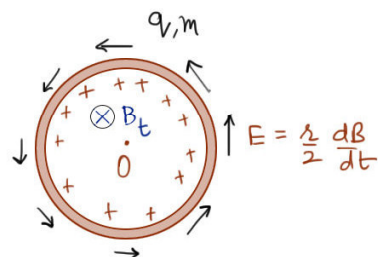
$$\vec{J} = 0$$

$$\Rightarrow \vec{E}_{nc} + \vec{E}_c = 0$$



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3.311. A thin non-conducting ring of mass m carrying a charge q can freely rotate about its axis. At the initial moment the ring was at rest and no magnetic field was present. Then a practically uniform magnetic field was switched on, which was perpendicular to the plane of the ring and increased with time according to a certain law $B(t)$. Find the angular velocity ω of the ring as a function of the induction $B(t)$.



$$\tau_0 = (qE)r = I\alpha$$

$$q \frac{1}{2} \frac{dB}{dt} \cdot r = m r^2 \frac{d\omega}{dt}$$

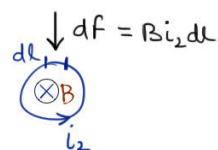
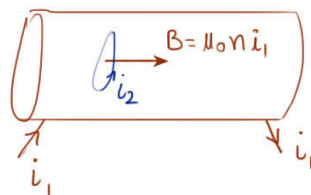
$$\Rightarrow d\omega = \frac{q}{2m} dB$$

$$\Rightarrow \int_{\omega_{t=0}}^{\omega_t} d\omega = \frac{q}{2m} \int_{B_{t=0}}^{B_t} dB$$

$$\Rightarrow \omega_t - 0 = \frac{q}{2m} (B_t - 0) \Rightarrow \omega_t = \frac{q B_t}{2m}$$

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3.312. A thin wire ring of radius a and resistance r is located inside a long solenoid so that their axes coincide. The length of the solenoid is equal to l , its cross-sectional radius, to b . At a certain moment the solenoid was connected to a source of a constant voltage V . The total resistance of the circuit is equal to R . Assuming the inductance of the ring to be negligible, find the maximum value of the radial force acting per unit length of the ring.



$$\frac{dF}{dl} = B i_2 \quad \phi = (\mu_0 n i_1) \pi a^2$$

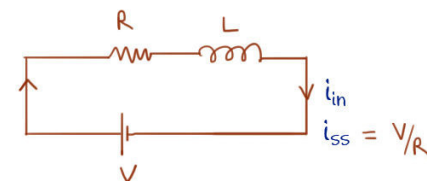
$$B = \mu_0 n i_1 \quad i_2 = \frac{1}{r} \frac{d\phi}{dt}$$

$$= \frac{1}{r} \mu_0 n \pi a^2 \frac{di_1}{dt}$$

$$\Rightarrow \frac{dF}{dl} = (\mu_0 n i_1) \left(\frac{1}{r} \mu_0 n \pi a^2 \frac{di_1}{dt} \right)$$

$$= \frac{\pi \mu_0^2 n^2 a^2}{r} i_1 \frac{di_1}{dt}$$

We know that in a RL circuit:



If initial current is i_{in} and steady state current is i_{ss} .

Then at any time t , current is:

$$i = i_{in} e^{-Rt/L} + i_{ss} (1 - e^{-Rt/L})$$

Current decay Current growth

Youtube / Arihant Senior

Sol. cont..

$$\frac{df}{dL} = \frac{\pi \mu_0 n^2 a^2}{2} i_1 \frac{di_1}{dt} \begin{cases} i_1 = i_{in} e^{-Rt/L} + \frac{V}{R} (1 - e^{-Rt/L}) \\ \frac{di_1}{dt} = \frac{V}{R} \left[\cancel{\left(\frac{R}{L} \right)} e^{-Rt/L} \right] = \frac{V}{L} e^{-Rt/L} \end{cases}$$

$$= \frac{\pi \mu_0 n^2 a^2}{2} \frac{V^2}{RL} \frac{e^{-Rt/L} (1 - e^{-Rt/L})}{x(1-x)}$$

$\frac{1}{2}$  $\downarrow$ max at $x = 1/2$

$\Rightarrow \frac{df}{dL}$ is maximum when $e^{-Rt/L} = 1/2$

(Which is possible as $e^{-Rt/L}$ varies from 1 to 0)

$$\Rightarrow \frac{df}{dL} \Big|_{\max} = \frac{\pi \mu_0 n^2 a^2}{2} \frac{V^2}{RL} \cdot \frac{1}{2} \cdot \frac{1}{2}$$

$\mu_0 n^2 \pi b^2 l$

$$= \frac{\mu_0 a^2 V^2}{4 \pi R l b^2}$$

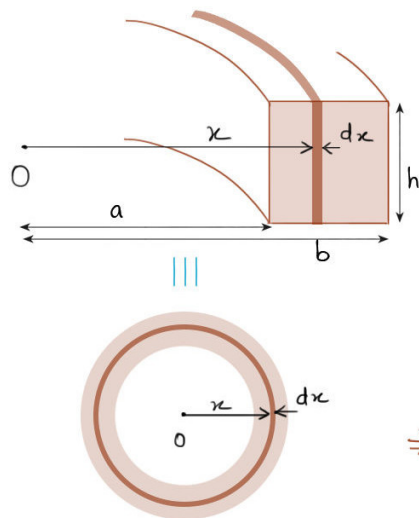
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3.313. A magnetic flux through a stationary loop with a resistance R varies during the time interval τ as $\Phi = at(\tau - t)$. Find the amount of heat generated in the loop during that time. The inductance of the loop is to be neglected.

$$dH = i^2 R dt \quad \Rightarrow \quad H = \int_0^\tau \left(\frac{a\tau - 2at}{R} \right)^2 R dt = \frac{a^2 \tau^3}{3R}$$

$i = \frac{1}{R} \frac{d\Phi}{dt} = \frac{1}{R} (a\tau - 2at)$

3.314. In the middle of a long solenoid there is a coaxial ring of square cross-section, made of conducting material with resistivity ρ . The thickness of the ring is equal to h , its inside and outside radii are equal to a and b respectively. Find the current induced in the ring if the magnetic induction produced by the solenoid varies with time as $B = \beta t$, where β is a constant. The inductance of the ring is to be neglected.



Resistance dR in elemental ring: $\frac{\rho l}{A} = \frac{\rho 2\pi x}{h dx}$

$di = \frac{e}{dR} \frac{d\phi}{dt} = \frac{e}{dR} \frac{d(\beta t \pi x^2)}{dt} = \beta \pi x^2$

$= \frac{\beta \pi x^2}{\frac{\rho 2\pi x}{h dx}} = \frac{\beta h x^2}{2\rho}$

$\Rightarrow i = \int di = \frac{\beta h}{2\rho} \int_a^b x dx = \frac{\beta h (b^2 - a^2)}{4\rho}$

Youtube / Arihant Senior

3.315. How many metres of a thin wire are required to manufacture a solenoid of length $l_0 = 100$ cm and inductance $L = 1.0$ mH if the solenoid's cross-sectional diameter is considerably less than its length?

Lets say length l is needed and radius of solenoid is a .

$\therefore$ Total turns $N = \frac{l}{2\pi a}$

$L = \mu_0 n^2 V$

$L = \mu_0 n^2 (\pi a^2) l_0$

$= \mu_0 \frac{N^2}{l_0} \pi a^2$

$L = \frac{\mu_0 l^2}{4\pi l_0} \Rightarrow l = \sqrt{\frac{4\pi l_0 L}{\mu_0}}$

Youtube / Arihant Senior

3.316. Find the inductance of a solenoid of length l_0 whose winding is made of copper wire of mass m . The winding resistance is equal to R . The solenoid diameter is considerably less than its length.

Wire needed:



In 3.315 we got: $l = \sqrt{\frac{4\pi l_0 L}{\mu_0}}$ (1)

Now we need to introduce R and m in this.

$R = \frac{\rho l}{A}$ (Resistivity)

$m = \rho_0 A l$ (Density)

} multiply

$mR = \rho \rho_0 l^2$ (2)

$1, 2 \Rightarrow \frac{mR}{\rho \rho_0} = \frac{4\pi l_0 L}{\mu_0} \Rightarrow L = \frac{\mu_0 m R}{4\pi \rho \rho_0}$ //

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3.317. A coil of inductance $L = 300 \text{ mH}$ and resistance $R = 140 \text{ m}\Omega$ is connected to a constant voltage source. How soon will the coil current reach $\eta = 50\%$ of the steady-state value?

We know in current growth:

$i = i_0 (1 - e^{-Rt/L})$

When $i = \eta i_0$

$\eta i_0 = i_0 (1 - e^{-Rt/L})$

$e^{-Rt/L} = 1 - \eta$

$-\frac{Rt}{L} = \ln(1 - \eta)$

$\Rightarrow t = -\frac{L}{R} \ln(1 - \eta)$ //

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3.318. Calculate the time constant τ of a straight solenoid of length $l_0 = 1.0$ m having a single-layer winding of copper wire whose total mass is equal to $m = 1.0$ kg. The cross-sectional diameter of the solenoid is assumed to be considerably less than its length.

Note. The time constant τ is the ratio L/R , where L is inductance and R is active resistance.

$$\frac{L}{R} = ?$$

From 3.315

$$L = \sqrt{\frac{4\pi l_0 L}{\mu_0}} \quad (1)$$

Now we need to introduce R and m in this.

$$R = \frac{\rho l}{A} \quad \text{Resistivity}$$

$$m = \rho_0 A l \quad \text{Density}$$

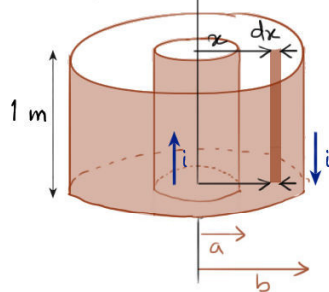
multiply

$$mR = \rho \rho_0 l^2 \quad (2)$$

$$1, 2 \Rightarrow \frac{mR}{\rho \rho_0} = \frac{4\pi l_0 L}{\mu_0} \Rightarrow \frac{L}{R} = \frac{\mu_0 m}{4\pi \rho \rho_0 l_0}$$

Youtube / Arihant Senior

3.319. Find the inductance of a unit length of a cable consisting of two thin-walled coaxial metallic cylinders if the radius of the outside cylinder is $\eta = 3.6$ times that of the inside one. The permeability of a medium between the cylinders is assumed to be equal to unity.



For a small area element between the two cables flux passing through is:

$$d\phi = \left(\frac{\mu_0 i}{2\pi x} \right) (1) dx$$

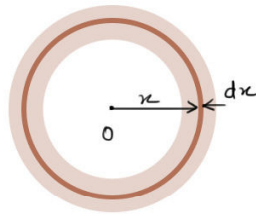
$$\phi = \frac{\mu_0 i}{2\pi} \int_a^b \frac{dx}{x} = \frac{\mu_0 i}{2\pi} \ln \frac{b}{a}$$

$$\phi = Li$$

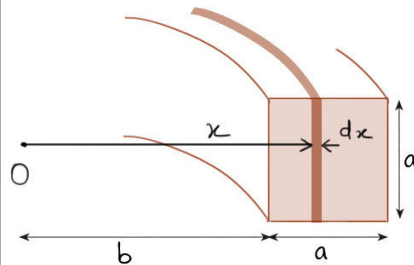
$$\Rightarrow L = \frac{\mu_0}{2\pi} \ln \frac{b}{a} = \frac{\mu_0}{2\pi} \ln \eta$$

Youtube / Arihant Senior

3.320. Calculate the inductance of a doughnut solenoid whose inside radius is equal to b and cross-section has the form of a square with side a . The solenoid winding consists of N turns. The space inside the solenoid is filled up with uniform paramagnetic having permeability μ .



|||



Finding B at distance x from center

$$\oint B \cdot d\mathbf{l} = \mu_0 i N$$

$$B(2\pi x) = \mu_0 i N \Rightarrow B = \frac{\mu_0 i N}{2\pi x}$$

Small Flux through elemental ring (through N loops)

$$d\phi = B \cdot dA = B \cdot (a dx \cdot N)$$

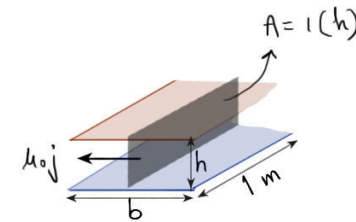
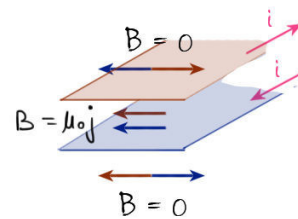
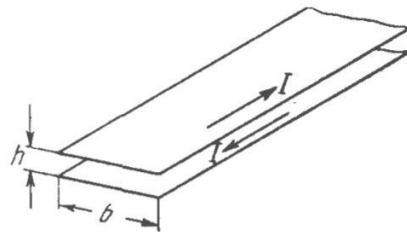
Total flux:

$$\phi = \int d\phi = \frac{\mu_0 i N}{2\pi} \cdot a N \int_b^{b+a} \frac{dx}{x} = \frac{\mu_0 a N^2 i}{2\pi} \ln \frac{b+a}{b}$$

$$\phi = Li \Rightarrow L = \frac{\mu_0 a N^2}{2\pi} \ln \frac{b+a}{b}$$

Youtube / Arihant / Senior

3.321. Calculate the inductance of a unit length of a double tape line (Fig. 3.92) if the tapes are separated by a distance h which is considerably less than their width b , namely, $b/h = 50$.

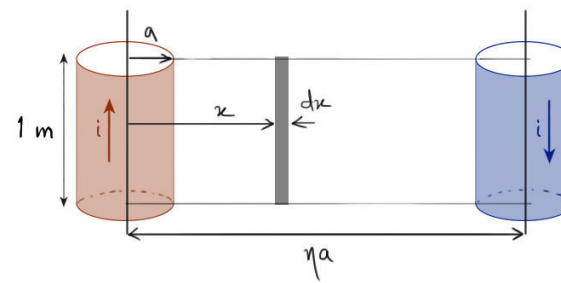


$$\begin{aligned} \phi &= BA \\ &= \mu_0 i \left(\frac{1}{b} \right) h \\ &= \mu_0 \left(\frac{i}{b} \right) h \end{aligned}$$

$$\phi = Li \Rightarrow L = \frac{\mu_0 h}{b}$$

Youtube / Arihant Senior

3.322. Find the inductance of a unit length of a double line if the radius of each wire is η times less than the distance between the axes of the wires. The field inside the wires is to be neglected, the permeability is assumed to be equal to unity throughout, and $\eta \gg 1$.



Flux due to one wire: $d\phi = \frac{\mu_0 i}{2\pi x} dx$

$$\phi = \int_a^{\eta a} d\phi = \frac{\mu_0 i}{2\pi} \ln \eta$$

Flux due to both wires by superposition:

$$\phi_{\text{net}} = 2 \frac{\mu_0 i}{2\pi} \ln \eta \Rightarrow \phi = Li \Rightarrow L = \frac{\mu_0}{\pi} \ln \eta //$$

Youtube / Arihant Senior

3.323. A superconducting round ring of radius a and inductance L was located in a uniform magnetic field of induction B . The ring plane was parallel to the vector B , and the current in the ring was equal to zero. Then the ring was turned through 90° so that its plane became perpendicular to the field. Find:

- the current induced in the ring after the turn;
- the work performed during the turn.

A In a superconducting ring: $\Delta\phi = 0$

(Otherwise emf $\frac{d\phi}{dt}$ will develop and current will be infinity)

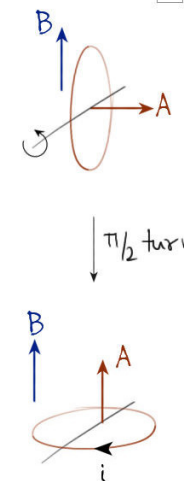
$$\phi_i = \phi_f$$

$$\Rightarrow 0 = B \cdot A - Li$$

$$\Rightarrow i = \frac{B(\pi a^2)}{L} //$$

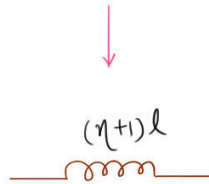
B Since $R=0$, no heat losses and loop comes to rest again.

$$W_{\text{ext}} = \Delta U = \frac{1}{2} L (i_f^2 - i_i^2) = \frac{1}{2} L i_f^2 = \frac{1}{2} L \frac{B^2 \pi^2 a^4}{L^2} = \frac{B^2 \pi^2 a^4}{2L} //$$



Youtube / Arihant Senior

3.324. A current $I_0 = 1.9$ A flows in a long closed solenoid. The wire it is wound of is in a superconducting state. Find the current flowing in the solenoid when the length of the solenoid is increased by $\eta = 5\%$.



In a superconducting solenoid: $\Delta\phi = 0$

$$\therefore \phi_i = \phi_f$$

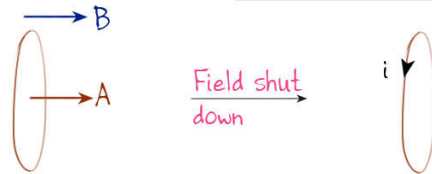
$$L i_0 = L_f i$$

$$\frac{\mu_0 n^2 A}{l_1} i_0 = \frac{\mu_0 n^2 A}{l_2} i$$

$$i = \frac{l_2}{l_1} i_0 = (1+\eta) i_0 //$$

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3.325. A ring of radius $a = 50$ mm made of thin wire of radius $b = 1.0$ mm was located in a uniform magnetic field with induction $B = 0.50$ mT so that the ring plane was perpendicular to the vector $\mathbf{B}$. Then the ring was cooled down to a superconducting state, and the magnetic field was switched off. Find the ring current after that. Note that the inductance of a thin ring along which the surface current flows is equal to $L = \mu_0 a \left(\ln \frac{8a}{b} - 2 \right)$.



In a superconducting ring: $\Delta\phi = 0$

$$\phi_1 = \phi_2$$

$$BA = Li$$

$$\Rightarrow i = \frac{BA}{L} = \frac{B\pi a^2}{L} = \frac{B\pi a^2}{\mu_0 a \left(\ln \frac{8a}{b} - 2 \right)} = \frac{\pi a B}{\mu_0 \left(\ln \frac{8a}{b} - 2 \right)} //$$

Youtube / Arihant Senior

3.326. A closed circuit consists of a source of constant emf $\mathcal{E}$ and a choke coil of inductance L connected in series. The active resistance of the whole circuit is equal to R . At the moment $t = 0$ the choke coil inductance was decreased abruptly η times. Find the current in the circuit as a function of time t .

Instruction. During a stepwise change of inductance the total magnetic flux (flux linkage) remains constant.

In a superconducting inductor flux never changes: $\Delta\phi = 0$
(Otherwise emf $\frac{d\phi}{dt}$ will develop and current will be infinity)

In a normal inductor flux changes but not immediately.

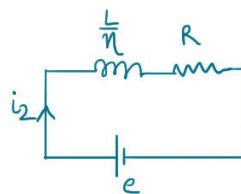
(Otherwise emf $\frac{d\phi}{dt} = \frac{\Delta\phi}{\Delta t} \rightarrow \text{finite}$ will be infinity and hence current too will be infinity)

Sol:

$$\phi_i = \phi_f$$

$$L_1 i_1 = L_2 i_2$$

$$L\left(\frac{e}{R}\right) = \frac{L}{\eta} i_2 \Rightarrow i_2 = \frac{\eta e}{R}$$



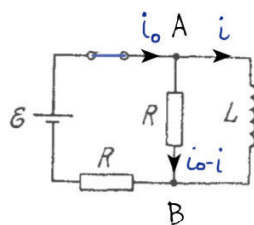
$$i_t = i_{in} e^{-Rt/L} + i_{ss} (1 - e^{-Rt/L})$$

$$\Rightarrow i = \frac{\eta e}{R} e^{-\frac{\eta R t}{L}} + \frac{e}{R} (1 - e^{-\frac{\eta R t}{L}})$$

$$= \frac{e}{R} \left[1 + (\eta - 1) e^{-\frac{\eta R t}{L}} \right]$$

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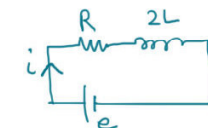
3.327. Find the time dependence of the current flowing through the inductance L of the circuit shown in Fig. 3.93 after the switch Sw is shorted at the moment $t = 0$.



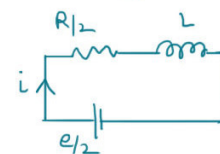
$$V_{AB} = (i_0 - i)R = L \frac{di}{dt} = e - i_0 R$$

eliminating i_0

$$e - iR - 2L \frac{di}{dt} = 0 \approx$$



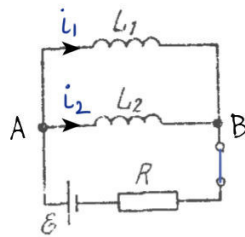
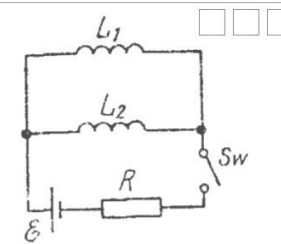
or



$$\Rightarrow i = \frac{e}{R} (1 - e^{-Rt/2L}) //$$

Youtube / Arihant Senior

3.328. In the circuit shown in Fig. 3.94 an emf $\mathcal{E}$, a resistance R , and coil inductances L_1 and L_2 are known. The internal resistance of the source and the coil resistances are negligible. Find the steady-state currents in the coils after the switch Sw was shorted.



At steady state:

$$L_1 i_1 = L_2 i_2 \quad (1) \quad \left\{ \begin{array}{l} \because V_{AB} = L_1 \frac{di_1}{dt} = L_2 \frac{di_2}{dt} \end{array} \right.$$

$$i_1 + i_2 = \frac{\mathcal{E}}{R} \quad (2)$$

$$i_1 \Rightarrow i_1 = \left(\frac{L_2}{L_1 + L_2} \right) \frac{\mathcal{E}}{R}$$

$$i_2 = \left(\frac{L_1}{L_1 + L_2} \right) \frac{\mathcal{E}}{R}$$

$$\Rightarrow L_1 di_1 - L_2 di_2 = 0$$

$$\Rightarrow d(L_1 i_1 - L_2 i_2) = 0$$

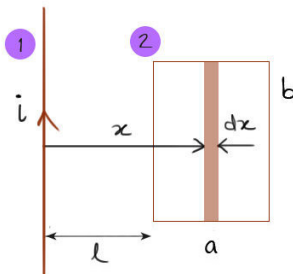
$$\Rightarrow L_1 i_1 - L_2 i_2 = \text{constant}$$

$$\text{at } t=0, i_1 = i_2 = 0 \Rightarrow \text{constant} = 0$$

$$\therefore L_1 i_1 = L_2 i_2$$

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3.329. Calculate the mutual inductance of a long straight wire and a rectangular frame with sides a and b . The frame and the wire lie in the same plane, with the side b being closest to the wire, separated by a distance l from it and oriented parallel to it.

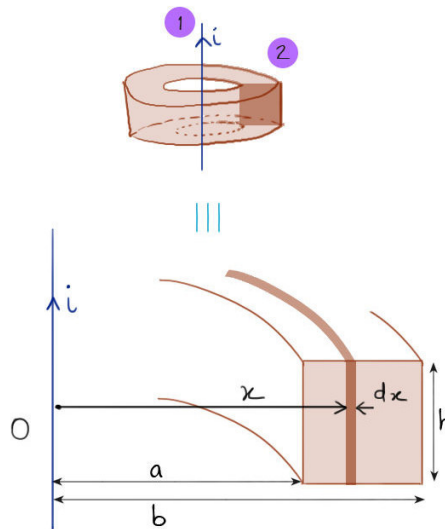


By definition of mutual induction,
 ϕ through 2 due to 1 = $M i$

$$\phi = \int d\phi = \int_l^{l+a} \left(\frac{\mu_0 i}{2\pi x} \right) b dx = \underbrace{\frac{\mu_0 b}{2\pi} \ln \frac{l+a}{l}}_M i$$

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3.330. Determine the mutual inductance of a doughnut coil and an infinite straight wire passing along its axis. The coil has a rectangular cross-section, its inside radius is equal to a and the outside one, to b . The length of the doughnut's cross-sectional side parallel to the wire is equal to h . The coil has N turns. The system is located in a uniform magnetic with permeability μ .



By definition of mutual induction,
 ϕ through 2 due to 1 = $M i$

$$d\phi = B \cdot dS = \left(\frac{\mu \mu_0 i}{2\pi x} \right) h dx \quad (\text{through one coil of 2})$$

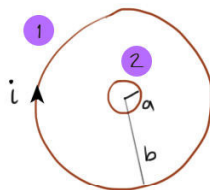
$$= \left(\frac{\mu \mu_0 i}{2\pi x} \right) h dx \cdot N \quad (\text{through } N \text{ coils of 2})$$

$$\phi = \int d\phi = \frac{\mu \mu_0 i h N}{2\pi} \int_a^b \frac{dx}{x} = \frac{\mu \mu_0 h N \ln \frac{b}{a}}{2\pi} i$$

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3.331. Two thin concentric wires shaped as circles with radii a and b lie in the same plane. Allowing for $a \ll b$, find:

- their mutual inductance;
- the magnetic flux through the surface enclosed by the outside wire, when the inside wire carries a current I .

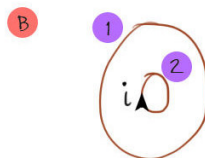


A By definition of mutual induction,
 ϕ through 2 due to current i in 1 = $M i$

$$\phi = B A_2$$

$$= \frac{\mu_0 i}{2b} (\pi a^2) = \frac{\mu_0 \pi a^2}{2b} i$$

Assumed uniform at center as $a \ll b$



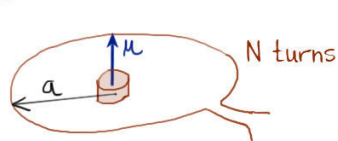
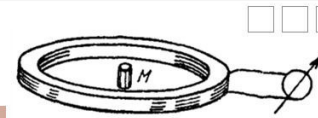
$$\phi \text{ through 1 due to 2} = M i$$

$$= \frac{\mu_0 \pi a^2}{2b} i$$

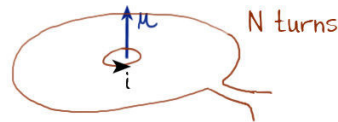
Note: Here without the reciprocity of magnetic inductance, calculation of flux will be very difficult.

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3.332. A small cylindrical magnet M (Fig. 3.95) is placed in the centre of a thin coil of radius a consisting of N turns. The coil is connected to a ballistic galvanometer. The active resistance of the whole circuit is equal to R . Find the magnetic moment of the magnet if its removal from the coil results in a charge q flowing through the galvanometer.



Replacing the magnet with a current carrying loop of radius x , current i



$$\therefore \mu = i \pi x^2$$

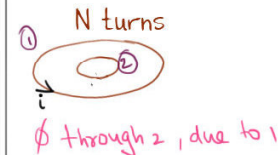
When loop is present $\phi_i = \mu i$

When loop is removed: $\phi_f = 0$

$$\text{We know that, } \Delta q = \frac{\Delta \phi}{R} = \frac{\mu i}{R}$$

$$\Rightarrow \Delta q = \frac{\mu_0 N}{2aR} (\pi x^2 i) \Rightarrow \mu = \frac{q \cdot 2aR}{\mu_0 N}$$

Calculating M

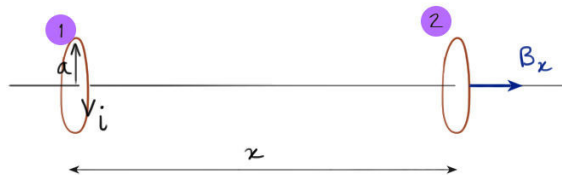


$$= \frac{\mu_0 (i \times N)}{2a} \cdot \pi x^2$$

$$\phi = \frac{\mu_0 N}{2a} \pi x^2 i$$

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3.333. Find the approximate formula expressing the mutual inductance of two thin coaxial loops of the same radius a if their centres are separated by a distance l , with $l \gg a$.



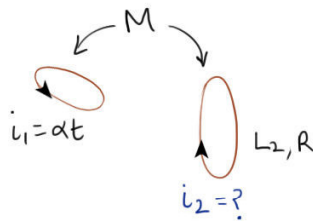
By definition of mutual induction,
 ϕ through 2 due to current i in 1 = μi

$$\text{We know that } B_x \text{ due to a loop axially is: } B_x = \frac{\mu_0}{4\pi} \frac{2\mu}{x^3}$$

$$\phi_2 = B_x \cdot A = \frac{\mu_0}{4\pi} \frac{2\mu}{x^3} \cdot \pi a^2 = \frac{\mu_0}{2} \frac{\pi a^4}{x^3} i$$

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3.334. There are two stationary loops with mutual inductance M . The current in one of the loops starts to be varied as $I_1 = \alpha t$, where α is a constant, t is time. Find the time dependence $I_2(t)$ of the current in the other loop whose inductance is L_2 and resistance R .

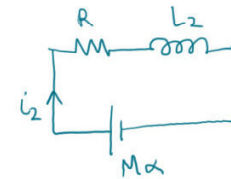


Flux through coil 2 due to current in coil 1: $\phi = M i_1$

$\therefore$ emf induced in coil 2 due to coil 1: $|e| = \frac{d\phi}{dt} = M \frac{di_1}{dt} = M\alpha$
A constant!

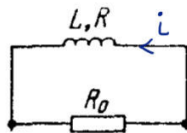
Situation is similar to an external battery of same emf. Which gives:

$$i_2 = \frac{M\alpha}{R} (1 - e^{-Rt/L_2}) //$$



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3.335. A coil of inductance $L = 2.0 \mu\text{H}$ and resistance $R = 1.0 \Omega$ is connected to a source of constant emf $\mathcal{E} = 3.0 \text{ V}$ (Fig. 3.96). A resistance $R_0 = 2.0 \Omega$ is connected in parallel with the coil. Find the amount of heat generated in the coil after the switch Sw is disconnected. The internal resistance of the source is negligible.

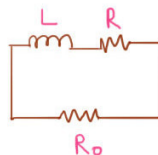


When switch is turned off, current in L doesn't drop immediately. Its value remains the same. So initial current in new circuit:

$$i = \frac{\mathcal{E}}{R}$$

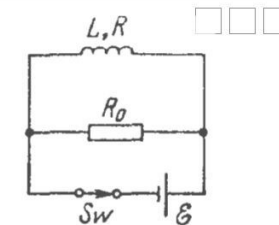
Eventually the current will be zero. So all the energy stored in inductor is turned into heat.

$\therefore$ In both resistors, $H_{\text{total}} = \frac{1}{2} Li^2$



$H \propto \text{Resistance}$ (As same current flows in both resistors)

$$\therefore H_R = \left(\frac{R}{R+R_0} \right) \frac{1}{2} Li^2 = \frac{R}{R+R_0} \cdot \frac{1}{2} L \frac{\mathcal{E}^2}{R^2} = \frac{L\mathcal{E}^2}{2(R+R_0)R} //$$



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3.336. An iron core supports $N = 500$ turns. Find the magnetic field energy if a current $I = 2.0$ A produces a magnetic flux across the core's cross-section equal to $\Phi = 1.0$ mWb.



ϕ = Flux through one turn

⇒ Flux through N turns = $N\phi = Li$

$$\text{Magnetic field energy} = \frac{1}{2} Li^2 = \frac{1}{2} \frac{N\phi}{i} \cdot i^2 = \frac{N\phi i}{2}$$

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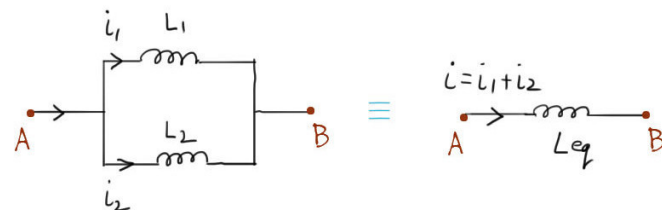
3.343. Two identical coils, each of inductance L , are interconnected (a) in series, (b) in parallel. Assuming the mutual inductance of the coils to be negligible, find the inductance of the system in both cases.

A



$$V_{AB} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} = L_{eq} \frac{di}{dt} \Rightarrow L_1 + L_2 = L_{eq} \quad (\text{Inductors in series})$$

B



$$V_{AB} = L_1 \frac{di_1}{dt} = L_2 \frac{di_2}{dt} = L_{eq} \frac{d(i_1 + i_2)}{dt} = K \Rightarrow \frac{K}{L_1} + \frac{K}{L_2} = \frac{di_1}{dt} + \frac{di_2}{dt} = \frac{d(i_1 + i_2)}{dt} = \frac{K}{L_{eq}}$$

(Let's say) $\Rightarrow \frac{1}{L_1} + \frac{1}{L_2} = \frac{1}{L_{eq}} \quad (\text{Inductors in parallel})$

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3.344. Two solenoids of equal length and almost equal cross-sectional area are fully inserted into one another. Find their mutual inductance if their inductances are equal to L_1 and L_2 .

Let the two coils have number of turns N_1 and N_2

If i_1 current flows through L_1 then,

Total flux through N_1 turns of L_1 : $\phi_1 = L_1 i_1$

Flux through cross section of L_1 : $\frac{\phi_1}{N_1} = \frac{L_1 i_1}{N_1}$ = Flux through cross section (single turn that is) of L_2

$\therefore$ Total flux through N_2 turns of L_2 : $\phi_2 = \phi_{cs} \cdot N_2 = \frac{L_1 i_1}{N_1} \cdot N_2$

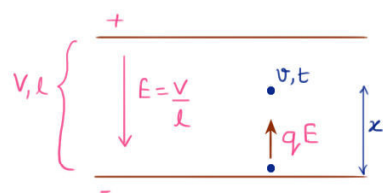
$$\Rightarrow \phi_2 = \frac{L_1 N_2}{N_1} i_1$$

$$\left(\begin{array}{l} L_1 \propto N_1^2 \\ L_2 \propto N_2^2 \\ \Rightarrow \frac{N_2}{N_1} = \sqrt{\frac{L_2}{L_1}} \end{array} \right) \Rightarrow \phi_2 = L_1 \sqrt{\frac{L_2}{L_1}} i_1 = \underbrace{\sqrt{L_1 L_2}}_{\text{(Mutual inductance)}} i_1$$

(Mutual inductance)

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3.372. At the moment $t = 0$ an electron leaves one plate of a parallel-plate capacitor with a negligible velocity. An accelerating voltage, varying as $V = at$, where $a = 100$ V/s, is applied between the plates. The separation between the plates is $l = 5.0$ cm. What is the velocity of the electron at the moment it reaches the opposite plate?



$$F = ma = qE = \frac{qV}{l} = \frac{qat}{l}$$

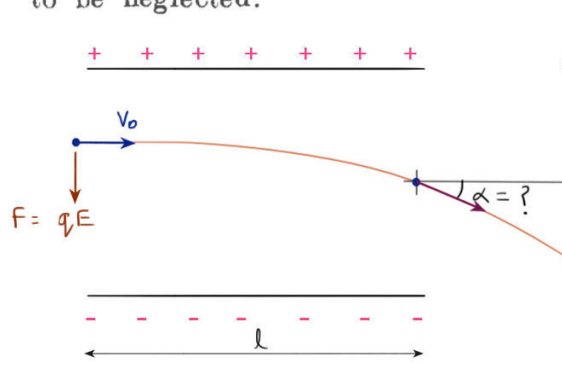
$$\Rightarrow m \frac{dv}{dt} = \frac{qa}{l} t \Rightarrow v = \int_0^v dv = \frac{qa}{ml} \int_0^t t dt = \frac{k}{2} t^2$$

$$\Rightarrow \frac{dx}{dt} = \frac{kt^2}{2} \Rightarrow l = \int_0^l dx = \int_0^{t_0} \frac{kt^2}{2} dt = \frac{kt_0^3}{6}$$

$$\therefore v_{t_0} = \frac{kt_0^2}{2} = \frac{k}{2} \left(\frac{6l}{k} \right)^{2/3} = \left[\frac{k^3}{2^3} \cdot \frac{2^2 \cdot 3^2 \cdot l^2}{k^2} \right]^{1/3} = \left(\frac{9kl^2}{2} \right)^{1/3} = \left(\frac{9qa}{2m} \right)^{1/3}$$

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3.373. A proton accelerated by a potential difference V gets into the uniform electric field of a parallel-plate capacitor whose plates extend over a length l in the motion direction. The field strength varies with time as $E = at$, where a is a constant. Assuming the proton to be non-relativistic, find the angle between the motion directions of the proton before and after its flight through the capacitor; the proton gets in the field at the moment $t = 0$. The edge effects are to be neglected.



Handwritten equations for problem 3.373:

$$\alpha = \tan^{-1} \frac{v_y}{v_x} \quad (1)$$

$$qV = \frac{1}{2} m v_0^2 \quad (2)$$

$$F_y = qE$$

$$m \frac{dv_y}{dt} = qat$$

$$v_y = \frac{qat^2}{2m}$$

$$t \rightarrow \frac{l}{v_0}$$

$$\Rightarrow v_y = \frac{qa l^2}{2m v_0^2} \quad (3)$$

$$1, 2, 3 \Rightarrow \alpha = \tan^{-1} \left(\frac{al^2}{4} \sqrt{\frac{m}{2qV^3}} \right)$$

Youtube / Arihant Senior

3.374. A particle with specific charge q/m moves rectilinearly due to an electric field $E = E_0 - ax$, where a is a positive constant, x is the distance from the point where the particle was initially at rest. Find:

- (a) the distance covered by the particle till the moment it came to a standstill;
 (b) the acceleration of the particle at that moment.

$$F = ma_c = qE$$

$$\Rightarrow m \cdot v \frac{dv}{dx} = q(E_0 - ax)$$

$$\Rightarrow m \int_0^v v dv = \int_0^{x_0} q(E_0 - ax) dx$$

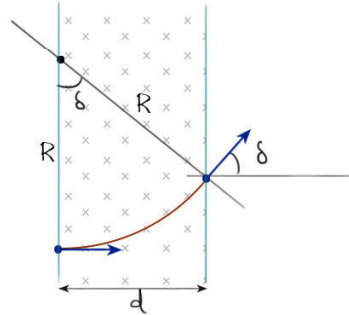
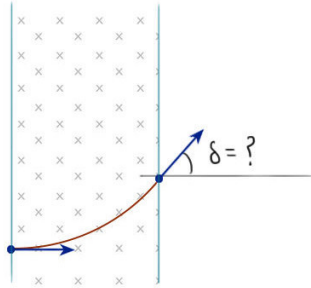
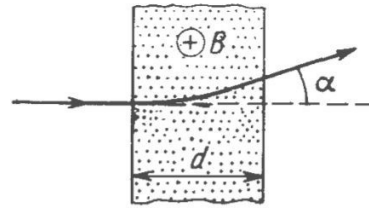
$$0 = q \left(E_0 x - \frac{ax^2}{2} \right)$$

$$x = 0, \frac{2E_0}{a} //$$

$$a_c = \frac{qE}{m} = \frac{q(E_0 - ax)}{m} = \frac{-qE_0}{m} //$$

Youtube / Arihant Senior

3.378. A proton accelerated by a potential difference $V = 500 \text{ kV}$ flies through a uniform transverse magnetic field with induction $B = 0.51 \text{ T}$. The field occupies a region of space $d = 10 \text{ cm}$ in thickness (Fig. 3.99). Find the angle α through which the proton deviates from the initial direction of its motion.



$$\delta = \sin^{-1} d/R$$

$$= \sin^{-1} \frac{d}{(mv/qB)}$$

$$= \sin^{-1} \frac{dqB}{mv}$$

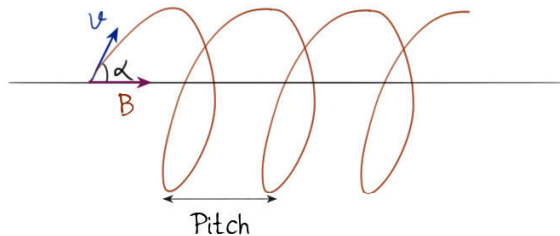
$$= \sin^{-1} dB \sqrt{\frac{q}{2mV}}$$

$$qV = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2qV}{m}}$$

Youtube/Arihant Senior

3.382. An electron accelerated by a potential difference $V = 1.0 \text{ kV}$ moves in a uniform magnetic field at an angle $\alpha = 30^\circ$ to the vector B whose modulus is $B = 29 \text{ mT}$. Find the pitch of the helical trajectory of the electron.

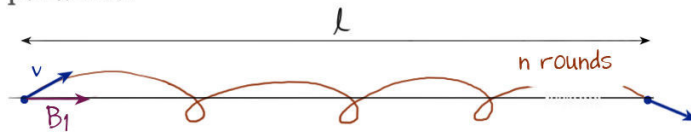


$$\text{Pitch} = v \cos \alpha \cdot T$$

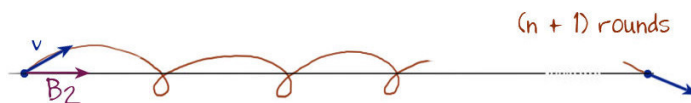
$$= \sqrt{\frac{2qV}{m}} \cos \alpha \cdot \frac{2\pi m}{qB}$$

$$= \frac{2\pi}{B} \sqrt{\frac{2mV}{q}} \cos \alpha //$$

3.383. A slightly divergent beam of non-relativistic charged particles accelerated by a potential difference V propagates from a point A along the axis of a straight solenoid. The beam is brought into focus at a distance l from the point A at two successive values of magnetic induction B_1 and B_2 . Find the specific charge q/m of the particles.



$$l = v_x (n T_1) \\ = v_x \left(n \cdot \frac{2\pi m}{q B_1} \right) \quad (1)$$



$$l = v_x (n+1) T_2 \\ = v_x (n+1) \left(\frac{2\pi m}{q B_2} \right) \quad (2)$$

Also, $\frac{1}{2} m v^2 = qV$
 $\Rightarrow v_x = \sqrt{\frac{2qV}{m}} \quad (3)$

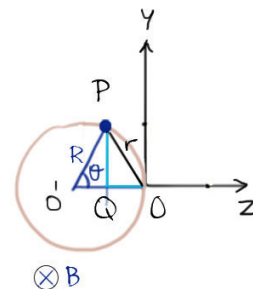
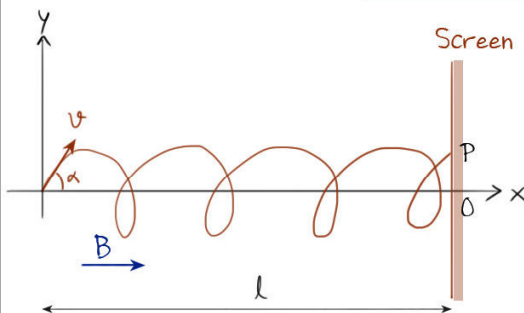
(Since the beam is only slightly divergent $v \approx v_x$)

Eliminating v_x and n from 1 and 2:

$$\frac{q}{m} = \frac{8\pi^2 V}{l^2 (B_2 - B_1)^2}$$

Youtube / Arihant Senior

3.384. A non-relativistic electron originates at a point A lying on the axis of a straight solenoid and moves with velocity v at an angle α to the axis. The magnetic induction of the field is equal to B . Find the distance r from the axis to the point on the screen into which the electron strikes. The screen is oriented at right angles to the axis and is located at a distance l from the point A .



$$L = \sqrt{PQ^2 + OQ^2} \\ = \sqrt{(R \sin \theta)^2 + R^2 (1 - \cos \theta)^2}$$

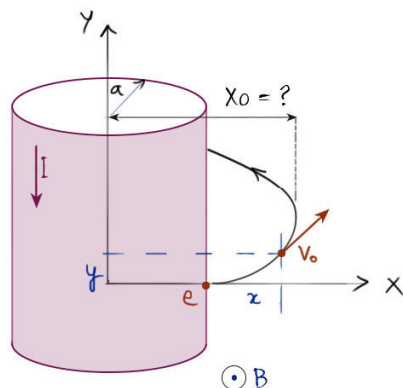
where, $\theta = \omega t$

$$= 2R \sin \frac{\omega t}{2} \\ \frac{mv}{qB} \leftarrow \omega = \frac{qB}{m} \quad v \cos \theta = l$$

$$\Rightarrow r = \frac{2mv}{qB} \sin \frac{qB}{m} \cdot \frac{l}{2}$$

Youtube / Arihant Senior

3.385. From the surface of a round wire of radius a carrying a direct current I an electron escapes with a velocity v_0 perpendicular to the surface. Find what will be the maximum distance of the electron from the axis of the wire before it turns back due to the action of the magnetic field generated by the current.



$$v_0 = \sqrt{v_x^2 + v_y^2}$$

$$B = \frac{\mu_0 I}{2\pi x} \hat{k}$$

$$F = -e(\vec{v} \times \vec{B})$$

$$= -e(v_x \hat{i} + v_y \hat{j}) \times B \hat{k}$$

$$= -e(-Bv_x \hat{j} + Bv_y \hat{i})$$

$$= \frac{\mu_0 I e}{2\pi x} (-v_y \hat{i} + v_x \hat{j})$$

$$a_x = -\frac{\mu_0 I e}{2\pi m} \frac{v_y}{x}$$

$$v_x \frac{dv_x}{dx} = -\frac{\mu_0 I e}{2\pi m} \frac{\sqrt{v_0^2 - v_x^2}}{x}$$

$$\int_{v_0}^0 \frac{v_x dv_x}{\sqrt{v_0^2 - v_x^2}} = -\frac{\mu_0 I e}{2\pi m} \int_a^{x_0} \frac{dx}{x}$$

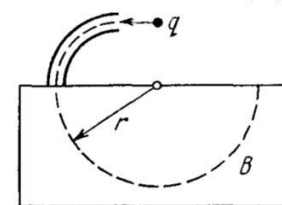
Substituting $v_0^2 - v_x^2 \rightarrow q$
and integrating:

$$v_0 = \frac{\mu_0 I e}{2\pi m} \ln \frac{x_0}{a}$$

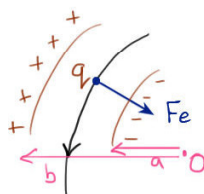
$$\Rightarrow x_0 = a e^{\frac{2\pi m v_0}{\mu_0 I e}}$$

Youtube / Arihant Senior

3.386. A non-relativistic charged particle flies through the electric field of a cylindrical capacitor and gets into a uniform transverse magnetic field with induction B (Fig. 3.100). In the capacitor the particle moves along the arc of a circle, in the magnetic field, along a semi-circle of radius r . The potential difference applied to the capacitor is equal to V , the radii of the electrodes are equal to a and b , with $a < b$. Find the velocity of the particle and its specific charge q/m .



Inside capacitor cylinder:



$$F_e = q \left(\frac{2k\lambda}{r} \right) = \frac{mv^2}{r}$$

$$\text{Also, } V = \int_a^b \frac{2k\lambda}{r} dr$$

$$\Rightarrow V = 2k\lambda \ln \frac{b}{a}$$

$$\Rightarrow V = \frac{mv^2}{q} \ln \frac{b}{a} \quad (1)$$

Inside magnetic field:

$$r = \frac{mv}{qB} \quad (2)$$

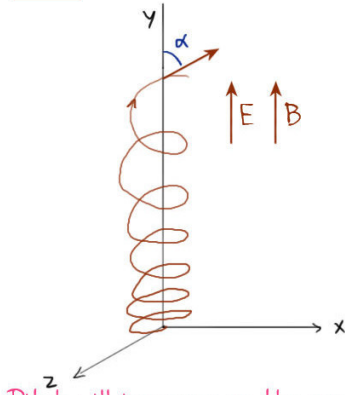
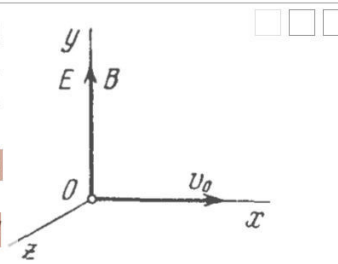
From 1 and 2:

$$\frac{q}{m} = \frac{V}{r^2 B^2 \ln b/a}, \quad v = \frac{V}{r B \ln b/a}$$

Youtube / Arihant Senior

3.387. Uniform electric and magnetic fields with strength E and induction B respectively are directed along the y axis (Fig. 3.101). A particle with specific charge q/m leaves the origin O in the direction of the x axis with an initial non-relativistic velocity v_0 . Find:

- (a) the coordinate y_n of the particle when it crosses the y axis for the n th time;
 (b) the angle α between the particle's velocity vector and the y axis at that moment.



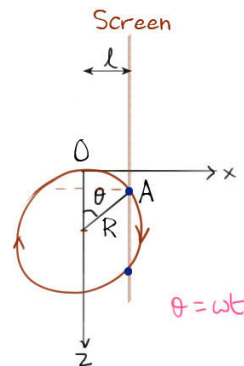
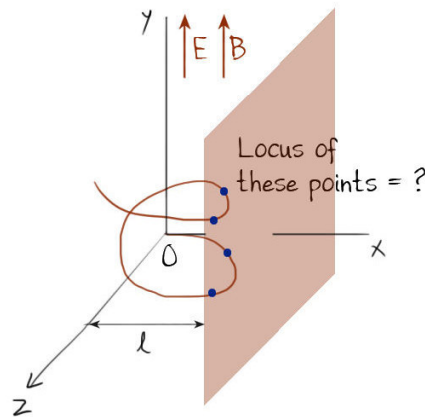
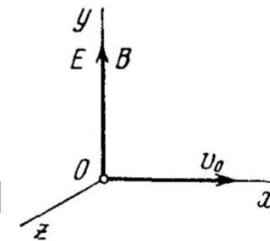
$$\begin{aligned} \text{A } y_n &= \frac{1}{2} a t^2 \\ &= \frac{1}{2} \frac{qE}{m} \cdot n^2 \cdot \frac{4\pi^2 m^2}{q^2 B^2} \\ &= \frac{2\pi^2 n^2 m E}{q B^2} // \end{aligned}$$

$$\begin{aligned} \text{B } \tan \alpha &= \frac{v_0}{v_y} = \frac{v_0}{\frac{qE}{m} t} \\ &= \frac{v_0}{\frac{qE}{m} \left(n \cdot \frac{2\pi m}{qB} \right)} \\ &= \frac{v_0 B}{2\pi n E} // \end{aligned}$$

Pitch will increase as the particle progresses due to E .
 Radius of helix won't change as velocity perpendicular to B is constant (v_0).

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3.388. A narrow beam of identical ions with specific charge q/m , possessing different velocities, enters the region of space, where there are uniform parallel electric and magnetic fields with strength E and induction B , at the point O (see Fig. 3.101). The beam direction coincides with the x axis at the point O . A plane screen oriented at right angles to the x axis is located at a distance l from the point O . Find the equation of the trace that the ions leave on the screen. Demonstrate that at $z \ll l$ it is the equation of a parabola.



Coordinates of point A:

$$\begin{aligned} x &= l = R \sin \theta \\ z &= R(1 - \cos \theta) \end{aligned} \Rightarrow \frac{z}{l} = \tan \frac{\theta}{2}$$

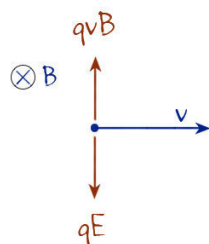
$$\begin{aligned} y &= \frac{1}{2} a t^2 \\ \Rightarrow z &= l \tan \frac{\omega t}{2} = l \tan \frac{qB}{2m} \cdot \frac{t}{2} \end{aligned}$$

$$\Rightarrow z = l \tan \frac{qB}{2m} \cdot \sqrt{\frac{2y}{(qE/m)}} = l \tan B \sqrt{\frac{qy}{2mE}}$$

$$\text{If } z \ll l, \quad \frac{z}{l} \approx B \sqrt{\frac{qy}{2mE}} \Rightarrow y = \frac{2mE}{B^2 l^2 q} z^2 //$$

Youtube / Arihant Senior

3.389. A non-relativistic proton beam passes without deviation through the region of space where there are uniform transverse mutually perpendicular electric and magnetic fields with $E = 120 \text{ kV/m}$ and $B = 50 \text{ mT}$. Then the beam strikes a grounded target. Find the force with which the beam acts on the target if the beam current is equal to $I = 0.80 \text{ mA}$.



$$qvB = qE$$

$$v = \frac{E}{B}$$

$$F = \frac{\Delta p}{\Delta t}$$

Lets consider beam hitting for one second

$$f = \frac{\Delta p}{\Delta t} = \left(\frac{icI}{e_p} \right) m_p v$$

Number of protons crossing in one second. Momentum of one proton.

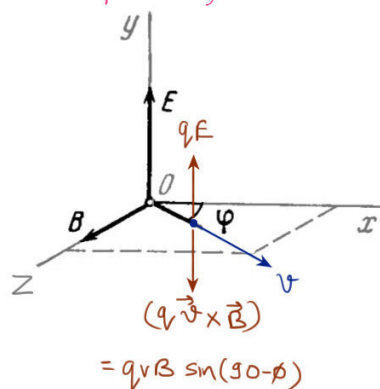
$$= \frac{icI}{e} m_p \cdot \frac{E}{B}$$

$$= \frac{m_p c I E}{e B} //$$

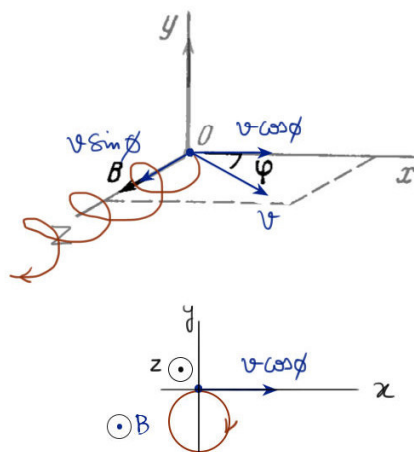
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3.390. Non-relativistic protons move rectilinearly in the region of space where there are uniform mutually perpendicular electric and magnetic fields with $E = 4.0 \text{ kV/m}$ and $B = 50 \text{ mT}$. The trajectory of the protons lies in the plane xz (Fig. 3.102) and forms an angle $\phi = 30^\circ$ with the x axis. Find the pitch of the helical trajectory along which the protons will move after the electric field is switched off.

Rectilinear motion
• Net force is zero.



After electric field is tured off



$$qE = qvB \sin(90 - \phi)$$

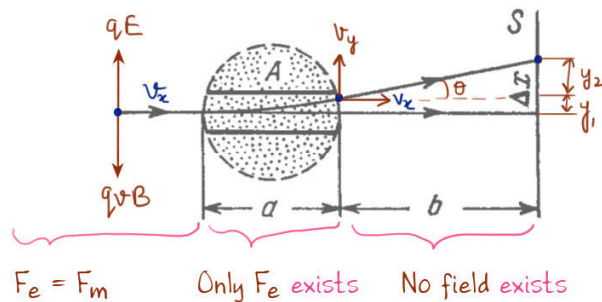
$$\text{Pitch} = v \sin \phi \cdot T$$

$$= \frac{E}{B \cos \phi} \cdot \sin \phi \cdot \frac{2\pi m}{qB}$$

$$= \frac{2\pi m E \tan \phi}{qB^2} //$$

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3.391. A beam of non-relativistic charged particles moves without deviation through the region of space A (Fig. 3.103) where there are transverse mutually perpendicular electric and magnetic fields with strength E and induction B . When the magnetic field is switched off, the trace of the beam on the screen S shifts by Δx . Knowing the distances a and b , find the specific charge q/m of the particles.



$$qE = qv_x B$$

$$\Rightarrow v_x = E/B$$

$$\Delta x = y_1 + y_2$$

$$= \frac{1}{2} a_c t^2 + b \tan \theta$$

$\frac{qE}{m}$ $\frac{a}{v_x}$ $\frac{v_y}{v_x} \rightarrow a_c t$

$$= \frac{1}{2} \frac{qE}{m} \cdot \frac{a^2}{E^2/B^2} + b \frac{qE}{m} \cdot \frac{a}{E/B}$$

$$\Rightarrow \Delta x = \frac{qB^2 a}{mE} \left(\frac{a}{2} + b \right)$$

$$\Rightarrow \frac{q}{m} = \frac{2E\Delta x}{B^2 a(a+2b)} \quad // \text{ Youtube / Arihant Senior}$$

3.392. A particle with specific charge q/m moves in the region of space where there are uniform mutually perpendicular electric and magnetic fields with strength E and induction B (Fig. 3.104). At the moment $t = 0$ the particle was located at the point O and had zero velocity. For the non-relativistic case find:

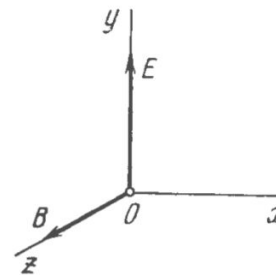


Fig. 3.104.

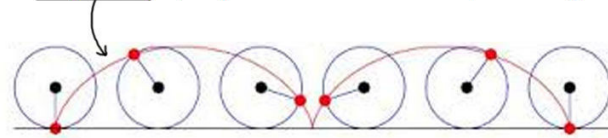
- the law of motion $x(t)$ and $y(t)$ of the particle; the shape of the trajectory;
- the length of the segment of the trajectory between two nearest points at which the velocity of the particle turns into zero;
- the mean value of the particle's velocity vector projection on the x axis (the drift velocity).

↓ Replacing constant force qE with mg . (Problem/solution otherwise remains exactly same)

Cycloid path in a magnetic field: A particle of mass m and charge q , is dropped in a uniform gravity g (vertically downwards) and uniform magnetic field B (horizontal). Show that its path is a cycloid.

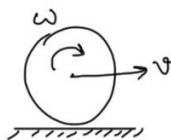
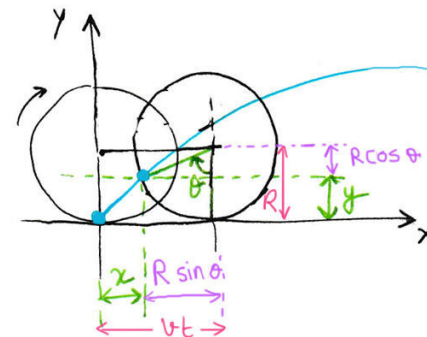
Cycloid path in a magnetic field: A particle of mass m and charge q , is dropped in a uniform gravity g (vertically downwards) and uniform magnetic field B (horizontal). Show that its path is a cycloid.

Cycloid Its trajectory of a particle on the rim of a rolling disc.



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Position of a particle in cycloid.



As it's rolling
 $v = \omega R$
 $\theta = \omega t$

When it rotates by θ , position of particle is given by:-

$$x = vt - R \sin \theta$$

$$= \omega R t - R \sin \omega t$$

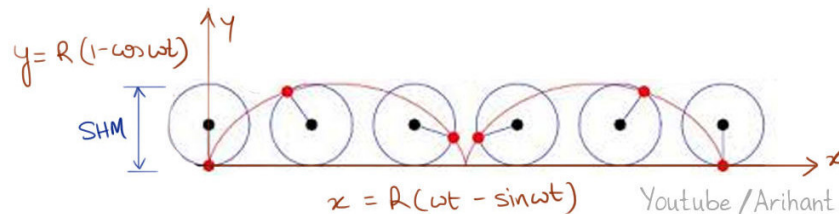
$$x = R (\omega t - \sin \omega t)$$

X coordinate wrt time

$$y = R - R \cos \theta$$

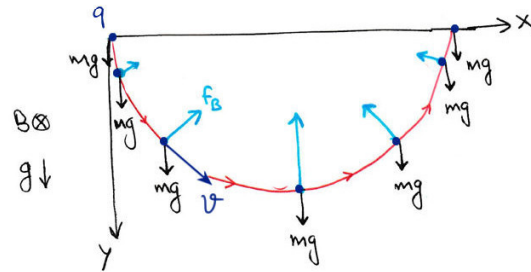
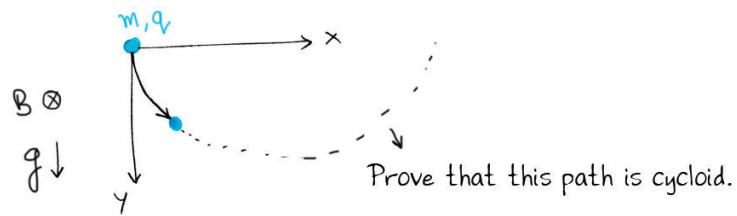
$$y = R (1 - \cos \omega t)$$

Y coordinate wrt time



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Our problem:

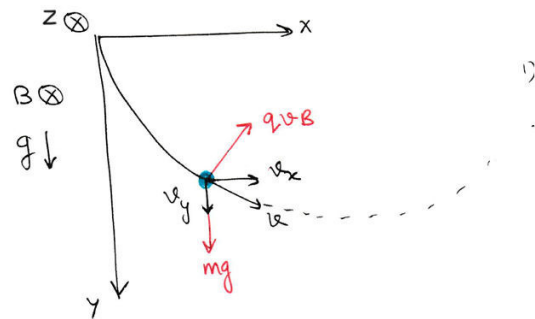


Approach:

- 1 We shall find the position of particle w.r.t time.
- 2 See, if its similar to that of a cycloid.

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Solution:



$$\begin{aligned}
 \vec{F} &= mg\hat{j} + q(\vec{v} \times \vec{B}) \\
 &= mg\hat{j} + q[(v_x\hat{i} + v_y\hat{j}) \times (B\hat{k})] \\
 &= (mg - qv_xB)\hat{j} + qv_yB\hat{i}
 \end{aligned}$$

$$\begin{aligned}
 m \frac{dv_y}{dt} &= mg - qBv_x \quad \text{--- } F_y \\
 m \frac{dv_x}{dt} &= qBv_y \quad \text{--- } F_x
 \end{aligned}$$

(continued on next slide)

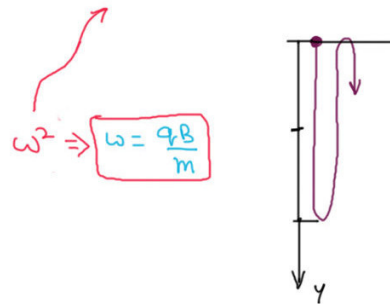
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differentiate

$$m \frac{dv_y}{dt} = -qB \frac{dv_x}{dt}$$

$$m \frac{d^2 y}{dt^2} = -qB \left(\frac{q v_y B}{m} \right)$$

$$\frac{d^2 y}{dt^2} = -\frac{q^2 B^2}{m^2} y \Rightarrow \boxed{\text{SHM}}$$



Recap of SHM

$$F = -ky$$

$$m \frac{dv_y}{dt} = -ky$$

differentiate

$$m \frac{d^2 y}{dt^2} = -k y$$

$$\frac{d^2 y}{dt^2} = -\frac{k}{m} y$$

$$\boxed{\frac{d^2 y}{dt^2} = -\omega^2 y}$$

General solution:

$$y - y_0 = A \sin(\omega t + \delta)$$

$y_0 \rightarrow$ equilibrium position

$\delta \rightarrow$ Initial Phase

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General solution:

Position: $y - y_0 = A \sin(\omega t + \delta)$

Velocity: $v = A\omega \cos(\omega t + \delta)$

Accⁿ: $a = -A\omega^2 \sin(\omega t + \delta)$

$\downarrow mg \Rightarrow a_{\max} = g = A\omega^2 \Rightarrow \boxed{A = \frac{g}{\omega^2}}$

$$x \Big|_0^x = \frac{g}{\omega} \left(t - \frac{\sin \omega t}{\omega} \right) \Big|_0^t$$

$$\boxed{x = \frac{g}{\omega^2} (\omega t - \sin \omega t)}$$

In Y direction

$$\omega = \frac{qB}{m}$$

Position: $y = \frac{g}{\omega^2} (1 - \cos \omega t)$

Velocity: $v_y = \frac{g}{\omega} \sin \omega t$

Accⁿ: $a_y = g \cos \omega t$ ②

$$m \frac{dv_y}{dt} = mg - qB v_x$$
 ①

① & ② $\Rightarrow m(g \cos \omega t) = mg - qB v_x$

$$v_x = \frac{mg}{qB} (1 - \cos \omega t)$$

integrate $v_x = \frac{g}{\omega} (1 - \cos \omega t)$

Our equations: $(\omega = \frac{qB}{m})$

$$x = \underbrace{\frac{g}{\omega^2}}_{\text{Radius } R} (\omega t - \sin \omega t)$$

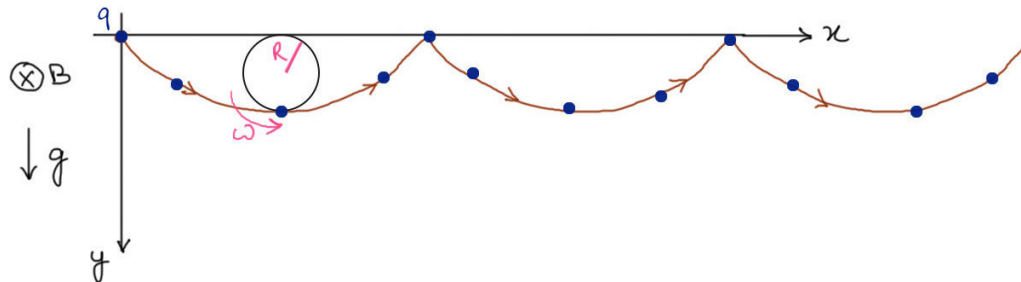
$$y = \frac{g}{\omega^2} (1 - \cos \omega t)$$

General cycloids equations:

$$x = R (\omega t - \sin \omega t)$$

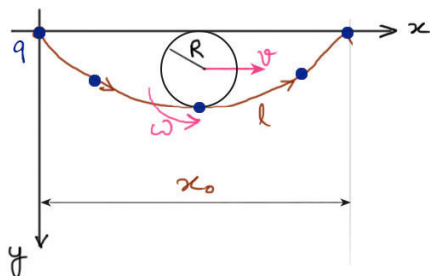
$$y = R (1 - \cos \omega t)$$

Hence q follows a cycloid path!



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- (a) the law of motion $x(t)$ and $y(t)$ of the particle; the shape of the trajectory;
 (b) the length of the segment of the trajectory between two nearest points at which the velocity of the particle turns into zero;
 (c) the mean value of the particle's velocity vector projection on the x axis (the drift velocity).



(c) $\langle v_x \rangle = \frac{x_0}{T} = \frac{v T}{T} = v$
 $= \omega R = \frac{q \cdot g}{\omega^2} = \frac{q m}{q B}$

A $x = \underbrace{\frac{g}{\omega^2}}_{\text{Radius } R} (\omega t - \sin \omega t)$

$$y = \frac{g}{\omega^2} (1 - \cos \omega t)$$

$$(\omega = \frac{qB}{m})$$

B $dx = R (\omega - \omega \cos \omega t) dt$
 $dy = R \omega \sin \omega t dt$

$$dl = \sqrt{dx^2 + dy^2}$$

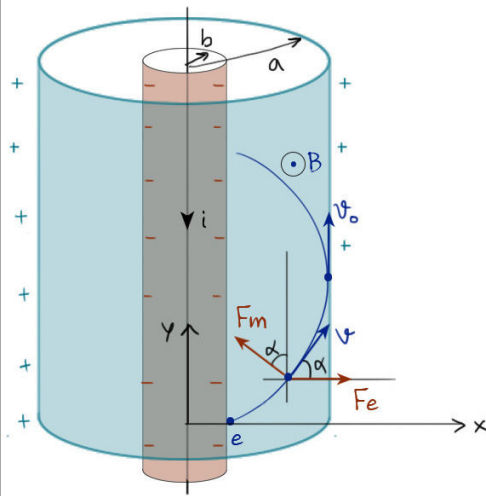
$$= R \omega \sqrt{(1 - \cos \omega t)^2 + \sin^2 \omega t} dt$$

$$l = \int dl = R \omega \int_0^{2\pi} 2 \sin \frac{\omega t}{2} dt$$

$$= 2R \omega \left(-\frac{\cos \frac{\omega t}{2}}{\frac{1}{2}} \right) \Big|_0^{2\pi} = 8R = \frac{8g}{\omega^2} = \frac{8qm^2}{q^2 B^2}$$

Note: To match books answers, replace g with qE everywhere

3.393. A system consists of a long cylindrical anode of radius a and a coaxial cylindrical cathode of radius b ($b < a$). A filament located along the axis of the system carries a heating current I producing a magnetic field in the surrounding space. Find the least potential difference between the cathode and anode at which the thermal electrons leaving the cathode without initial velocity start reaching the anode.



$$F_y = F_m \cos \alpha$$

$$= qvB \cos \alpha$$

$$m \cdot \frac{dv_y}{dt} = q \frac{\mu_0 i}{2\pi r} \cdot v \cos \alpha$$

$$m \frac{dv_y}{dx} \cdot \frac{dx}{dt} = q \frac{\mu_0 i}{2\pi r} \cdot v \cos \alpha$$

$$v_0 \int_0^a dv_y = \frac{q \mu_0 i}{2\pi m} \int_b^a \frac{dx}{x}$$

$$\Rightarrow v_0 = \frac{q \mu_0 i}{2\pi m} \ln \frac{a}{b} = \sqrt{\frac{2qV}{m}} \Rightarrow V = \frac{2 \left(\frac{\mu_0 i}{4\pi} \right)^2 q}{m} \ln^2 \frac{a}{b}$$

$$qV = \frac{1}{2} m v_0^2$$

$$\Rightarrow v_0 = \sqrt{\frac{2qV}{m}}$$

$$V = \frac{2 \left(\frac{\mu_0 i}{4\pi} \right)^2 q}{m} \ln^2 \frac{a}{b}$$

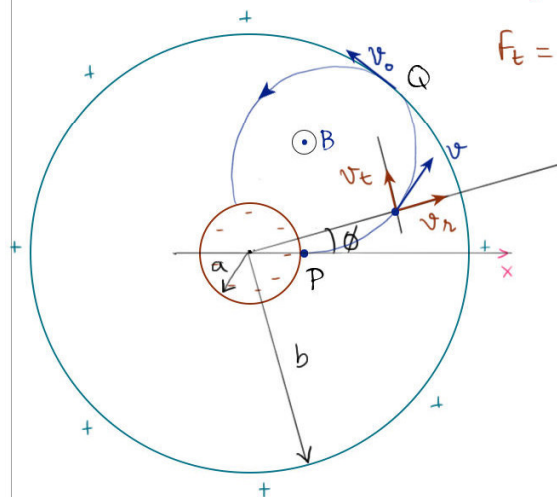
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3.394. Magnetron is a device consisting of a filament of radius a and a coaxial cylindrical anode of radius b which are located in a uniform magnetic field parallel to the filament. An accelerating potential difference V is applied between the filament and the anode. Find the value of magnetic induction at which the electrons leaving the filament with zero velocity reach the anode.

$$a_t = r \frac{d^2 \phi}{dt^2} + 2 \frac{dr}{dt} \cdot \frac{d\phi}{dt}$$

$$= \frac{1}{r} \cdot \frac{d}{dt} \left(r^2 \frac{d\phi}{dt} \right)$$

We know that general tangential acceleration of a particle is given by



$$F_t = q(\vec{v}_t \times \vec{B}) = m \cdot \frac{1}{r} \frac{d}{dt} \left(r^2 \frac{d\phi}{dt} \right)$$

$$q \left(\frac{dr}{dt} \right) B = m \cdot \frac{1}{r} \frac{d}{dt} \left(r^2 \frac{d\phi}{dt} \right)$$

$$\frac{qB}{m} \int_a^b r dr = \int_a^b \left(r^2 \frac{d\phi}{dt} \right)$$

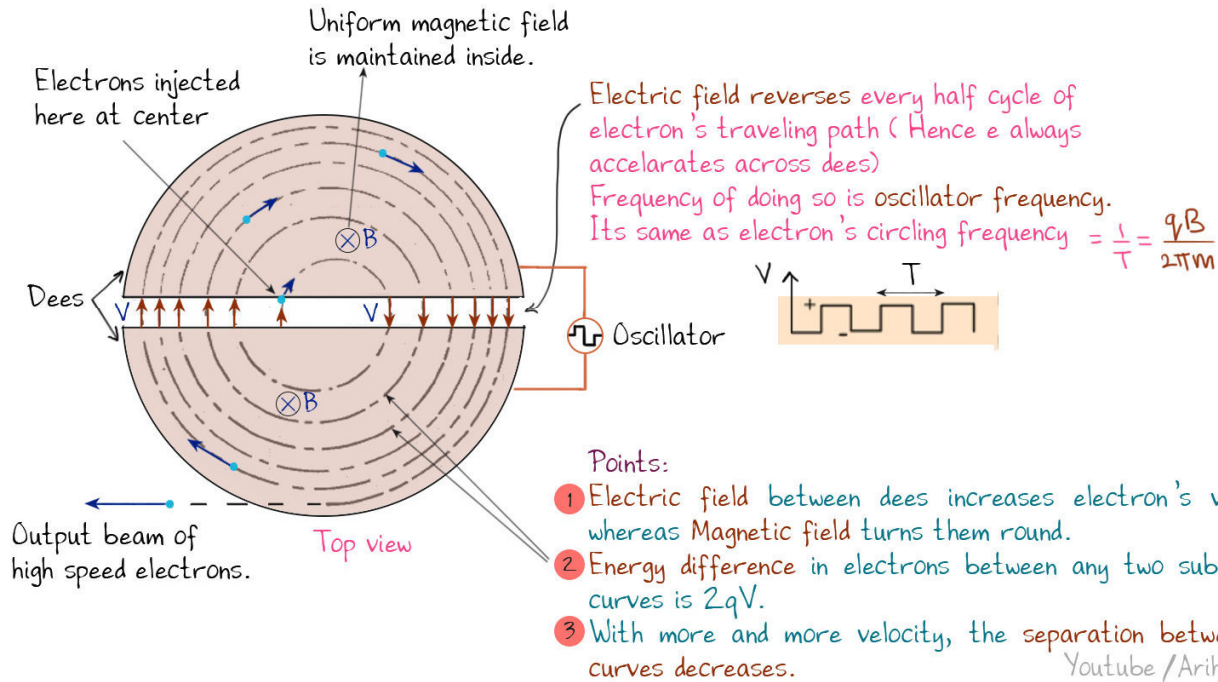
$$\Rightarrow \frac{qB(b^2 - a^2)}{2m} = \left. r^2 \frac{d\phi}{dt} \right|_a = \frac{b^2}{a} \sqrt{\frac{2qV}{m}} \Rightarrow B = \frac{2b}{b^2 - a^2} \sqrt{\frac{2mV}{q}}$$

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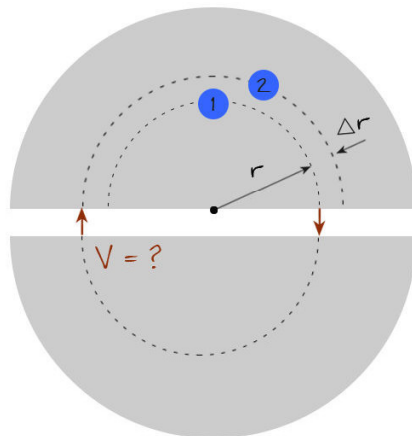
$$\frac{1}{2} m v_0^2 = qV$$

$$\left. \frac{d\phi}{dt} \right|_a = \frac{v_0}{b}$$

Cyclotron construction and working



3.396. The cyclotron's oscillator frequency is equal to $f = 10 \text{ MHz}$. Find the effective accelerating voltage applied across the dees of that cyclotron if the distance between the neighbouring trajectories of protons is not less than $\Delta r = 1.0 \text{ cm}$, with the trajectory radius being equal to $r = 0.5 \text{ m}$.



$$E_2 - E_1 = dE$$

$$dE = 2qV = d\left(\frac{1}{2}mv^2\right) \rightarrow v = \omega r = 2\pi f r$$

$$= d\left(\frac{1}{2}m \cdot 4\pi^2 f^2 r^2\right)$$

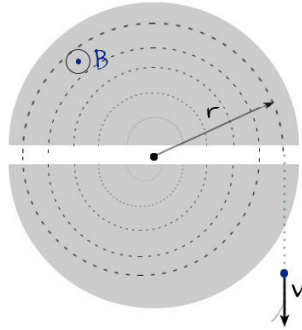
$$= 2m f^2 \pi^2 dr^2$$

$$2qV = 2m f^2 \pi^2 2r dr$$

$$\Rightarrow V \approx \frac{2\pi^2 m f^2 r \Delta r}{q} \quad \because \Delta r \ll r$$

3.397. Protons are accelerated in a cyclotron so that the maximum curvature radius of their trajectory is equal to $r = 50$ cm. Find:

- (a) the kinetic energy of the protons when the acceleration is completed if the magnetic induction in the cyclotron is $B = 1.0$ T;
 (b) the minimum frequency of the cyclotron's oscillator at which the kinetic energy of the protons amounts to $T = 20$ MeV by the end of acceleration.



$$\text{A } KE = \frac{1}{2}mv^2 \quad \omega = \frac{mv}{qB}$$

$$= \frac{1}{2}m \frac{q^2 B^2 r^2}{m^2} = \frac{q^2 B^2 r^2}{2m} //$$

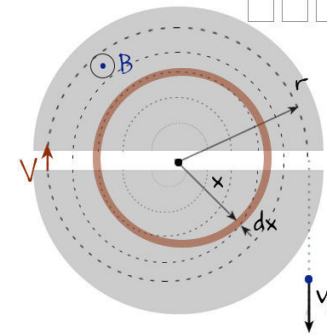
$$\text{B } T = \frac{1}{2}mv^2 \quad \omega R \rightarrow 2\pi f$$

$$T = \frac{1}{2}m(2\pi f)^2 R^2 \Rightarrow f = \frac{1}{2\pi R} \sqrt{\frac{2T}{m}} //$$

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3.398. Singly charged ions He^+ are accelerated in a cyclotron so that their maximum orbital radius is $r = 60$ cm. The frequency of a cyclotron's oscillator is equal to $\nu = 10.0$ MHz, the effective accelerating voltage across the dees is $V = 50$ kV. Neglecting the gap between the dees, find:

- (a) the total time of acceleration of the ion;
 (b) the approximate distance covered by the ion in the process of its acceleration.



$$\text{A } \text{Total time } t_0 = \frac{N_0}{f} \cdot T = \frac{N_0}{f} \quad 1$$

(Total number of rounds)

$$\text{Total energy} = N_0 \cdot 2qV = \frac{1}{2}mv_0^2 \quad v_0 = \omega r = 2\pi f r$$

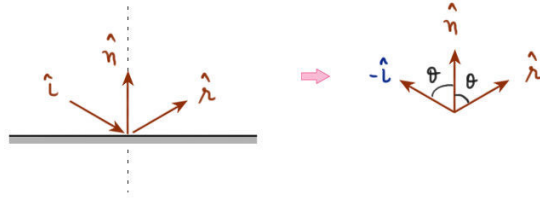
$$N_0 = \frac{\frac{1}{2}m(2\pi f r)^2}{2qV} \quad 1, 2 \Rightarrow t_0 = \frac{m\pi f r^2}{qV} \quad 3$$

$$\text{B } \text{From 1 and 3, at general distance } x \text{ from center: } \frac{N}{f} = \frac{m\pi f x^2}{qV} \xrightarrow{\text{diff.}} dN = \frac{m\pi f}{qV} \cdot 2x dx$$

$$\text{Small distance covered } dl = 2\pi x dN \Rightarrow l = \frac{4\pi^2 m f^2}{qV} \int_0^r x^2 dx = \frac{4\pi^2 m f^2 r^3}{3qV}$$

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5.13. Write the law of reflection of a light beam from a mirror in vector form, using the directing unit vectors $\hat{i}$ and $\hat{r}$ of the incident and reflected beams and the unit vector $\hat{n}$ of the outside normal to the mirror surface.



Let, $-\hat{i} + \hat{r} = k\hat{n}$ ①

Finding k :

$$(-\hat{i} + \hat{r}) \cdot \hat{n} = (k\hat{n}) \cdot \hat{n}$$

$$\Rightarrow -\hat{i} \cdot \hat{n} + \hat{r} \cdot \hat{n} = k$$

$$\Rightarrow 2(\hat{i} \cdot \hat{n}) = k \quad (2) \quad (\because -\hat{i} \cdot \hat{n} = \hat{r} \cdot \hat{n} = \cos\theta)$$

Eliminating k from 1,2:

$$-\hat{i} + \hat{r} = -2(\hat{i} \cdot \hat{n})\hat{n}$$

$$\Rightarrow \hat{r} = \hat{i} - 2(\hat{i} \cdot \hat{n})\hat{n}$$

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5.14. Demonstrate that a light beam reflected from three mutually perpendicular plane mirrors in succession reverses its direction.

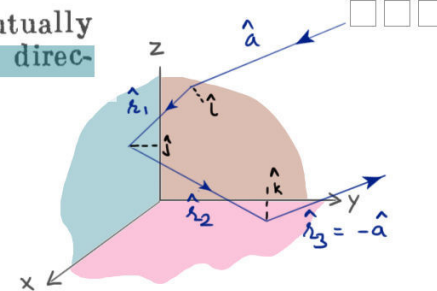
In 5.13 we derived

$$\hat{r} = \hat{a} - 2(\hat{a} \cdot \hat{n})\hat{n}$$

Unit vector along reflected ray.

Unit vector along incident ray

Unit vector along normal to mirror



Sol: Let incident ray be: $\hat{a} = x\hat{i} + y\hat{j} + z\hat{k}$ and mirror's normals be $\hat{i}, \hat{j}$ and $\hat{k}$.

$$\begin{aligned} \text{After first reflection: } \hat{r}_1 &= \hat{a} - 2(\hat{a} \cdot \hat{i})\hat{i} \\ &= (x\hat{i} + y\hat{j} + z\hat{k}) - 2(x)\hat{i} \\ &= -x\hat{i} + y\hat{j} + z\hat{k} \end{aligned}$$

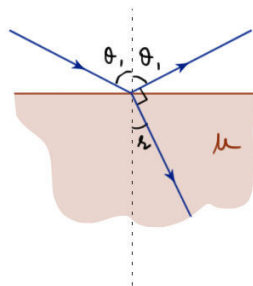
$$\text{After second reflection: } \hat{r}_2 = \hat{r}_1 - 2(\hat{r}_1 \cdot \hat{j})\hat{j} = -x\hat{i} - y\hat{j} + z\hat{k}$$

$$\text{After third reflection: } \hat{r}_3 = \hat{r}_2 - 2(\hat{r}_2 \cdot \hat{k})\hat{k} = -x\hat{i} - y\hat{j} - z\hat{k} = -\hat{a}$$

Hence proved

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5.15. At what value of the angle of incidence θ_1 is a shaft of light reflected from the surface of water perpendicular to the refracted shaft?



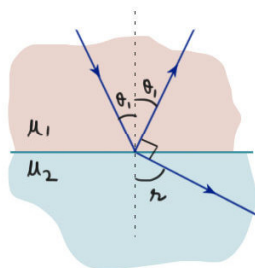
$$\sin \theta_1 = \mu \sin r \quad (1)$$

$$\theta_1 + r = \frac{\pi}{2} \quad (2)$$

$$\text{From 1,2: } \tan \theta_1 = \mu //$$

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5.16. Two optical media have a plane boundary between them. Suppose θ_{1cr} is the critical angle of incidence of a beam and θ_1 is the angle of incidence at which the refracted beam is perpendicular to the reflected one (the beam is assumed to come from an optically denser medium). Find the relative refractive index of these media if $\sin \theta_{1cr} / \sin \theta_1 = \eta = 1.28$.



$$\text{Let, } \frac{\mu_1}{\mu_2} = \mu$$

$$\text{Snell's law: } \mu_1 \sin \theta_1 = \mu_2 \sin r \quad (1)$$

$$\text{Given: } \theta_1 + r = \frac{\pi}{2} \quad (2)$$

$$\text{From 1,2: } \tan \theta_1 = \frac{\mu_2}{\mu_1} = \frac{1}{\mu}$$

$$\Rightarrow \frac{\sqrt{1+\mu^2}}{\mu} \quad (A)$$

$$\text{Also, } \mu_1 \sin \theta_{1cr} = \mu_2 \sin 90^\circ$$

$$\Rightarrow \sin \theta_{1cr} = \frac{1}{\mu} \quad (B)$$

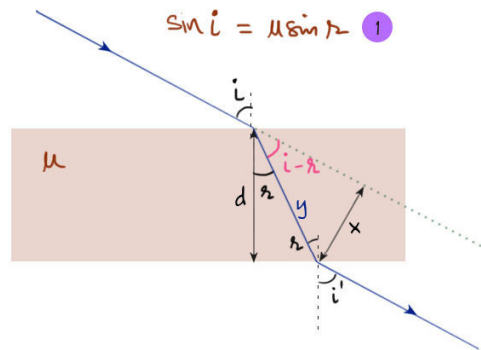
$$\text{Given: } \frac{\sin \theta_{1cr}}{\sin \theta_1} = \eta$$

$$\Rightarrow \frac{\frac{1}{\mu}}{\frac{1}{\sqrt{1+\mu^2}}} = \eta$$

$$\Rightarrow \mu = \frac{1}{\sqrt{\eta^2 - 1}} //$$

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5.17. A light beam falls upon a plane-parallel glass plate $d=6.0$ cm in thickness. The angle of incidence is $\theta = 60^\circ$. Find the value of deflection of the beam which passed through that plate.



$$\sin i = \mu \sin r \quad (1)$$

$$\sin i = \mu \sin r = \sin i'$$

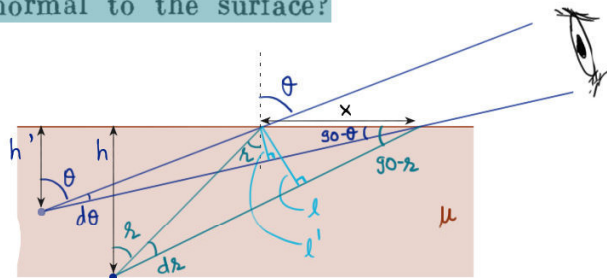
Refraction at 1st surface. Refraction at 2nd surface.

$\Rightarrow i = i' \Rightarrow$ Incoming and outgoing rays are parallel.

$$\begin{aligned} y &= \frac{d}{\cos r} = \frac{x}{\sin(i-r)} \\ \Rightarrow x &= \frac{\sin(i-r)}{\cos r} d \\ &= \left(\frac{\sin i \cos r - \cos i \sin r}{\cos r} \right) d \quad (2) \\ \text{From 1,2: } x &= \left[\sin i - \cos i \frac{\sin i / \mu}{\sqrt{1 - \frac{\sin^2 i}{\mu^2}}} \right] d \\ &= d \sin i \left(1 - \frac{\cos i}{\sqrt{\mu^2 - \sin^2 i}} \right) \end{aligned}$$

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5.18. A man standing on the edge of a swimming pool looks at a stone lying on the bottom. The depth of the swimming pool is equal to h . At what distance from the surface of water is the image of the stone formed if the line of vision makes an angle θ with the normal to the surface?



We want h' in terms of h .

Common link between h and h' is x .

So let's get x in terms of h and h' . And then eliminate it.

$$x' = \left(\frac{h'}{\cos \theta} \right) d\theta = x \sin(90 - \theta)$$

$$x = \left(\frac{h}{\cos r} \right) dr = x \sin(90 - r)$$

$$\text{Divide: } \frac{h'}{h} \frac{d\theta}{dr} = \frac{\cos^2 \theta}{\cos^2 r} \quad (1)$$

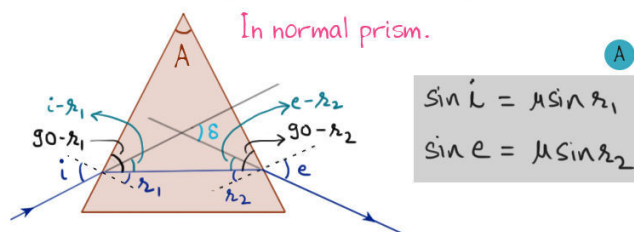
$$\text{Also, } \sin \theta = \mu \sin r$$

$$\Rightarrow \cos \theta d\theta = \mu \cos r dr \quad (2)$$

$$\text{From 1,2: } h' = \frac{h \cos^3 \theta}{\mu \cos^3 r} = \frac{h \cos^3 \theta}{\mu (1 - \sin^2 \theta)^{3/2}}$$

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5.19. Demonstrate that in a prism with small refracting angle θ the shaft of light deviates through the angle $\alpha \simeq (n - 1) \theta$ regardless of the angle of incidence, provided that the latter is also small.



In normal prism.

$$\sin i = \mu \sin r_1$$

$$\sin e = \mu \sin r_2$$

$$A + (90 - r_1) + (90 - r_2) = 180^\circ$$

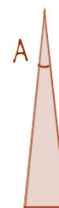
$$\Rightarrow r_1 + r_2 = A$$

$$\text{Also, } \delta = i - r_1 + e - r_2$$

$$= i + e - (r_1 + r_2)$$

$$\Rightarrow \delta = i + e - A$$

Sol:



In thin prism (small A) with small i, r.

$$\sin i = \mu \sin r_1$$

$$\Rightarrow i \simeq \mu r_1$$

$$\text{and, } \sin e = \mu \sin r_2$$

$$\Rightarrow e \simeq \mu r_2$$

$$\delta = i + e - A$$

$$= \mu r_1 + \mu r_2 - A$$

$$= \mu (r_1 + r_2) - A$$

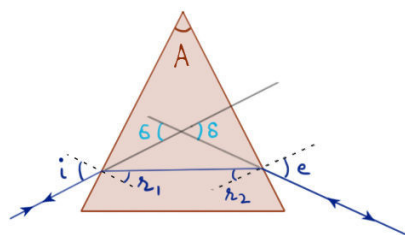
$$= \mu A - A$$

$$= (\mu - 1) A //$$

(constant, independent of i)

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Minimum deviation by a prism



$$\sin i = \mu \sin r_1$$

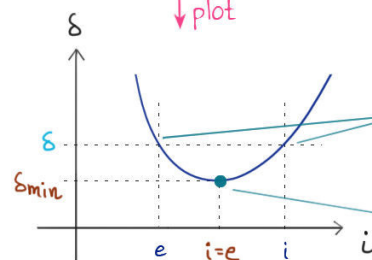
$$\sin e = \mu \sin r_2$$

$$r_1 + r_2 = A$$

$$\delta = i + e - A$$

$$\delta = i + \sin^{-1} \left[\mu \sin \left(A - \sin^{-1} \left(\frac{\sin i}{\mu} \right) \right) \right] - A$$

plot

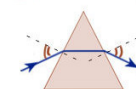


When ray is reversed, deviation is same.

Two values on X-axis correspond to i and e of same ray.

At minimum deviation, $i = e$
That is ray passes symmetrically

Finding μ of glass from measuring δ_m



$$i = e$$

$$\Rightarrow \text{From D: } \delta_m = 2i - A \Rightarrow i = \frac{\delta_m + A}{2}$$

$$\Rightarrow \text{From C: } r_1 = r_2 = A/2$$

$$\Rightarrow \text{From A: } \sin i = \mu \sin A/2$$

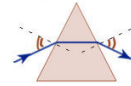
$$\mu = \frac{\sin \left(\frac{\delta_m + A}{2} \right)}{\sin A/2}$$

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5.20. A shaft of light passes through a prism with refracting angle θ and refractive index n . Let α be the diffraction angle of the shaft. Demonstrate that if the shaft of light passes through the prism symmetrically,

- (a) the angle α is the least;
 (b) the relationship between the angles α and θ is defined by Eq. (5.1e).

$$\mu = \frac{\sin \left(\frac{\delta_{\min} + A}{2} \right)}{\sin A/2} //$$



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5.21. The least deflection angle of a certain glass prism is equal to its refracting angle. Find the latter.

We know, $\mu = \frac{\sin \left(\frac{\delta_{\min} + A}{2} \right)}{\sin A/2} //$ (derived in 5.20)

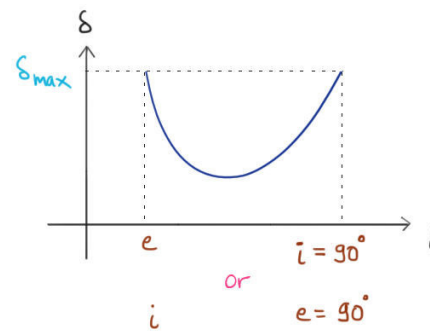
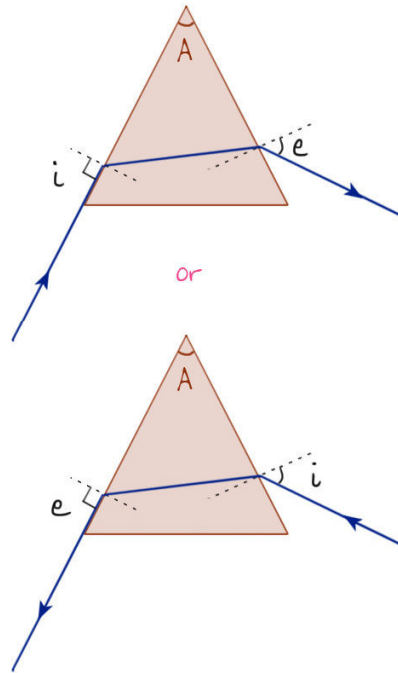
Sol: Given: $\delta_{\min} = A$

$$\therefore \mu = \frac{\sin \frac{A+A}{2}}{\sin A/2} = 2 \cos A/2$$

$$\Rightarrow A = 2 \cos^{-1} \frac{\mu}{2} // = 2 \cos^{-1} \frac{3/2}{2} = 83^\circ$$

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Maximum deviation by a prism



i can't be more than 90°

∴ From graph, at maximum deviation, $i = 90^\circ$, or $e = 90^\circ$.

That is, either incident or emergent ray is almost parallel to the surface of prism.

$$\delta_{\max} = i + e - A$$

$$= 90^\circ + e - A$$

$$\sin 90^\circ = \mu \sin r_1 \Rightarrow r_1 = \sin^{-1} \frac{1}{\mu}$$

$$r_1 + r_2 = A \Rightarrow r_2 = A - \sin^{-1} \frac{1}{\mu}$$

$$\sin e = \mu \sin r_2 \Rightarrow e = \sin^{-1} \left[\mu \sin \left(A - \sin^{-1} \frac{1}{\mu} \right) \right]$$

$$\delta_{\max} = 90^\circ + \sin^{-1} \left[\mu \sin \left(A - \sin^{-1} \frac{1}{\mu} \right) \right] - A //$$

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5.22. Find the minimum and maximum deflection angles for a light ray passing through a glass prism with refracting angle $\theta = 60^\circ$.

Minimum deviation:

Rearranging A: $\delta_{\min} = 2 \sin^{-1} \left(\mu \sin \frac{A}{2} \right) - A$

$$= 2 \sin^{-1} \left(\frac{3}{2} \sin \frac{60^\circ}{2} \right) - 60^\circ$$

$$= 37^\circ //$$

Using

A $\mu = \frac{\sin \left(\frac{\delta_{\min} + A}{2} \right)}{\sin \frac{A}{2}}$

B $\delta_{\max} = 90^\circ + \sin^{-1} \left[\mu \sin \left(A - \sin^{-1} \frac{1}{\mu} \right) \right] - A$

Maximum deviation:

Using B: $\delta_{\max} = 90^\circ + \sin^{-1} \left[\frac{3}{2} \sin \left(60^\circ - \sin^{-1} \frac{2}{3} \right) \right] - 60^\circ$

$$= 58^\circ //$$

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5.23. A trihedral prism with refracting angle 60° provides the least deflection angle 37° in air. Find the least deflection angle of that prism in water.

$$\mu = \frac{\sin\left(\frac{\delta_{\min} + A}{2}\right)}{\sin A/2}$$

For glass prism in air: $\mu_g = \frac{\sin\left(\frac{37^\circ + 60^\circ}{2}\right)}{\sin 60/2}$ (1)

For glass prism in water: $\frac{\mu_g}{\mu_w} = \frac{\sin\left(\frac{\delta_{\min} + 60}{2}\right)}{\sin 60/2}$ (2)

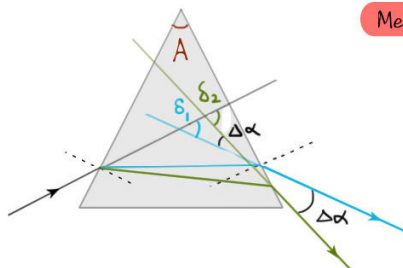
Dividing 1 and 2: $\frac{\mu_w}{\mu_g} = \frac{\sin 37/2}{\sin\left(\frac{\delta_{\min} + 60}{2}\right)} \Rightarrow \frac{4/3}{1} = \frac{\sin 37/2}{\sin\left(\frac{\delta_{\min} + 60}{2}\right)}$

$\delta_{\min} = 8^\circ$ As expected, deflection is less when prism and surrounding mediums refractive indices are comparable.

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5.24. A light ray composed of two monochromatic components passes through a trihedral prism with refracting angle $\theta = 60^\circ$. Find the angle $\Delta\alpha$ between the components of the ray after its passage through the prism if their respective indices of refraction are equal to 1.515 and 1.520. The prism is oriented to provide the least deflection angle.

$$\mu = \frac{\sin\left(\frac{\delta_{\min} + A}{2}\right)}{\sin A/2}$$



Method 1 $\delta_2 = \delta_1 + \Delta\alpha$

$$\Rightarrow \Delta\alpha = \delta_2 - \delta_1 = \delta_{2m} - \delta_{1m}$$

$$\delta_{\min} = 2 \sin^{-1}(\mu \sin A/2) - A$$

$$= 2 \sin^{-1}\left(1.520 \sin \frac{60^\circ}{2}\right) - 2 \sin^{-1}\left(1.515 \sin \frac{60^\circ}{2}\right)$$

$$= 2 \left(\sin^{-1} \frac{1.52}{2} - \sin^{-1} \frac{1.515}{2} \right) = 0.44^\circ //$$

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Method 2 ∴ μ difference is small

$$\mu = \frac{\sin\left(\frac{\delta_{\min} + A}{2}\right)}{\sin A/2}$$

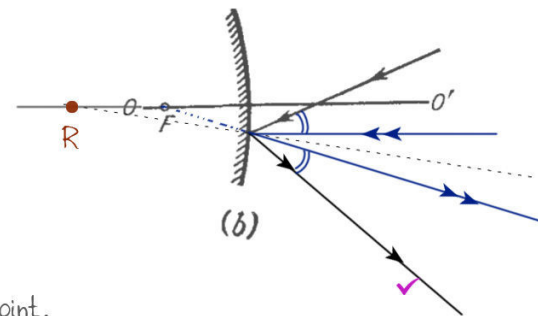
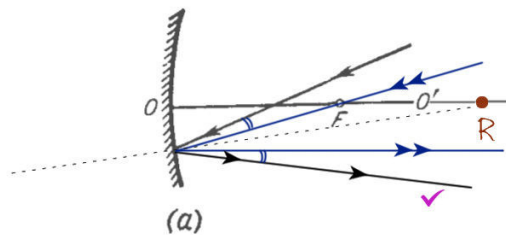
↓ differentiate

$$d\mu = \frac{1}{2} \frac{\cos\left(\frac{\delta_m + A}{2}\right)}{\sin A/2} d\delta_m$$

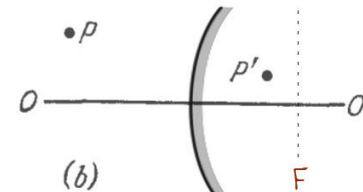
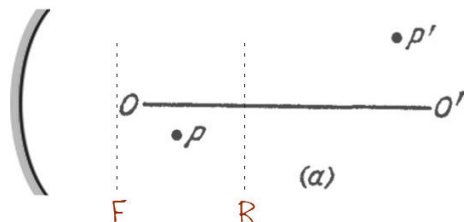
$$d\delta_m = \Delta\alpha = \frac{2 d\mu \sin A/2}{\cos\left(\frac{\delta_m + A}{2}\right)} = \frac{2 \sin A/2}{\sqrt{1 - \mu^2 \sin^2 A/2}} d\mu = \frac{2 \sin 30^\circ (1.520 - 1.515)}{\sqrt{1 - (1.515)^2 \sin^2 30^\circ}} = 7.6 \times 10^{-3} \text{ rad} = 0.44^\circ //$$

Youtube / Arihant Senior

5.26. A plot the path of ray



B Draw approximate position of mirror and focal point.



Object P is in between F and R of concave mirror
Image P' forms between R and ∞

Object P is in front of convex mirror.
Image P' forms between pole and focus.

Youtube / Arihant Senior

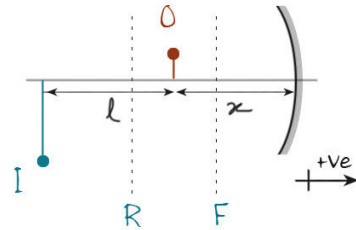
5.27. Determine the focal length of a concave mirror if:
 (a) with the distance between an object and its image being equal
 to $l = 15$ cm, the transverse magnification $\beta = -2.0$;

A $\beta = -2$

→ -ve means real image

→ $|\beta| > 1$ means enlarged image.

∴ object is between F and R. And image is beyond R.



$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{-(l+x)} + \frac{1}{-x} = \frac{1}{f} \quad (1)$$

magnification: $\beta = \frac{-v}{u} = -\left[\frac{-(l+x)}{-x}\right]$

$$\Rightarrow x = \frac{-l}{\beta + 1} \quad (2)$$

From 1,2: $f = \frac{\beta l}{\beta^2 - 1} //$

Youtube / Arihant Senior

5.27(b) in a certain position of the object the transverse magnification is $\beta_1 = -0.50$ and in another position displaced with respect to the former by a distance $l = 5.0$ cm the transverse magnification $\beta_2 = -0.25$.

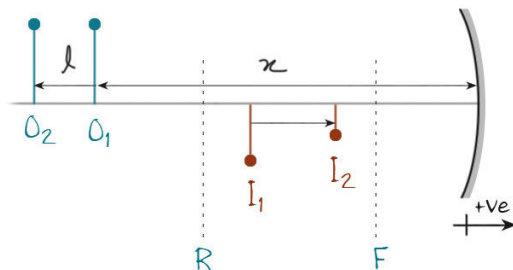
$$\beta_1 = -0.5 \rightarrow \beta_2 = -0.25$$

→ -ve means real images.

→ $|\beta| < 1$ means diminished images.

∴ objects are beyond R. And images are between F and R.

→ $|\beta|$ reduces, so object is moved further away from R.



magnification: $\beta = \frac{-v}{u} = \frac{f}{f-u}$ $v = \frac{uf}{u-f}$

$$\therefore \beta_1 = \frac{f}{f - (-x)} \quad (1)$$

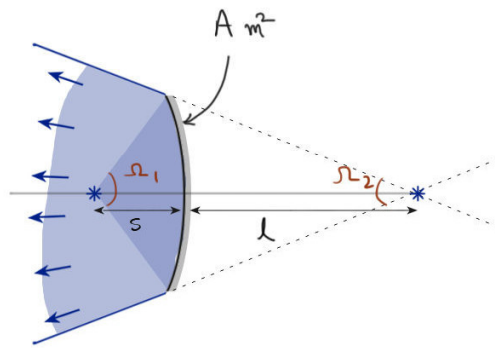
$$\beta_2 = \frac{f}{f - [-(l+x)]} \quad (2)$$

From 1,2:

$$f = \frac{\beta_1 \beta_2 l}{\beta_1 - \beta_2} //$$

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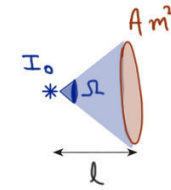
5.28. A point source with luminous intensity $I_0 = 100 \text{ cd}$ is positioned at a distance $s = 20.0 \text{ cm}$ from the crest of a concave mirror with focal length $f = 25.0 \text{ cm}$. Find the luminous intensity of the reflected ray if the reflection coefficient of the mirror is $\rho = 0.80$.



$$v = \frac{uf}{u-f} = \frac{(-20)(-25)}{-20 - (-25)} = 100 = l$$

Luminous intensity: Light energy emitted per second per unit solid angle.

$$\therefore \text{Energy falling on area } A \text{ per second} = I_0 \Omega = I_0 \frac{A}{l^2}$$



Sol:

Energy reflected by mirror = η Energy incident on mirror

$$\therefore I_2 \Omega_2 = \eta I_0 \Omega_1$$

$$\Rightarrow I_2 \frac{A}{l^2} = \eta I_0 \frac{A}{s^2}$$

$$\Rightarrow I_2 = \eta \frac{l^2}{s^2} I_0 = (0.8) \frac{100^2}{20^2} \cdot 100 = 2000 \text{ cd}$$

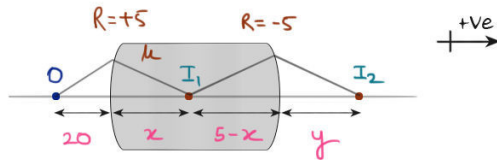
This is how intensity of vehicle headlights and torches is increased

Youtube / Arihant Senior

5.31. A point source is located at a distance of 20 cm from the front surface of a symmetrical glass biconvex lens. The lens is 5.0 cm thick and the curvature radius of its surfaces is 5.0 cm . How far beyond the rear surface of this lens is the image of the source formed?

Lets assume I_1 and I_2 as shown in diagram.

glass $\mu = 1.5$



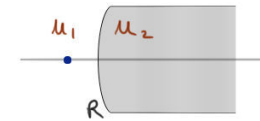
Refraction on first surface:

$$\frac{\mu}{x} - \frac{1}{(-20)} = \frac{\mu-1}{5} \Rightarrow x = 30$$

Refraction on second surface:

$$\frac{1}{y} - \frac{\mu}{-(5-x)} = \frac{1-\mu}{(-5)} \Rightarrow \frac{1}{y} = \frac{8}{50} \Rightarrow y = 6.25$$

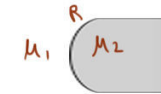
We know,



$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

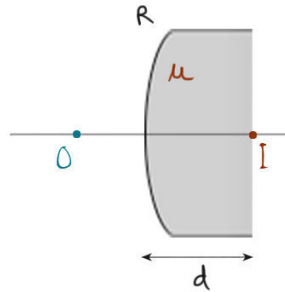
5.32. An object is placed in front of convex surface of a glass plano-convex lens of thickness $d = 9.0$ cm. The image of that object is formed on the plane surface of the lens serving as a screen. Find:
 (a) the transverse magnification if the curvature radius of the lens's convex surface is $R = 2.5$ cm;

For a medium change: ☐ ☐ ☐



$$\rightarrow \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\rightarrow m = \frac{\mu_1 v}{\mu_2 u}$$



$$\frac{\mu}{d} - \frac{1}{u} = \frac{\mu - 1}{R}$$

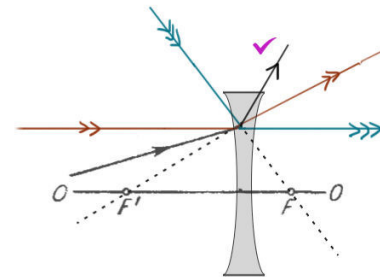
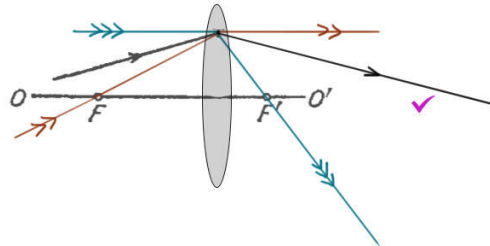
$$\Rightarrow u = \frac{dR}{\mu R - d(\mu - 1)}$$

$$\text{Also, } m = \frac{(1)v}{\mu u} = \frac{(1)d}{\mu u} = \left[\frac{\mu R - d(\mu - 1)}{\mu d R} \right] d = 1 - \frac{d}{R} \left(\frac{\mu - 1}{\mu} \right) //$$

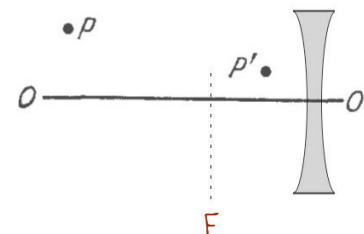
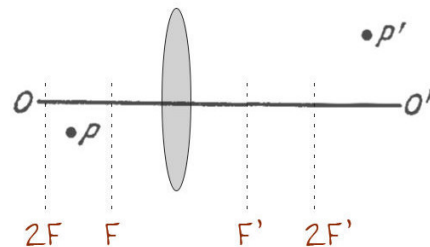
Youtube / Arihant Senior

5.34. plot

A



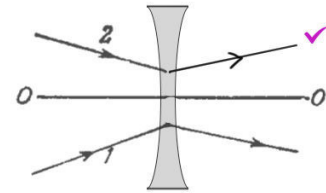
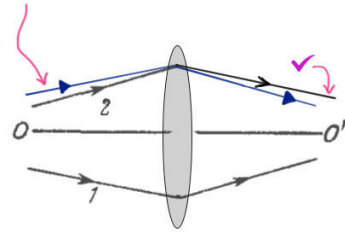
B



Youtube / Arihant Senior

5.34. C

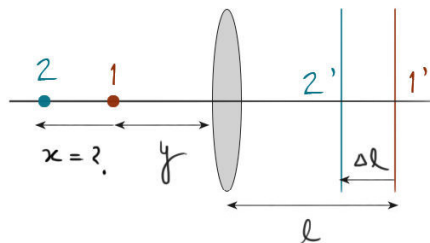
Mirror of 1



∴ incident 1 and 2 are symmetric

Youtube / Arihant Senior

5.35. A thin converging lens with focal length $f = 25$ cm projects the image of an object on a screen removed from the lens by a distance $l = 5.0$ m. Then the screen was drawn closer to the lens by a distance $\Delta l = 18$ cm. By what distance should the object be shifted for its image to become sharp again?

Method 2 ∴ $\Delta l \ll l$ Differentiating ① $-\frac{1}{l^2} dl - \frac{1}{y^2} dy = 0$

$$dy = -y^2 \frac{dl}{l^2} = -\left(\frac{fl}{l-f}\right)^2 \frac{dl}{l^2} = \frac{-f^2 dl}{(l-f)^2} = +0.5 \text{ mm}$$

y increases

Method 1

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

On initial screen:

$$\frac{1}{l} - \frac{1}{-y} = \frac{1}{f} \quad (1)$$

After moving screen:

$$\frac{1}{l-\Delta l} - \frac{1}{-(x+y)} = \frac{1}{f} \quad (2)$$

From 1, 2: $x = 0.5 \text{ mm}$

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Displacement method to find focal length of convex lens:

A Finding f in terms of D and x .

$$D = u_0 + v_0$$

$$x = D - 2u_0$$

$$u_0 = \frac{D-x}{2}$$

$$v_0 = \frac{D+x}{2}$$

For 1

$$f = \frac{uv}{u-v} = \frac{(-u_0)v_0}{(-u_0)-v_0} = \frac{u_0v_0}{u_0+v_0} = \frac{D^2-x^2}{4D}$$

B Finding f in terms of D and magnification ratio I_1/I_2

$$m = \frac{v}{u} \Rightarrow m_1 = \frac{v_0}{(-u_0)}, m_2 = \frac{u_0}{(-v_0)}$$

$$\therefore m_1 m_2 = 1 //$$

$$\Rightarrow \left(\frac{I_1}{O}\right)\left(\frac{I_2}{O}\right) = 1 \Rightarrow O = \sqrt{I_1 I_2} //$$

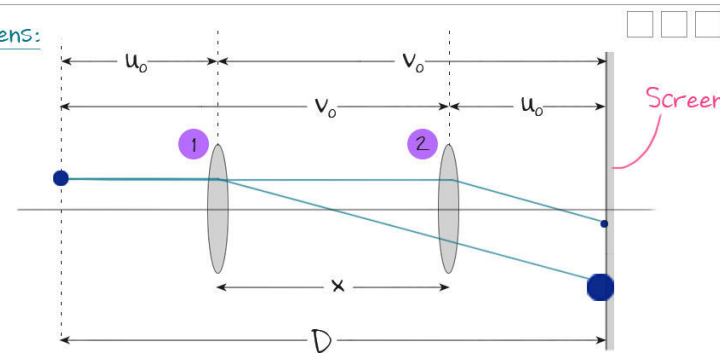
$$\text{Let, } \frac{m_1}{m_2} = \eta = \frac{v_0^2}{u_0^2} \Rightarrow \sqrt{\eta} = \frac{v_0}{u_0}$$

$$D = u_0 + v_0$$

$$u_0 = \frac{D}{1+\sqrt{\eta}}, v_0 = \frac{\sqrt{\eta} D}{1+\sqrt{\eta}}$$

$$f = \frac{u_0 v_0}{u_0 + v_0} = \frac{D \sqrt{\eta}}{(1+\sqrt{\eta})^2} //$$

Youtube/Arihant Senior



5.36. A source of light is located at a distance $D = 90 \text{ cm}$ from a screen. A thin converging lens provides the sharp image of the source when placed between the source of light and the screen at two positions. Determine the focal length of the lens if

(a) the distance between the two positions of the lens is $x = 30 \text{ cm}$;

(b) the transverse dimensions of the image at one position of the lens are $\eta = 4.0$ greater than those at the other position.

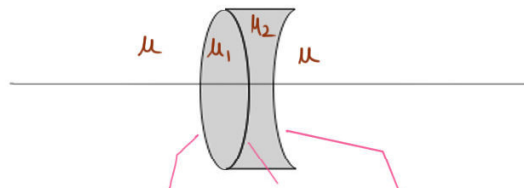
$$\text{A } f = \frac{D^2 - x^2}{4D} = \frac{90^2 - 30^2}{4(90)} = 20 \text{ cm} //$$

$$\text{B } f = \frac{D \sqrt{\eta}}{(1 + \sqrt{\eta})^2} = \frac{90 \sqrt{4}}{(1 + \sqrt{4})^2} = 20 \text{ cm} //$$

5.37. A thin converging lens is placed between an object and a screen whose positions are fixed. There are two positions of the lens at which the sharp image of the object is formed on the screen. Find the transverse dimension of the object if at one position of the lens the image dimension equals $h' = 2.0 \text{ mm}$ and at the other, $h'' = 4.5 \text{ mm}$.

$$O = \sqrt{I_1 I_2} = \sqrt{2(4.5)} = 3 \text{ mm} //$$

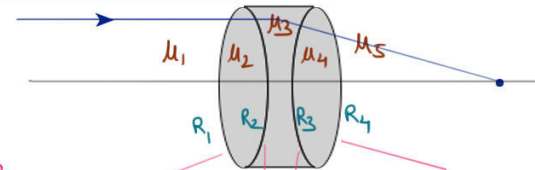
5.40. There are two thin symmetrical lenses: one is converging, with refractive index $n_1 = 1.70$, and the other is diverging with refractive index $n_2 = 1.51$. Both lenses have the same curvature radius of their surfaces equal to $R_0 = 10$ cm. The lenses were put close together and submerged into water. What is the focal length of this system in water?



$$\frac{\mu}{f_{eq}} = \frac{\mu_1 - \mu}{R_0} + \frac{\mu_2 - \mu_1}{-R_0} + \frac{\mu - \mu_2}{R_0}$$

$$\Rightarrow \frac{\mu}{f_{eq}} = \frac{\mu_1 - \mu - \mu_2 + \mu_1 + \mu - \mu_2}{R_0}$$

$$\Rightarrow f_{eq} = \frac{\mu R_0}{2(\mu_1 - \mu_2)} = \frac{\frac{4}{3} \cdot 10}{2(1.7 - 1.51)} = 35 \text{ cm} //$$



We know,

$$\frac{\mu_5}{f_{eq}} = \frac{\mu_2 - \mu_1}{R_1} + \frac{\mu_3 - \mu_2}{R_2} + \frac{\mu_4 - \mu_3}{R_3} + \frac{\mu_5 - \mu_4}{R_4}$$

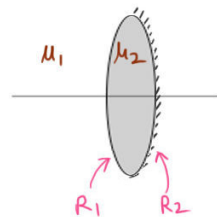
Put with appropriate signs

Note: Focal length will be different if incident ray is striking from right side.

As μ_1 is not same as μ_5 Youtube / Arihant Senior

5.41. Determine the focal length of a concave spherical mirror which is manufactured in the form of a thin symmetric biconvex glass lens one of whose surfaces is silvered. The curvature radius of the lens surface is $R_0 = 40$ cm.

Method 1 Using formula:

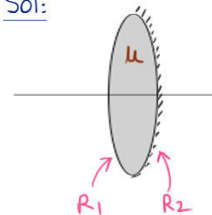


Put with appropriate signs

ex: $R_1 = +5 \text{ cm}$
 $R_2 = -10 \text{ cm}$

$$\frac{\mu_1}{f_{eq}} = \frac{2\mu_2}{R_2} - \frac{2(\mu_2 - \mu_1)}{R_1}$$

Sol:



$$\frac{\mu_1}{f_{eq}} = \frac{2\mu_2}{R_2} - \frac{2(\mu_2 - \mu_1)}{R_1}$$

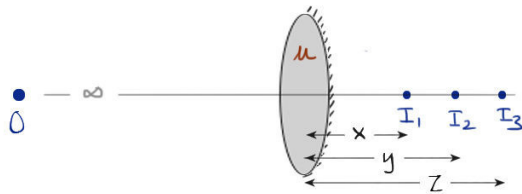
$\begin{matrix} \nearrow \mu & \nearrow \mu \\ \searrow -R_0 & \searrow +R_0 \end{matrix}$

$$\frac{1}{f_{eq}} = \frac{-2\mu}{R_0} - \frac{2(\mu - 1)}{R_0}$$

$$\Rightarrow f_{eq} = \frac{-R_0}{2(2\mu - 1)} //$$

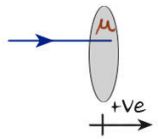
Method 2

$\xrightarrow{+ve}$ (Sign convention I follow. Its always +ve towards right)



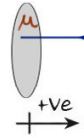
- Let the object be at infinity at left side of system.
- After passing through lens first time, let the image be I_1 at distance x .
- After reflection from mirror, let the image be I_2 at distance y .
- After passing through lens again, let the final image be I_3 at distance z . This will be at focus of equivalent mirror
- $z = f_{eq}$ (as per our sign convention $\xrightarrow{+ve}$)

For a biconvex lens:



$$\frac{1}{f_L} = \frac{\mu-1}{+R_o} + \frac{1-\mu}{-R_o}$$

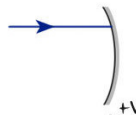
$$\Rightarrow f_L = \frac{R_o}{2(\mu-1)}$$



$$\frac{1}{f_L} = \frac{\mu-1}{-R_o} + \frac{1-\mu}{+R_o}$$

$$f_L = \frac{-R_o}{2(\mu-1)}$$

For a concave mirror:

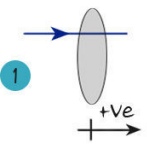


$$f_m = \frac{-R_o}{2}$$

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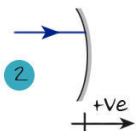
1



$$f_L = \frac{R_o}{2(\mu-1)}$$

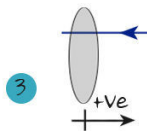
$$x = \frac{R_o}{2(\mu-1)}$$

2



$$f_m = -\frac{R_o}{2}$$

3



$$f_L = \frac{-R_o}{2(\mu-1)}$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{y} + \frac{1}{x} = \frac{1}{-R_o/2}$$

$$\Rightarrow \frac{1}{y} + \frac{1}{\frac{R_o}{2(\mu-1)}} = \frac{1}{-\frac{R_o}{2}}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{z} - \frac{1}{y} = \frac{2(\mu-1)}{-R_o}$$

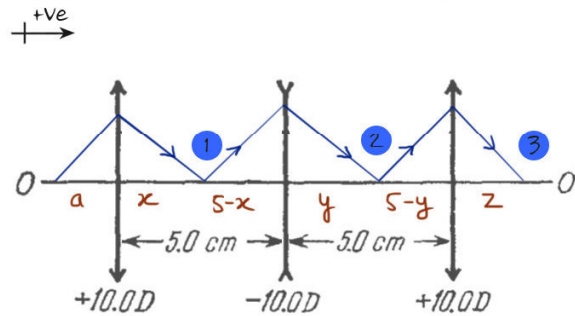
$$\Rightarrow \frac{1}{f_{eq}} + \frac{2(\mu-1)}{R_o} + \frac{2}{R_o} = \frac{2(\mu-1)}{-R_o}$$

$$\Rightarrow f_{eq} = \frac{-R_o}{2(2\mu-1)} //$$

Youtube / Arihant Senior

5.42. Figure 5.9 illustrates an aligned system consisting of three thin lenses. The system is located in air. Determine:

- (a) the position of the point of convergence of a parallel ray incoming from the left after passing through the system;
 (b) the distance between the first lens and a point lying on the axis to the left of the system, at which that point and its image are located symmetrically with respect to the lens system.



$\Rightarrow f = 10 \text{ cm}$ $f = -10$ $f = 10$
 $(\because f = \frac{1}{p})$

$$\frac{1}{x} - \frac{1}{-a} = \frac{1}{10}$$

$$\frac{1}{y} - \frac{1}{-(5-x)} = \frac{1}{-10}$$

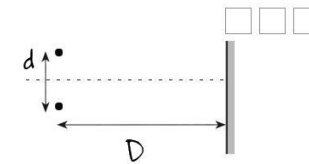
$$\frac{1}{z} - \frac{1}{-(5-y)} = \frac{1}{10}$$

A $a = -\infty$
 $\Rightarrow x = 10$
 $\Rightarrow y = 10$
 $\Rightarrow z = 10/3 //$

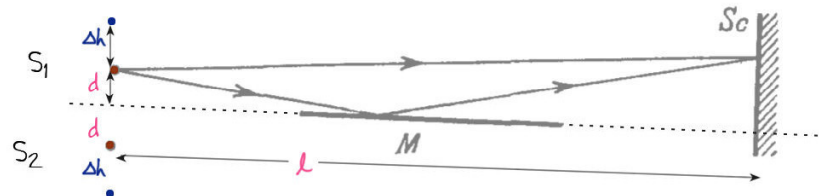
B $a = z$
 $\Rightarrow a = 50/3 //$

Youtube / Arihant Senior

5.69. In Lloyd's mirror experiment (Fig. 5.13) a light wave emitted directly by the source S (narrow slit) interferes with the wave reflected from a mirror M . As a result, an interference fringe pattern is formed on the screen Sc . The source and the screen are separated by a distance $l = 100 \text{ cm}$. At a certain position of the source the fringe width on the screen was equal to $\Delta x = 0.25 \text{ mm}$, and after the source was moved away from the mirror plane by $\Delta h = 0.60 \text{ mm}$, the fringe width decreased $\eta = 1.5$ times. Find the wavelength of light.



In YDSE,
 Fringe width = $\frac{\lambda D}{d}$



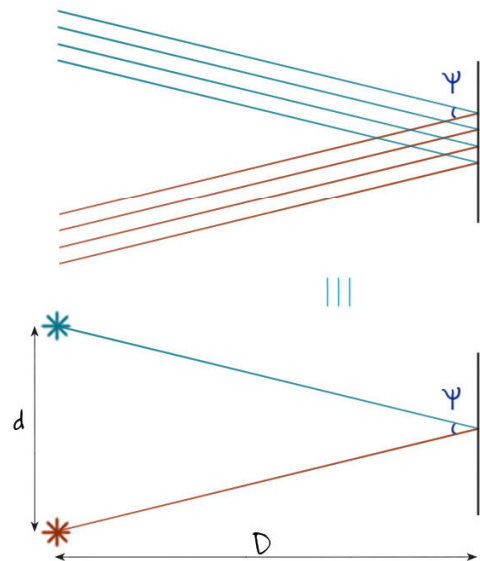
Initial fringe width: $\Delta x = \frac{\lambda l}{2d}$

Final fringe width: $\frac{\Delta x}{\eta} = \frac{\lambda l}{2(d+\Delta h)}$

Eliminating d $\lambda = \frac{2 \Delta x \Delta h}{l (\eta - 1) //$

Youtube / Arihant Senior 206

5.70. Two coherent plane light waves propagating with a divergence angle $\psi \ll 1$ fall almost normally on a screen. The amplitudes of the waves are equal. Demonstrate that the distance between the neighbouring maxima on the screen is equal to $\Delta x = \lambda/\psi$, where λ is the wavelength.



$$\psi \ll 1 \Rightarrow \psi = \frac{d}{D}$$

$$\text{Fringe width} = \frac{\lambda D}{d} = \frac{\lambda}{\psi}$$

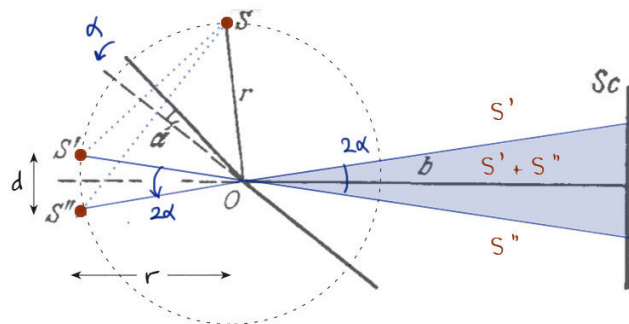
Youtube / Arihant Senior

5.71. Figure 5.14 illustrates the interference experiment with Fresnel mirrors. The angle between the mirrors is $\alpha = 12'$, the distances from the mirrors' intersection line to the narrow slit S and the screen Sc are equal to $r = 10.0$ cm and $b = 130$ cm respectively. The wavelength of light is $\lambda = 0.55$ μm . Find:

(a) the width of a fringe on the screen and the number of possible maxima;

If mirror rotates by α , image rotates by 2α in the same sense

Also, S, S', S'', all lie on the same circle (center O about which mirror is rotated)



$$\text{Fringe width} = \beta = \frac{\lambda D}{d} = \frac{\lambda (r+b)}{r(2\alpha)} = 1.1 \text{ mm}$$

Region where light from both S' and S'' will fall.

$$\text{No. of maxima} = 2 \left[\frac{\frac{b(2\alpha)}{2}}{\beta} \right] + 1 = 2 [4.15] + 1$$

$$= 8 + 1 = 9$$

We know that,

Total maxima =

$$2[n] + 1$$

Central maxima

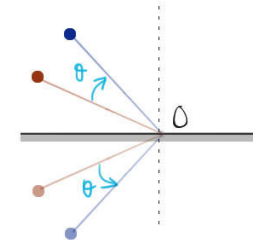
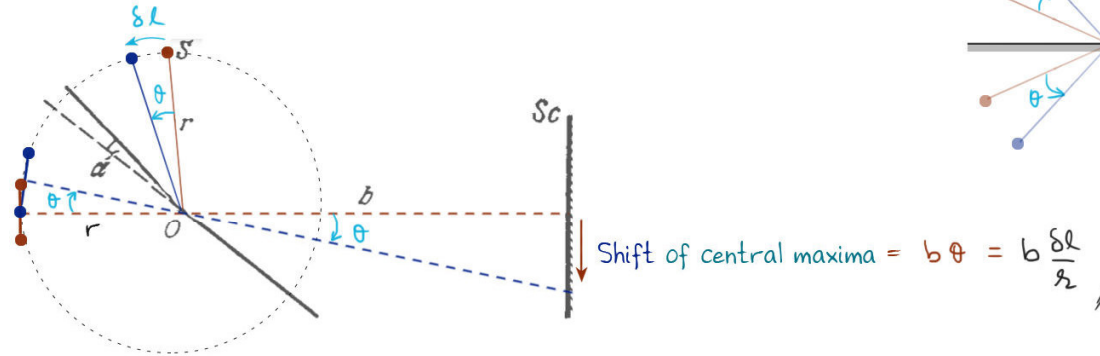
No. of fringes on half the screen

Greatest Integer Function

Youtube / Arihant Senior 207

(b) the shift of the interference pattern on the screen when the slit is displaced by $\delta l = 1.0 \text{ mm}$ along the arc of radius r with centre at the point O ;

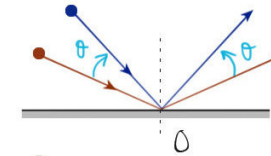
If object rotates by θ , image rotates by θ in the opposite sense, about O .



Another way to analyse:

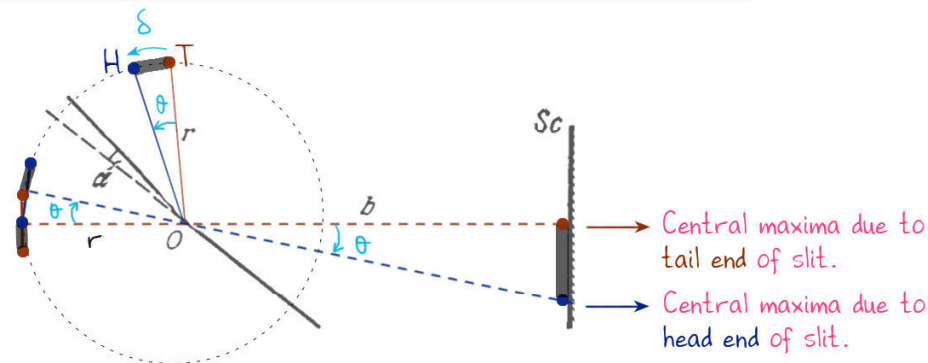
If object rotates by θ about O , reflected rays rotate by θ in the opposite sense, about O .

And line of central maxima passes through O . Therefore central maxima will shift by $b\theta$



Youtube / Arihant Senior

(c) at what maximum width δ_{max} of the slit the interference fringes on the screen are still observed sufficiently sharp.



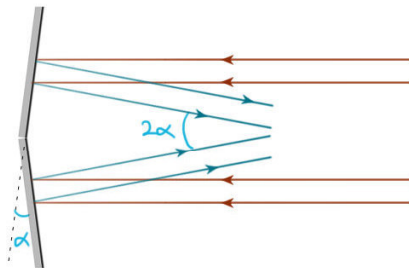
For sharp fringes, Maxima of head must not coincide with minima of tail.

$$\therefore b\theta < \frac{\text{fringe width}}{2}$$

$$\Rightarrow b \frac{\delta_{max}}{r} < \frac{1}{2} \left(\frac{\lambda(r+b)}{\lambda(2a)} \right) \Rightarrow \delta_{max} < \frac{\lambda(r+b)}{4b} \propto //$$

5.72. A plane light wave falls on Fresnel mirrors with an angle $\alpha = 2.0'$ between them. Determine the wavelength of light if the width of the fringe on the screen $\Delta x = 0.55$ mm.

If mirrors are at angle α , then reflected rays are at angle 2α to each other.



So we have interference due to two set of rays at angle 2α with each other.

$$\therefore \text{Fringe width } \Delta x = \frac{\lambda}{2\alpha} \quad (\text{derived in 5.70})$$

$$\Rightarrow \lambda = 2\alpha \Delta x //$$

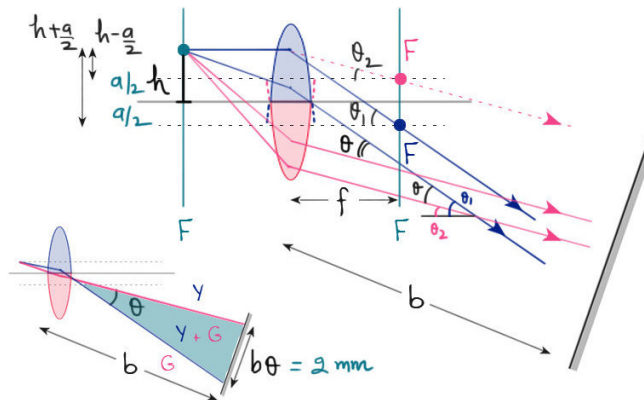
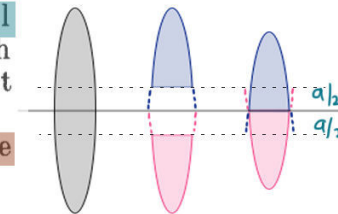
Youtube / Arihant Senior

5.73. A lens of diameter 5.0 cm and focal length $f = 25.0$ cm was cut along the diameter into two identical halves. In the process, the layer of the lens $a = 1.00$ mm in thickness was lost. Then the halves were put together to form a composite lens. In this focal plane a narrow slit was placed, emitting monochromatic light with wavelength $\lambda = 0.60$ μm . Behind the lens a screen was located at a distance $b = 50$ cm from it. Find:

(a) the width of a fringe on the screen and the number of possible maxima;

Let the slit be at height h in focal plane.

Prerequisite:
5.70, 5.71



$$\theta = \theta_1 - \theta_2 = \frac{h + \frac{a}{2}}{f} - \frac{h - \frac{a}{2}}{f} = \frac{a}{f} = 4 \times 10^{-3} \text{ rad}$$

$$\therefore \text{Fringe width} = \beta = \frac{\lambda}{\theta} = \frac{\lambda f}{a} = 0.15 \text{ mm} //$$

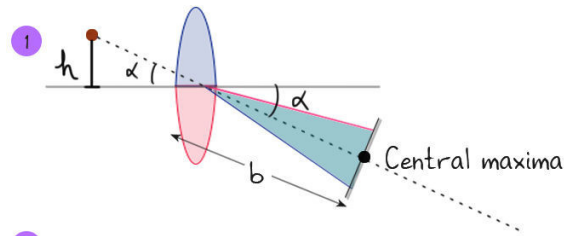
$$\therefore \text{Possible maxima} = 2 \left[\frac{b\theta}{\beta} \right] + 1$$

$$= 2[6.67] + 1$$

$$= 2(6) + 1 = 13 //$$

Youtube / Arihant Senior

(b) the maximum width of the slit δ_{max} at which the fringes on the screen will be still observed sufficiently sharp.



When instead of point source, slit becomes δ long:
Central maxima spreads out.

From 1: $\alpha = h/f$

From 2: $\alpha + d\alpha = \frac{\delta + h}{f}$

$\Rightarrow d\alpha = \delta/f$

In order to still see sufficient contrast, central maxima of head should not spread out till minima of tail.

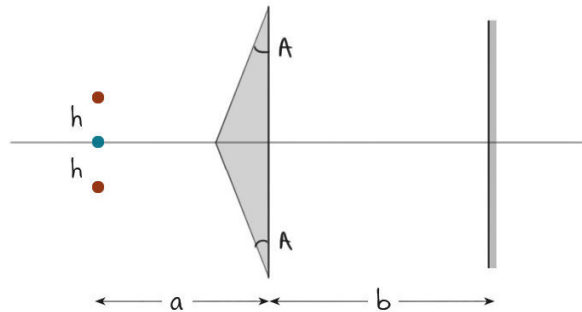
$b d\alpha < \frac{b}{2}$

$\Rightarrow b \frac{\delta}{f} < \frac{\lambda f}{2a} \Rightarrow \delta_{max} = \frac{\lambda f^2}{2ab} = 37 \mu m$

Youtube / Arihant Senior

5.74. The distances from a Fresnel biprism to a narrow slit and a screen are equal to $a = 25$ cm and $b = 100$ cm respectively. The refracting angle of the glass biprism is equal to $\theta = 20'$. Find the wavelength of light if the width of the fringe on the screen is $\Delta x = 0.55$ mm.

$1^\circ = 60' = 3600''$
deg min sec

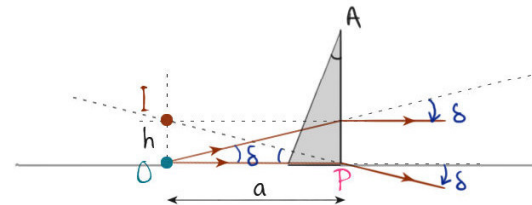


Fringe width: $\Delta x = \frac{\lambda D}{d} = \frac{\lambda (a+b)}{2h} = \frac{\lambda (a+b)}{2a(\mu-1)A}$
 $\Rightarrow \lambda = \frac{2a(\mu-1)\Delta x A}{a+b}$

Image of an axial object by a thin prism:

For small angled prism:

deviation $\delta = (\mu-1)A$

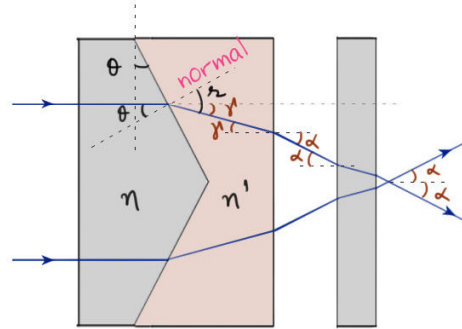
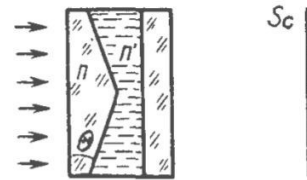


$\triangle IOP \quad \tan \delta = \frac{h}{a}$

$\Rightarrow h = a \tan \delta \approx a \delta = a(\mu-1)A$

Youtube / Arihant Senior

5.75. A plane light wave with wavelength $\lambda = 0.70 \mu\text{m}$ falls normally on the base of a biprism made of glass ($n = 1.520$) with refracting angle $\theta = 5.0^\circ$. Behind the biprism (Fig. 5.15) there is a plane-parallel plate, with the space between them filled up with benzene ($n' = 1.500$). Find the width of a fringe on the screen S_c placed behind this system.



Fringe width $\Delta x = \frac{\lambda}{2\alpha}$ (from 5.70)

$$\Rightarrow \Delta x = \frac{\lambda}{2 \left[n' \cdot \theta \left(\frac{n}{n'} - 1 \right) \right]}$$

$$= \frac{\lambda}{2\theta(n-n')} //$$

$$n' \sin \delta = 1 \sin \alpha$$

$$\Rightarrow n' \delta = \alpha$$

$$n \sin \theta = n' \sin \delta$$

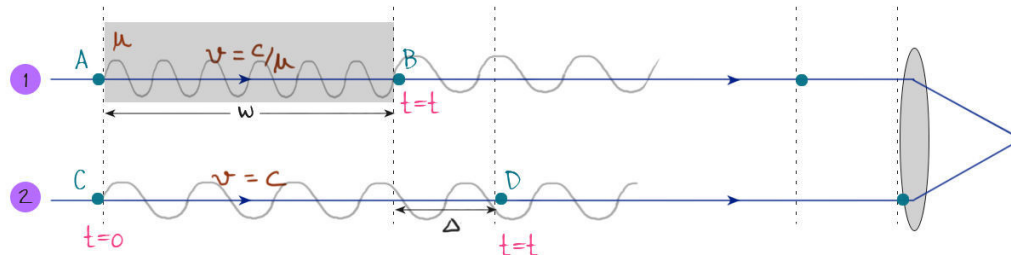
$$n \sin \theta = n' \sin (\theta + \delta)$$

$$n \theta = n' (\theta + \delta)$$

$$\delta = \theta \left(\frac{n}{n'} - 1 \right)$$

Youtube / Arihant Senior

5.76 Optical path length due to medium = $\mu \omega$

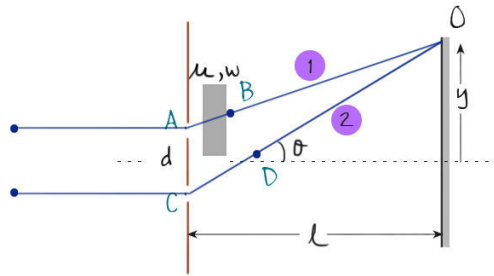


$$ct = \cancel{c} \left(\frac{\omega}{\cancel{c}/\mu} \right) = \mu \omega \text{ (Equivalent optical path length in free space for light ray 1)}$$

For interference at screen

Path difference between 1 and 2: $\Delta = \mu \omega - \omega = (\mu - 1) \omega$

YDSE due to a film covering one slit:



For ray 2 from C to O,
Optical path length = CO (In air only)

For ray 1 from A to O,
Optical path length = $(AO - w) + \mu w$
In air In medium

Path difference at O due to two rays:

$$\Delta x = CO - [(AO - w) + \mu w]$$

$$= [CO - AO] - (\mu - 1)w$$

$$\Rightarrow \Delta x = \frac{dy}{D} - (\mu - 1)w$$

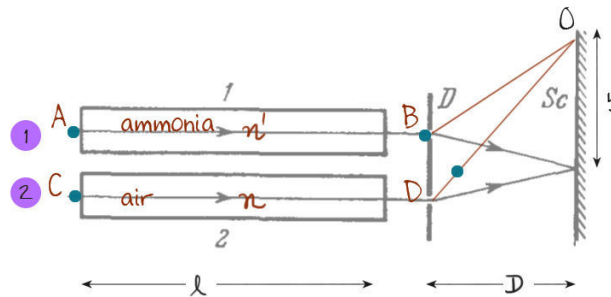
Location on screen

$$y = \frac{(\mu - 1)wD}{d}$$

O for central maxima
 $N\lambda$ for constructive interference
 $(2N+1)\frac{\lambda}{2}$ for destructive interference

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5.77 Find n' if central maxima shifts N times fringe width.



For ray 2 from C to O,
Optical path length = $n\ell + DO$

For ray 1 from A to O,
Optical path length = $n'\ell + BO$

Path difference at O due to two rays:

$$\Delta x = (n\ell + DO) - (n'\ell + BO)$$

$$= (n - n')\ell + (DO - BO)$$

$$\Rightarrow \Delta x = (n - n')\ell + \frac{dy}{D}$$

For central maxima:

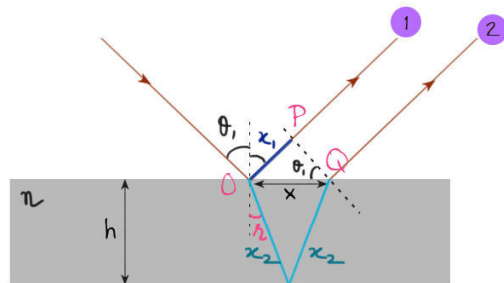
$$\Delta x = (n - n')\ell + \frac{dy}{D}$$

$N\left(\frac{\lambda D}{d}\right)$ given

$$\Rightarrow (n' - n)\ell = \frac{d}{D} \frac{N\lambda D}{d} \Rightarrow n' = \frac{N\lambda}{\ell} + n //$$

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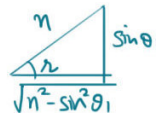
5.79. A parallel beam of white light falls on a thin film whose refractive index is equal to $n = 1.33$. The angle of incidence is $\theta_1 = 52^\circ$. What must the film thickness be equal to for the reflected light to be coloured yellow ($\lambda = 0.60 \mu\text{m}$) most intensively?



$$x_2 = \frac{h}{\cos r_2}$$

$$\begin{aligned} x_1 &= x \sin \theta_1 \\ &= (2x_2 \sin r_2) \sin \theta_1 \\ &= \left(2 \frac{h}{\cos r_2} \sin r_2\right) \sin \theta_1 \\ &= 2h \tan r_2 \sin \theta_1 \end{aligned}$$

$$\sin \theta_1 = n \sin r_2$$



Path traveled by 1, from O to P:

$$\delta_1 = x_1 + \frac{\lambda}{2} \rightarrow \text{Due to reflection from denser medium glass.}$$

Path traveled by 2, from O to Q:

$$\delta_2 = n(2x_2)$$

For constructive interference: $\delta_1 - \delta_2 = N\lambda$

$$\Rightarrow 2h \tan r_2 \sin \theta_1 + \frac{\lambda}{2} - \frac{n 2h}{\cos r_2} = N\lambda$$

$$\Rightarrow 2h \frac{\sin^2 \theta_1}{\sqrt{n^2 - \sin^2 \theta_1}} - \frac{2nh}{\sqrt{n^2 - \sin^2 \theta_1}} = \frac{(2N+1)\lambda}{2}$$

$$\Rightarrow h = \frac{\lambda (2N+1)}{4 \sqrt{n^2 - \sin^2 \theta_1}} \quad // \text{ Youtube / Arihant Senior}$$

5.80. Find the minimum thickness of a film with refractive index 1.33 at which light with wavelength $0.64 \mu\text{m}$ experiences maximum reflection while light with wavelength $0.40 \mu\text{m}$ is not reflected at all. The incidence angle of light is equal to 30° .

For constructive interference: $h = \frac{\lambda (2N_1+1)}{4 \sqrt{n^2 - \sin^2 \theta_1}}$ (1) (Derived in 5.79)

For destructive interference: $h = \frac{\lambda}{2 \sqrt{n^2 - \sin^2 \theta_1}}$ (2)

By modifying formula 1 as:

$$\frac{(2N_1+1)\lambda}{2} \rightarrow N_2 \lambda$$

$$\text{Sol: } \frac{\lambda_1 (2N_1+1)}{2 \sqrt{n^2 - \sin^2 \theta_1}} = \frac{\lambda_2 N_2}{2 \sqrt{n^2 - \sin^2 \theta_1}}$$

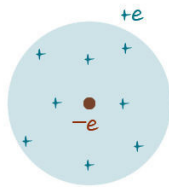
$$\Rightarrow (0.64)(2N_1+1) = 2(0.4) N_2$$

By hit and trial, minimum values are: $N_1 = 2, N_2 = 4$

$\therefore$ From 2,

$$\begin{aligned} h &= \frac{0.4 \cdot 10^{-6}}{2} \cdot \frac{4}{\sqrt{\left(\frac{4}{3}\right)^2 - \frac{1}{4}}} \\ &= 0.65 \mu\text{m} \end{aligned}$$

6.1. Employing Thomson's model, calculate the radius of a hydrogen atom and the wavelength of emitted light if the ionization energy of the atom is known to be equal to $I_e = 13.6 \text{ eV}$.



Potential inside uniformly charged sphere is given by:

$$V = \frac{kq}{R} \left(\frac{3}{2} - \frac{r^2}{2R^2} \right) \quad \text{(At center)}$$

$$\Rightarrow V = \frac{3ke}{2R}$$

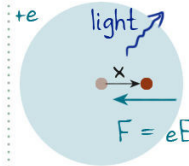
∴ Ionisation energy of electron:

$$|I_e| = eV$$

$$\Rightarrow I_e = \frac{3ke^2}{2R}$$

$$\Rightarrow R = \frac{3ke^2}{2I_e} \quad (k=1 \text{ in gaussian system})$$

f of vibrating electron = f of emitted light.



$$E = -\frac{kq}{R^3} x \Rightarrow F = -\frac{ke^2}{R^3} x$$

$$\Rightarrow a = -\left(\frac{ke^2}{m_e R^3}\right) x$$

Comparing with standard SHM equation: $a = -\omega^2 x$

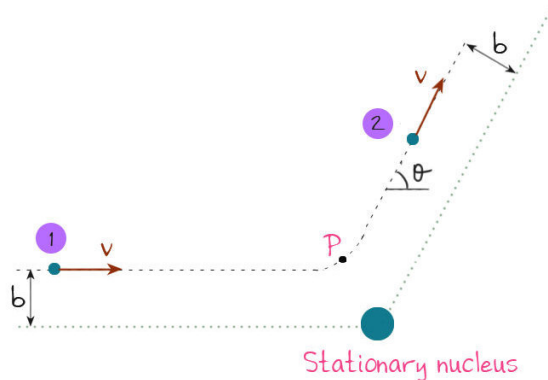
(Equation of SHM)

$$\omega = \sqrt{\frac{ke^2}{m_e R^3}} \Rightarrow \frac{2\pi v}{\lambda} = \sqrt{\frac{ke^2}{m_e R^3}}$$

$$\Rightarrow \lambda = 2\pi c \sqrt{\frac{m_e R^3}{ke^2}}$$

Youtube / Arihant Senior

6.2. An alpha particle with kinetic energy 0.27 MeV is deflected through an angle of 60° by a golden foil. Find the corresponding value of the aiming parameter.



By conservation of energy: $\frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2$
 $\Rightarrow v_1 = v_2$

By conservation of angular momentum about nucleus: $mv_1 b_1 = mv_2 b_2$
 $\Rightarrow b_1 = b_2$

Deviation is given by: $\tan \frac{\theta}{2} = \frac{kq_1 q_2}{2bT}$
 $\rightarrow KE = \frac{1}{2}mv^2$

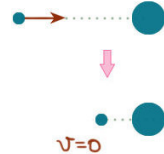
$$\Rightarrow b = 0.73 \text{ pm} \quad \left(\begin{matrix} q_1 = 2e \\ q_2 = 79e \end{matrix} \right)$$

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6.3. To what minimum distance will an alpha particle with kinetic energy $T = 0.40$ MeV approach in the case of a head-on collision to

- (a) a stationary Pb nucleus; ($q = 82e$)
 (b) a stationary free Li⁷ nucleus? ($q = 3e$)
 (to move)

A Energy conservation:
 when alpha stops.

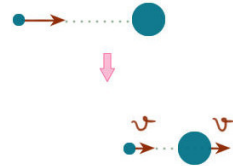


$$\Delta KE + \Delta U = 0$$

$$(K_f - K_i) + (U_f - U_i) = 0$$

$$\Rightarrow K_i = \frac{k q_1 q_2}{r} \Rightarrow r = \frac{(9 \times 10^9) (2e) (82e)}{0.4 \text{ MeV}} = 0.6 \text{ pm} \quad [1 \text{ pm} = 10^{-12} \text{ m}]$$

B When both alpha and Li have same velocity,
 distance between them is minimum.



E conservation: $\frac{1}{2} m_\alpha v^2 + \frac{1}{2} m_L v^2 + \frac{k(2e)(3e)}{r} = T$ ①

p conservation: $\sqrt{2m_\alpha T} = (m_\alpha + m_L) v$ ②

From 1, 2 $r = \frac{2kze^2}{T} \left(1 + \frac{m_\alpha}{m_L}\right) = 0.034 \text{ pm}$

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6.20. Demonstrate that the frequency ω of a photon emerging when an electron jumps between neighbouring circular orbits of a hydrogen-like ion satisfies the inequality $\omega_n > \omega > \omega_{n+1}$, where ω_n and ω_{n+1} are the frequencies of revolution of that electron around the nucleus along the circular orbits. Make sure that as $n \rightarrow \infty$ the frequency of the photon $\omega \rightarrow \omega_n$.

For the orbiting electron:

$$\frac{mv^2}{r} = \frac{kze^2}{r^2} \quad v_n = \frac{kze^2}{n\hbar} \quad \omega_n = \frac{v}{r} = \frac{(kze^2)^2 m}{n^3 \hbar^3}$$

$$mv r = n\hbar \quad r_n = \frac{n^2 \hbar^2}{kze^2 m}$$

$$E_n = \frac{1}{2} mv^2 - \frac{kze^2}{r} = \frac{1}{2} m \frac{(kze^2)^2}{n^2 \hbar^2} - \frac{m(kze^2)^2}{n^2 \hbar^2} = -\frac{1}{2} m \frac{(kze^2)^2}{n^2 \hbar^2}$$

For the emitted photon:

$$E_{n+1} - E_n = -\frac{1}{2} m \frac{(kze^2)^2}{\hbar^2} \left[\frac{1}{(n+1)^2} - \frac{1}{n^2} \right] = h\nu = \frac{h\omega}{2\pi} = \hbar\omega$$

$$\omega = \frac{m(kze^2)^2}{2\hbar^3} \left(\frac{1}{n^2} - \frac{1}{(n+1)^2} \right)$$

$\omega \rightarrow$ of photon
 $\omega_n \rightarrow$ of revolving e

(reduced plancks constant) $\hbar = \frac{h}{2\pi}$

Constant C_0

C_0

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To prove

$$\omega_{n+1} < \omega < \omega_n$$

$$\frac{1}{(n+1)^3} < \frac{1}{2} \left(\frac{1}{n^2} - \frac{1}{(n+1)^2} \right) < \frac{1}{n^3}$$

ie. $\frac{1}{(n+1)^3} < \frac{1}{2} \left(\frac{1}{n^2} - \frac{1}{(n+1)^2} \right) < \frac{1}{n^3}$

multiply by $n^2(n+1)^2$

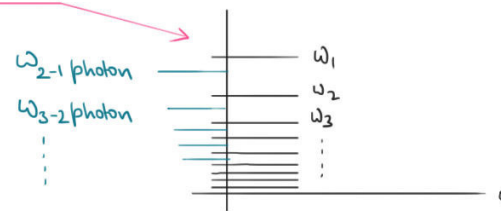
or $\frac{n^2}{(n+1)} < \frac{1}{2} \left[(n+1)^2 - n^2 \right] < \frac{(n+1)^2}{n}$

or $n^{-1} + \frac{1}{n+1} < n + \frac{1}{2} < n + 2 + \frac{1}{n}$

True as LHS < 0 True as RHS > 2

Hence proved

$$\omega_n \propto \frac{1}{n^3}$$



Clearly as $n \rightarrow \infty$

$$\omega_n \approx \omega_{n+1} \approx \omega_{(n+1)} \rightarrow n \text{ photon}$$

Hence proved

(Their values will ofcourse be close to zero.)

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6.21. A particle of mass m moves along a circular orbit in a central symmetrical potential field $U(r) = \frac{kr^2}{2}$. Using the Bohr quantization condition, find the permissible orbital radii and energy levels of that particle.

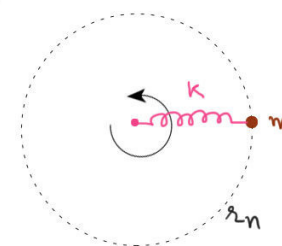
$F = -\frac{dU}{dr} = -kr$
central attractive

$F = ma \Rightarrow kr = \frac{mv^2}{r}$ (1)

L quantization: $\Rightarrow mvr = \frac{nh}{2\pi} = n\hbar$ (2) reduced plancks constant

From 1,2: $r_n = \sqrt{\frac{n\hbar}{\sqrt{mk}}}$, $v_n = \frac{\sqrt{n\hbar\sqrt{mk}}}{m}$

$E_n = K + U$
 $= \frac{1}{2}mv^2 + \frac{1}{2}kr^2 = n\hbar\sqrt{\frac{k}{m}}$



Youtube / Arihant Senior

6.22. Calculate for a hydrogen atom and a He^+ ion:

- (a) the radius of the first Bohr orbit and the velocity of an electron moving along it;
 (b) the kinetic energy and the binding energy of an electron in the ground state;
 (c) the ionization potential, the first excitation potential and the wavelength of the resonance line ($n' = 2 \rightarrow n = 1$).

$$E_n = KE_n + PE_n$$

$$-\frac{1}{2} \frac{kze^2}{r_n} = \frac{1}{2} \frac{kze^2}{r_n} - \frac{kze^2}{r_n}$$

$$-13.6 \frac{z^2}{n^2} = 13.6 \frac{z^2}{n^2} - 2 \frac{13.6 z^2}{n^2}$$

$$r_n = 0.529 \frac{n^2}{Z} \text{ \AA}$$

A H: $z=1, n=1$

He: $z=2, n=1$

$$v_n = 2.2 \times 10^6 \frac{z}{n} \text{ m/s}$$

$$r_1 = 53 \text{ pm}$$

$$r_1 = 53/2 \text{ pm}$$

$$v_1 = 2.2 \times 10^6 \text{ m/s}$$

$$v_1 = (2.2 \times 10^6) \times 2 \text{ m/s}$$

$$E_n = -13.6 \frac{z^2}{n^2} \text{ eV}$$

B $KE_n = E_b = 13.6 \text{ eV}$

$$KE = E_b = (13.6) \times 2^2 \text{ eV}$$

$$|E_n| = |KE_n| = \frac{|U_n|}{2}$$

Energy needed to take the bounded electron away = ionisation energy

$$E_{\text{photon}} = \frac{hc}{\lambda} = 13.6 z^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \text{ (when e jumps from } n_i \text{ to } n_f) \Rightarrow \lambda = \frac{12400 \text{ \AA}}{\Delta E} = \frac{1}{z^2} \frac{912 \text{ \AA}}{\left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)}$$

$\downarrow$ in eV

$$\frac{13.6}{hc} = \text{Rydbergs constant} \approx 1.1 \times 10^7 \text{ m}^{-1} \Rightarrow \frac{1}{\lambda} = R z^2 \left(\frac{1}{n_2^2} - \frac{1}{n_1^2} \right)$$

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C Ionisation energy = $eV \rightarrow$ Ionisation potential (unit Volts)

H: $z=1$

He: $z=2$ & $IP \propto z^2$

$$IP = \frac{13.6 \text{ eV}}{e} = 13.6 \text{ V}$$

$$\therefore IP = (13.6) \times 2^2 \text{ V}$$

$$\text{First excitation potential} = \frac{E_2 - E_1}{e} \text{ V}$$

$$EP \propto z^2$$

$$EP_1 = \frac{13.6 (1)^2 \left(\frac{1}{1} - \frac{1}{2^2} \right)}{e} \text{ eV} = 10.2 \text{ V}$$

$$\therefore EP_1 = 10.2 (2)^2 \text{ V}$$

$$\lambda = \frac{12400 \text{ \AA}}{\Delta E}$$

$$\lambda \propto \frac{1}{z^2}$$

$$= \frac{12400 \text{ \AA}}{10.2} = 1215 \text{ \AA}$$

$$\therefore \lambda = \frac{1215 \text{ \AA}}{(2)^2}$$

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6.23. Calculate the angular frequency of an electron occupying the second Bohr orbit of He^+ ion.

$\downarrow$ $n=2$ $\downarrow$ $Z=2$

$$\omega = \frac{v}{r} = \frac{2.2 \times 10^6 \frac{Z}{n} \text{ m/s}}{0.529 \frac{n^2}{Z} \text{ \AA}} = \frac{2.2 \times 10^6}{0.529 \times 10^{-10}} \cdot \frac{Z^2}{n^3} = 2 \times 10^{16} \text{ rad/s}$$

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6.24. For hydrogen-like systems find the magnetic moment μ_n corresponding to the motion of an electron along the n -th orbit and the ratio of the magnetic and mechanical moments μ_n/M_n . Calculate the magnetic moment of an electron occupying the first Bohr orbit.

$r_n = 0.529 \frac{n^2}{Z} \text{ \AA}$
 $v_n = 2.2 \times 10^6 \frac{Z}{n} \text{ m/s}$

$$\mu_n = \frac{iA}{\downarrow} \rightarrow \pi r^2$$

$$(i = \frac{e}{T} = \frac{e}{2\pi/\omega} = \frac{e\omega}{2\pi})$$

$mvr = n\hbar$

$$= \frac{e\omega r}{2} = \frac{e}{2} \cdot \frac{n\hbar}{m} \quad (\text{Independent of } Z)$$

$$\Rightarrow \mu_1 = \frac{(1.6 \times 10^{-19})}{2} (2.2) 10^6 (0.529) 10^{-10} \text{ n}$$

$$\Rightarrow \mu_1 = 9.3 \times 10^{-24} \text{ A m}^2$$

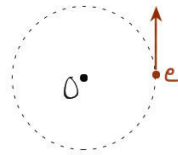
Mechanical moment: $M_n = mvr$

From 1,2:

$$\frac{\mu_n}{M_n} = \frac{e\hbar}{2m} = \frac{e}{2m}$$

(Independent of $n \neq Z$)

6.25. Calculate the magnetic field induction at the centre of a hydrogen atom caused by an electron moving along the first Bohr orbit.



$$B_0 = \frac{\mu_0 i}{2r} \quad \rightarrow \quad i = \frac{e}{T} = \frac{e}{2\pi/\omega} = \frac{e\omega}{2\pi} = \frac{e}{2\pi} \frac{v}{r}$$

$$\Rightarrow B_0 = \frac{\mu_0}{2r} \left(\frac{e}{2\pi} \frac{v}{r} \right) = \frac{\mu_0 e}{4\pi} \frac{v}{r^2}$$

$$r_n = 0.529 \frac{n^2}{Z} \text{ \AA}$$

$$v_n = 2.2 \times 10^6 \frac{Z}{n} \text{ m/s}$$

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6.26. Calculate and draw on the wavelength scale the spectral intervals in which the Lyman, Balmer, and Paschen series for atomic hydrogen are confined. Show the visible portion of the spectrum.

$$\lambda = \frac{1}{Z^2} \frac{912}{\left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)} \text{ \AA}$$

Name of the spectral series	Lower state (n_1)	Upper state (n_2)	Wavelength (λ) in $\text{\AA}$
Lyman	1	2	1216 (longest wavelength)
	1	∞	912 (shortest wavelength)
Balmer	2	3	6563
	2	∞	3646
Paschen	3	4	18751
	3	∞	8220
Brackett	4	5	40533
	4	∞	14592
Pfund	5	6	74618
	5	∞	22800

→ UV rays

→ Visible

} Infrared

6.27. To what series does the spectral line of atomic hydrogen belong if its wave number is equal to the difference between the wave numbers of the following two lines of the Balmer series: 486.1 and 410.2 nm? What is the wavelength of that line?

$$\lambda = \frac{1}{Z^2} \frac{912}{\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)} \text{ \AA}$$

$$4102 = \frac{912}{\left(\frac{1}{4} - \frac{1}{n_2^2}\right)} \Rightarrow n_2 = 6$$

$$4861 = \frac{912}{\left(\frac{1}{4} - \frac{1}{n_2^2}\right)} \Rightarrow n_2 = 4$$

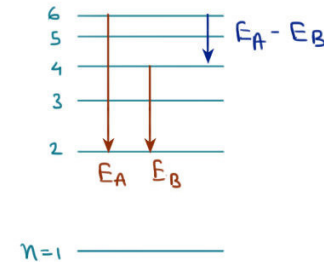
$$\frac{1}{\lambda} = \frac{1}{\lambda_A} - \frac{1}{\lambda_B}$$

$$\Rightarrow \frac{1}{\lambda} = \frac{1}{\lambda_A} - \frac{1}{\lambda_B}$$

$$\Rightarrow E = E_A - E_B$$

(6 → 4) (6 → 2) (4 → 2)

Brackett series



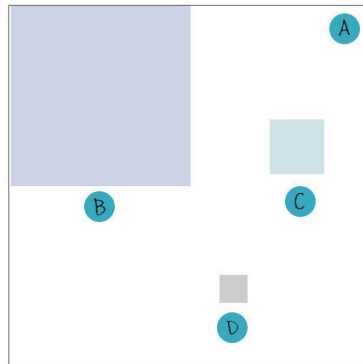
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6.28. For the case of atomic hydrogen find:

(a) the wavelengths of the first three lines of the Balmer series;

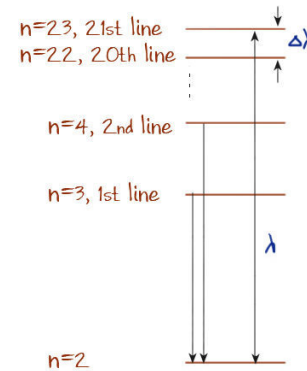
(b) the minimum resolving power $\lambda/\delta\lambda$ of a spectral instrument capable of resolving the first 20 lines of the Balmer series.

$$\lambda = \frac{1}{Z^2} \frac{912}{\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)} \text{ \AA}$$



Minimum resolution of screen (A) to see the smallest pixel (D) clearly is given by:

$$\text{Resolution} = \frac{\text{Total area (A)}}{\text{Area of smallest pixel (D)}}$$



Balmer series

Similarly, minimum resolving power of instrument to observe the difference between closest spectral lines is given by Resolving power = $\frac{\lambda}{\Delta\lambda}$

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Method 1

$$\frac{\lambda}{\Delta \lambda} = \frac{1}{\frac{1}{2^2} - \frac{1}{23^2}} \quad \left(\lambda = \frac{1}{z^2} \frac{912}{\left(\frac{1}{n^2} - \frac{1}{n'^2}\right)} \text{ \AA} \right)$$

$$= \frac{1}{\frac{1}{2^2} - \frac{1}{22^2} - \frac{1}{2^2} - \frac{1}{23^2}} = 1411 //$$

Method 2

$$c = f \lambda$$

$$\Rightarrow 0 = \Delta f \lambda + f \Delta \lambda$$

$$\Rightarrow \left| \frac{\lambda}{\Delta \lambda} \right| = \frac{f}{\Delta f} = \frac{1}{K \left(\frac{1}{2^2} - \frac{1}{23^2} \right) - K \left(\frac{1}{2^2} - \frac{1}{22^2} \right)} = \frac{\frac{1}{2^2} - \frac{1}{23^2}}{\frac{1}{22^2} - \frac{1}{23^2}} = 1411 //$$

Method 3

$$\text{Let, } \frac{1}{\lambda} = K \left(\frac{1}{4} - \frac{1}{n^2} \right)$$

$$\Rightarrow -\frac{1}{\lambda^2} d\lambda = K \left(0 + \frac{2}{n^3} dn \right)$$

$$\Rightarrow \left| \frac{\lambda}{d\lambda} \right| = \frac{n^3}{2K \lambda dn}$$

$$= \frac{n^3}{2K} K \left(\frac{1}{4} - \frac{1}{n^2} \right)$$

$$= \frac{n(n^2 - 4)}{8}$$

$$= \frac{23(23^2 - 4)}{8} \approx 1518 //$$

(Book answer)
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6.30. What element has a hydrogen-like spectrum whose lines have wavelengths four times shorter than those of atomic hydrogen?

$$\lambda \propto \frac{1}{Z^2} \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{Z_2^2}{Z_1^2} \Rightarrow \frac{\lambda_H}{(\lambda_H/4)} = \frac{Z^2}{(1)^2} \Rightarrow Z = 2 //$$

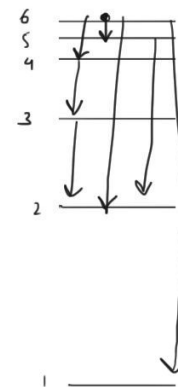
6.31. How many spectral lines are emitted by atomic hydrogen excited to the n -th energy level?

□□□

Between any two (out of n) energy levels, a spectral line will be produced

∴ Total possible pairs of energy levels = nC_2

$$= \frac{n(n-1)}{2} //$$



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6.32. What lines of atomic hydrogen absorption spectrum fall within the wavelength range from 94.5 to 130.0 nm?

□□□

Hydrogen spectrum for given wavelength falls in Lyman series (91.2 nm - 121.6 nm)

$$\lambda = \frac{1}{Z^2} \frac{912}{\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)} \text{ \AA}$$

$$\Rightarrow \lambda = \frac{912}{1 - \frac{1}{n^2}}$$

For $\lambda = 945$ $n \rightarrow 5.35$

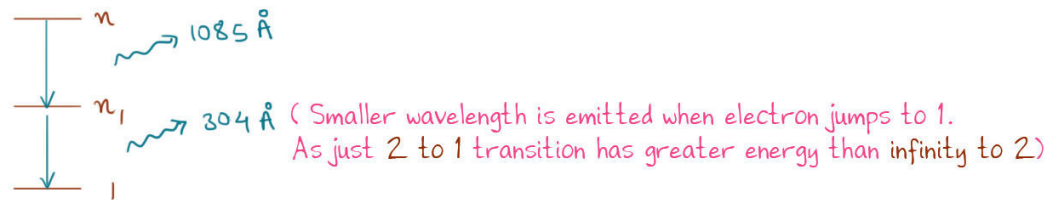
For $\lambda = 1300$ $n \rightarrow 2$
(As $1300 > 1216$)

∴ n is integer, permissible values of n : 2, 3, 4, 5 //

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6.33. Find the quantum number n corresponding to the excited state of He^+ ion if on transition to the ground state that ion emits two photons in succession with wavelengths 108.5 and 30.4 nm.

$$\lambda = \frac{1}{Z^2} \frac{912}{\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)} \text{ \AA}$$



$$1085 = \frac{912}{4\left(\frac{1}{n_1^2} - \frac{1}{n^2}\right)} \quad (1)$$

$$304 = \frac{912}{4\left(1 - \frac{1}{n_1^2}\right)} \quad (2)$$

From 1, 2: $n_1 = 2$
 $n = 5 //$

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6.34. Calculate the Rydberg constant R if He^+ ions are known to have the wavelength difference between the first (of the longest wavelength) lines of the Balmer and Lyman series equal to $\Delta\lambda = 133.7 \text{ nm}$.

$$\frac{1}{\lambda} = R Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\lambda = \frac{1}{R Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}$$

$$\lambda_1 - \lambda_2 = 1337 \times 10^{-10} = \frac{1}{R \cdot 2^2} \left[\frac{1}{\frac{1}{2^2} - \frac{1}{3^2}} - \frac{1}{1 - \frac{1}{2^2}} \right]$$

$$\Rightarrow R = 1.1 \times 10^7 // \text{ [SI units]}$$

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6.35. What hydrogen-like ion has the wavelength difference between the first lines of the Balmer and Lyman series equal to 59.3 nm?

$$\frac{1}{\lambda} = R Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$\lambda = \frac{1}{R Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)}$$

$$\lambda_1 - \lambda_2 = 593 \times 10^{-10} = \frac{1}{R Z^2} \left[\frac{1}{2^2 - 3^2} - \frac{1}{1 - \frac{1}{2^2}} \right]$$

$\downarrow$
 $1.1 \times 10^7 \text{ [SI units]}$

$$\Rightarrow Z = 3 \text{ (Li}^{+2}\text{)}$$

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6.36. Find the wavelength of the first line of the He⁺ ion spectral series whose interval between the extreme lines is $\Delta\omega = 5.18 \cdot 10^{15} \text{ s}^{-1}$.

$$\lambda = \frac{1}{Z^2} \frac{912}{\left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)} \text{ \AA}$$

$$E = h f = \frac{h \omega}{2\pi} = \frac{h c}{\lambda} = h c R Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$\omega = 2\pi f$

$$\frac{1}{\lambda} = R Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

Let the required series be n-series.
For extreme lines:

$$\frac{h(\omega_2 - \omega_1)}{2\pi} = h c R Z^2 \left[\left(\frac{1}{n^2} - \frac{1}{\infty^2} \right) - \left(\frac{1}{n^2} - \frac{1}{(n+1)^2} \right) \right]$$

$$\Rightarrow (n+1)^2 = \sqrt{\frac{R c (2)^2 \cdot 2\pi}{\Delta\omega}} = 4$$

$$\Rightarrow n = 3$$

∴ First line:

$$\lambda = \frac{912}{4} \left(\frac{1}{3^2} - \frac{1}{4^2} \right) \text{ \AA}$$

$$= 4690 \text{ \AA}$$

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6.37. Find the binding energy of an electron in the ground state of hydrogen-like ions in whose spectrum the third line of the Balmer series is equal to 108.5 nm.

$$\frac{1}{\lambda} = R z^2 \left(\frac{1}{2^2} - \frac{1}{5^2} \right) \Rightarrow z = 2$$

$$E_b = 13.6 z^2 = 54.4 \text{ eV}$$

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6.38. The binding energy of an electron in the ground state of He atom is equal to $E_0 = 24.6 \text{ eV}$. Find the energy required to remove both electrons from the atom.

$$E_b \text{ of first electron } E_{01} = 24.6 \text{ eV}$$

$$E_b \text{ of second electron } E_{02} = 13.6 z^2 = 54.4 \text{ eV}$$

$$\therefore \text{total BE to remove both: } E_b = E_{01} + E_{02} = 79 \text{ eV}$$

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6.39. Find the velocity of photoelectrons liberated by electromagnetic radiation of wavelength $\lambda = 18.0$ nm from stationary He^+ ions in the ground state.

$$\frac{hc}{\lambda} = \frac{12420}{\lambda(\text{\AA})} \text{ eV}$$

Energy conservation

$$E_{\text{in}} - E_b = \frac{1}{2}mv^2$$

$$\Rightarrow \frac{hc}{\lambda} - E_b = \frac{1}{2}mv^2$$

$$\Rightarrow v^2 = \frac{2}{m} \left(\frac{hc}{\lambda} - 13.6 Z^2 \right) \text{ eV}$$

$$= \sqrt{\frac{2}{9.1 \times 10^{-31}} \left[\frac{12420}{180} - (13.6) 4 \right] 1.6 \times 10^{-19}}$$

$$\Rightarrow v = 2.3 \times 10^6 \text{ m/s}$$

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6.40. At what minimum kinetic energy must a hydrogen atom move for its inelastic head-on collision with another, stationary, hydrogen atom to make one of them capable of emitting a photon? Both atoms are supposed to be in the ground state prior to the collision.

To emit a photon on deexcitation, electron must jump atleast one level above first. It will get that energy from inelastic collisions energy loss.

$$\therefore \Delta KE = 13.6 \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = 10.2 \text{ eV}$$

For inelastic collision, by momentum conservation:

$$mv = 2mv'$$

$$\Rightarrow v' = v/2$$

$$\begin{aligned} \therefore \text{Loss of Kinetic energy: } \Delta KE &= \frac{1}{2}mv^2 - \frac{1}{2}(2m)v'^2 \\ &= \frac{1}{4}mv^2 \\ &= \frac{KE}{2} \end{aligned}$$

From 1,2:

$$KE = 2(10.2)$$

$$= 20.4 \text{ eV}$$

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6.41. A stationary hydrogen atom emits a photon corresponding to the first line of the Lyman series. What velocity does the atom acquire?

For a photon:

$$h = p\lambda$$

Also,

$$\frac{hc}{\lambda} = E$$

$$\Rightarrow p = \frac{E}{c}$$

Sol: Momentum conservation:

$$\frac{E}{c} = mv$$

$$\Rightarrow v = \frac{E}{mc} = \frac{13.6 \left(1 - \frac{1}{2^2}\right) \times 1.6 \times 10^{-19}}{(1.67 \times 10^{-27}) \times 3 \times 10^8}$$

$$= 3.25 \text{ m/s} //$$

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6.42. From the conditions of the foregoing problem find how much (in per cent) the energy of the emitted photon differs from the energy of the corresponding transition in a hydrogen atom.

$$\frac{E_{\text{transition}} - E_{\text{photon}}}{E_{\text{transition}}} \times 100$$

$$= \frac{k \cdot E_{\text{atom}}}{E_{\text{transition}}} \times 100$$

$$= \frac{\frac{1}{2} (1.67 \times 10^{-27}) \times (3.25)^2 \times 100}{13.6 \left(1 - \frac{1}{4}\right) \times 1.6 \times 10^{-19}}$$

From 6.41

$$= 5.4 \times 10^{-7} \%$$

(So major energy is taken up by the photon, or energy of recoiled atom is negligible compared to photon)

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6.43. A stationary He^+ ion emitted a photon corresponding to the first line of the Lyman series. That photon liberated a photoelectron from a stationary hydrogen atom in the ground state. Find the velocity of the photoelectron.

E released from He:

$$E_1 = 13.6 Z^2 \left(1 - \frac{1}{4}\right) = 40.8 \text{ eV}$$

∴ E of escaped electron of H:

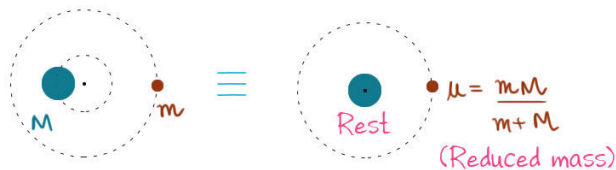
$$\frac{1}{2} m v^2 = E_1 - 13.6 Z^2 \text{ eV}$$

$$\Rightarrow v = \sqrt{\frac{2 (40.8 - 13.6) \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}}$$

$$= 3.6 \times 10^6 \text{ m/s}$$

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Bohrs equations, when we consider the nucleus to be flexible and not fixed



Both rotating about center of mass.

A $r_n \propto \frac{n^2}{Z m_e}$

B $E \propto \frac{Z^2}{n^2} m_e$

In this case:

$$r_n \propto \frac{n^2 (m_e + m_n)}{Z m_e m_n}$$

In this case:

$$\therefore E \propto \frac{Z^2 m_e m_n}{n^2 (m_e + m_n)}$$

$$\Rightarrow r_n = 0.529 \left(\frac{m_e + m_n}{m_n} \right) \frac{n^2}{Z} \text{ \AA}$$

$$\Rightarrow E = -13.6 \frac{Z^2}{n^2} \left(\frac{m_n}{m_e + m_n} \right) \text{ eV}$$

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6.46. Taking into account the motion of the nucleus of a hydrogen atom, find the expressions for the electron's binding energy in the ground state and for the Rydberg constant. How much (in per cent) do the binding energy and the Rydberg constant, obtained without taking into account the motion of the nucleus, differ from the more accurate corresponding values of these quantities?

$$E_b = -13.6 \frac{Z^2}{n^2} \left(\frac{m_n}{m_e + m_n} \right) \text{ eV}$$

$$\frac{\Delta E_b}{E_b} \times 100 = \left(\frac{1 - \frac{m_n}{m_e + m_n}}{\frac{m_n}{m_e + m_n}} \right) \times 100 = \frac{m_e}{m_n} \times 100 = 0.054\%$$

$\underbrace{m_n}_{1837 m_e}$

$$E_b = \text{constant } R \Rightarrow \frac{\Delta E_b}{E_b} = \frac{\Delta R}{R} = 0.054\%$$

$\Rightarrow \Delta E_b = \text{constant } \Delta R$

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6.47. For atoms of light and heavy hydrogen (H and D) find the difference

- (a) between the binding energies of their electrons in the ground state;
 (b) between the wavelengths of first lines of the Lyman series.

$$m_H = 1837 m_e$$

$$m_D = 2(1837) m_e$$

A $E_b = -13.6 \frac{Z^2}{n^2} \left(\frac{m_n}{m_e + m_n} \right) \text{ eV}$

$$\Delta E_b = -13.6 (1)^2 \left[\frac{m_H}{m_e + m_H} - \frac{m_D}{m_e + m_D} \right]$$

$$= -13.6 \frac{(m_H - m_D) m_e}{(m_e + m_H)(m_e + m_D)}$$

$$\approx 13.6 \frac{(1837) m_e^2}{(1837)(2 \times 1837) m_e^2}$$

$$= 3.7 \times 10^{-3} \text{ eV} //$$

B $\lambda = \frac{912}{Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)} \frac{(m_e + m_n)}{m_n} \text{ \AA}$

$$\Delta \lambda = \frac{912}{1^2 \left(1 - \frac{1}{4} \right)} \left[\frac{m_e + m_H}{m_H} - \frac{m_e + m_D}{m_D} \right]$$

$$= 1216 \left[\frac{(m_D - m_H) m_e}{(1837)(2 \times 1837) m_e^2} \right]$$

$$= \frac{1216}{2 \times 1837} = 0.33 \text{ \AA} //$$

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6.48. Calculate the separation between the particles of a system in the ground state, the corresponding binding energy, and the wavelength of the first line of the Lyman series, if such a system is

(a) a mesonic hydrogen atom whose nucleus is a proton (in a mesonic atom an electron is replaced by a meson whose charge is the same and mass is 207 that of an electron);

(b) a positronium consisting of an electron and a positron revolving around their common centre of masses.

$$r_n \propto \frac{1}{m_e}$$

$$E \propto m_e$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

Radius $r_n \propto \frac{1}{m_e} \Rightarrow r_n = 0.529 \frac{n^2}{z} \text{ \AA}$

$$\therefore r_n \propto \frac{1}{\mu} \Rightarrow r_n = 0.529 \times \frac{\mu}{m_e} \frac{n^2}{z} \text{ \AA} \quad (1)$$

Energy $E_n \propto m_e \Rightarrow E_n = 13.6 \frac{z^2}{n^2}$

$$\therefore E_n \propto \mu \Rightarrow E_n = \frac{13.6 \mu}{m_e} \frac{z^2}{n^2} \text{ eV} \quad (2)$$

Wavelength

$$\lambda \propto \frac{1}{m_e} \Rightarrow \lambda = \frac{912}{z^2} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$\therefore \lambda \propto \frac{1}{\mu} \Rightarrow \lambda = \frac{912 m_e}{z^2 \mu} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \quad (3)$$

A $m_1 \rightarrow 207 m_e$
 $m_2 \rightarrow 1837 m_e \Rightarrow \mu = \frac{(207)(1837)}{(207+1837)} = 186$

B $m_1 \rightarrow m_e$
 $m_2 \rightarrow m_e \Rightarrow \mu = \frac{m_e \cdot m_e}{m_e(m_e+m_e)} = \frac{1}{2}$

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A $r = \frac{0.529}{186} \text{ \AA} = 2.84 \times 10^{-13} \text{ m}$

$$E_b = (13.6)(186) = 2526 \text{ eV}$$

$$\lambda = \frac{912}{186 \left(1 - \frac{1}{4}\right)} = 6.53 \text{ \AA}$$

B $r = \frac{0.529}{1/2} = 1.05 \text{ \AA}$

$$E_b = (13.6) \frac{1}{2} = 6.8 \text{ eV}$$

$$\lambda = \frac{912}{1/2 \left(1 - \frac{1}{4}\right)} = 2420 \text{ \AA}$$

6.49. Calculate the de Broglie wavelengths of an electron, proton, and uranium atom, all having the same kinetic energy $T = 100 \text{ eV}$.

□ □ □

$$\begin{aligned}
 p\lambda &= h \\
 \Rightarrow \lambda &= \frac{h}{\sqrt{2mT}}
 \end{aligned}$$

For electron (e):

$$\lambda_e = \frac{h}{\sqrt{2 \times 9.1 \times 10^{-31} \times 100 \times 1.6 \times 10^{-19}}} = 1.22 \text{ \AA} = 122 \text{ pm} //$$

For proton (p):

$$\lambda_p = \frac{122 \text{ pm}}{\sqrt{1837}} = 2.86 \text{ pm} //$$

For uranium atom (U_{238}):

$$\lambda_{U_{238}} = \frac{2.86}{\sqrt{238}} = 0.19 \text{ pm} //$$

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6.50. What amount of energy should be added to an electron to reduce its de Broglie wavelength from 100 to 50 pm?

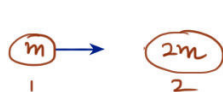
□ □ □

$$\begin{aligned}
 \lambda &= \frac{h}{p} = \frac{h}{\sqrt{2mE}} \\
 \Rightarrow E &= \frac{h^2}{2m\lambda^2} \\
 \Rightarrow \Delta E &= \frac{h^2}{2m} \left(\frac{1}{\lambda_1^2} - \frac{1}{\lambda_2^2} \right) = \frac{(6.6 \times 10^{-34})^2}{2 \times (9.1 \times 10^{-31})} \left[\frac{1}{50^2} - \frac{1}{100^2} \right] \frac{1}{10^{-24}} \cdot \frac{1}{1.6 \times 10^{-19}} \text{ eV} \\
 &= 449 \text{ eV} //
 \end{aligned}$$

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6.51. A neutron with kinetic energy $T = 25 \text{ eV}$ strikes a stationary deuteron (heavy hydrogen nucleus). Find the de Broglie wavelengths of both particles in the frame of their centre of inertia.

Before collision:



$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{m_1 v_1}{m_1 + m_2} = \frac{\sqrt{2m_1 T}}{m_1 + m_2}$$

$$v_{2,cm} = v_{cm} \text{ (As 2 is at rest)}$$

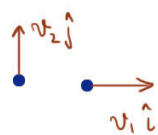
$$\Rightarrow p_{2,cm} = m_2 v_{2,cm} = m_2 v_{cm} = \frac{m_2 \sqrt{2m_1 T}}{m_1 + m_2}$$

$$\text{Also, } p_{1,cm} = p_{2,cm}$$

$$\Rightarrow \lambda_{1,cm} = \lambda_{2,cm} = \frac{h}{p_{1,cm}} = \frac{h(m_1 + m_2)}{m_2 \sqrt{2m_1 T}} = 8.6 \text{ pm} //$$

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6.52. Two identical non-relativistic particles move at right angles to each other, possessing de Broglie wavelengths λ_1 and λ_2 . Find the de Broglie wavelength of each particle in the frame of their centre of inertia.



$$v_{cm} = \frac{m\vec{v}_1 + m\vec{v}_2}{2m} = \frac{v_1 \hat{i} + v_2 \hat{j}}{2}$$

$$\Rightarrow v_{1,cm} = \vec{v}_1 - \vec{v}_{cm} = v_1 \hat{i} - \left(\frac{v_1 \hat{i} + v_2 \hat{j}}{2} \right) = \frac{v_1 \hat{i} - v_2 \hat{j}}{2}$$

$$mv_1 = \frac{h}{\lambda_1}$$

$$mv_2 = \frac{h}{\lambda_2}$$

$$\text{Also, } p_{1,cm} = p_{2,cm}$$

$$\begin{aligned} \Rightarrow \lambda_{1,cm} = \lambda_{2,cm} &= \frac{h}{p_{1,cm}} = \frac{h}{m \left(\frac{v_1 \hat{i} - v_2 \hat{j}}{2} \right)} \\ &= \frac{h}{\frac{m}{2} \left[\frac{h}{m} \left(\frac{1}{\lambda_1} \hat{i} - \frac{1}{\lambda_2} \hat{j} \right) \right]} = \frac{2\lambda_1 \lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}} // \end{aligned}$$

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6.53. Find the de Broglie wavelength of hydrogen molecules, which corresponds to their most probable velocity at room temperature.

$$\sqrt{\frac{2RT}{M}}$$

Molar mass in kg/mol

$$\lambda = \frac{h}{p} = \frac{h}{m \sqrt{\frac{2RT}{M}}} = \frac{6.6 \times 10^{-34}}{2 \times 1.67 \times 10^{-27} \sqrt{\frac{2(8.314)(300)}{2 \times 10^{-3}}}} = 125 \text{ pm} //$$

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6.133. Find the wavelength of the K_α line in copper ($Z = 29$) if the wavelength of the K_α line in iron ($Z = 26$) is known to be equal to 193 pm.

$$\frac{1}{\lambda} = R(Z - \sigma)^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

For Copper: $\frac{1}{\lambda_{Cu}} = R (Z - 1)^2 \left(1 - \frac{1}{4} \right)$

For K_α line, $\sigma = 1$

For Iron: $\frac{1}{\lambda_{Fe}} = R (Z - 1)^2 \left(1 - \frac{1}{4} \right)$

$$\Rightarrow \frac{\lambda_{Cu}}{\lambda_{Fe}} = \frac{25^2}{28^2}$$

$$\Rightarrow \lambda_{Cu} = \frac{25^2}{28^2} \cdot 193 = 154 \text{ pm} //$$

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6.134. Proceeding from Moseley's law find:

(a) the wavelength of the K_α line in aluminium and cobalt:

(b) the difference in binding energies of K and L electrons in vanadium.

$$\lambda = \frac{912}{z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)} \text{ \AA}$$

$$\lambda_{K_\alpha} (\text{Al}) = \frac{912 \cdot 4}{12^2 \cdot 3} = 8.44 \text{ \AA} = 844 \text{ pm}$$

$$\lambda_{K_\alpha} (\text{Co}) = 844 \times \frac{12^2}{26^2} = 179 \text{ pm}$$

Difference in binding energy:

$$\begin{aligned} E_K - E_L &= E_{K_\alpha} = 13.6 (Z-1)^2 \frac{3}{4} \\ &= 13.6 (22)^2 \cdot \frac{3}{4} \\ &= 5 \text{ keV} \end{aligned}$$

$$\lambda_{K_\alpha} = \frac{912}{(Z-1)^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)} \text{ \AA}$$

(for K_α)

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6.135. How many elements are there in a row between those whose wavelengths of K_α lines are equal to 250 and 179 pm?

$$250 = \frac{912}{(Z_1-1)^2} \left(\frac{4}{3} \right) \Rightarrow Z_1 = 23$$

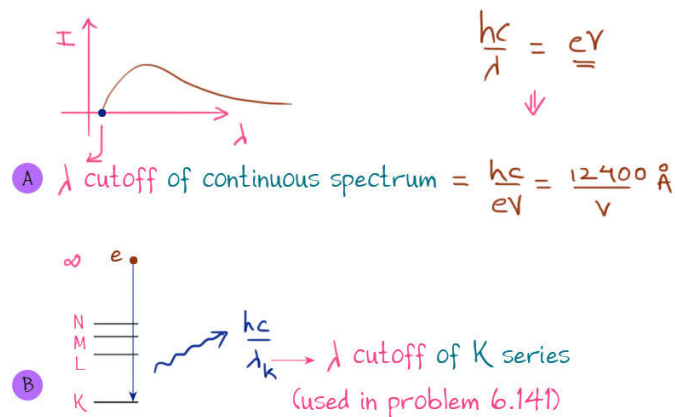
$$179 = \frac{912}{(Z_2-1)^2} \left(\frac{4}{3} \right) \Rightarrow Z_2 = 27$$

In between elements: 24, 25, 26 : 3

$$\lambda_{K_\alpha} = \frac{912}{(Z-1)^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)} \text{ \AA}$$

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6.136. Find the voltage applied to an X-ray tube with nickel anticathode if the wavelength difference between the K_α line and the short-wave cut-off of the continuous X-ray spectrum is equal to 84 pm.



Sol:

Given:

$$\lambda_{K\alpha} - \lambda_{\text{cutoff}} = 84 \text{ pm}$$

$$\Rightarrow \frac{912}{(Z-1)^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)} \text{ \AA} - \frac{12400 \text{ \AA}}{V} = 84 \times 10^{-2} \text{ \AA}$$

$\begin{matrix} 28 & 1 & 2 \end{matrix}$

$$\Rightarrow V = 15 \text{ kV}$$

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6.137. At a certain voltage applied to an X-ray tube with aluminium anticathode the short-wave cut-off wavelength of the continuous X-ray spectrum is equal to 0.50 nm. Will the K series of the characteristic spectrum whose excitation potential is equal to 1.56 kV be also observed in this case?

$$\lambda_{\text{cutoff}} = \frac{12400}{V}$$

$$\Rightarrow V = \frac{12400}{0.5 \times 10^{-9} \text{ m}}$$

$$\Rightarrow V = 2.5 \text{ kV} > 1.56 \text{ kV}$$

Potential that electrons are accelerated through. Potential needed to get K series.

$\therefore$ Yes, K series will be observed.

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6.138. When the voltage applied to an X-ray tube increased from $V_1 = 10$ kV to $V_2 = 20$ kV, the wavelength interval between the K_α line and the short-wave cut-off of the continuous X-ray spectrum increases by a factor $n = 3.0$. Find the atomic number of the element of which the tube's anticathode is made.

$$\frac{hc}{\lambda_{\text{cutoff}}} = eV$$

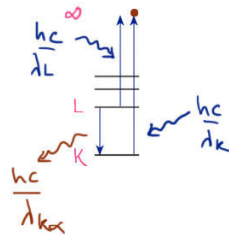
$$\frac{1}{\lambda_{K_\alpha}} = R(z-1)^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$\therefore \frac{\lambda_{K_\alpha} - \lambda_{\text{cutoff}_1}}{\lambda_{K_\alpha} - \lambda_{\text{cutoff}_2}} = \frac{1}{n}$$

$$\Rightarrow \frac{\frac{4}{R(z-1)^2} - \frac{hc}{eV_1}}{\frac{4}{R(z-1)^2} - \frac{hc}{eV_2}} = \frac{1}{n} \Rightarrow z = \sqrt{\frac{4(n-1)eV_1V_2}{3Rhc(nV_2 - V_1)}} + 1 = 29 //$$

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6.139. What metal has in its absorption spectrum the difference between the frequencies of X-ray K and L absorption edges equal to $\Delta\nu = 6.85 \cdot 10^{18} \text{ s}^{-1}$?

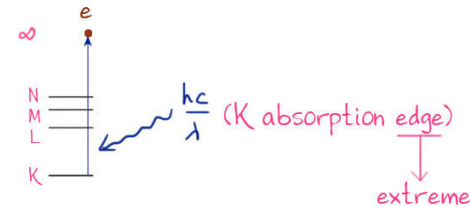


$$\frac{hc}{\lambda_K} - \frac{hc}{\lambda_L} = \frac{hc}{\lambda_{K_\alpha}}$$

$$\Rightarrow h\nu_K - h\nu_L = \frac{hc}{\lambda_{K_\alpha}}$$

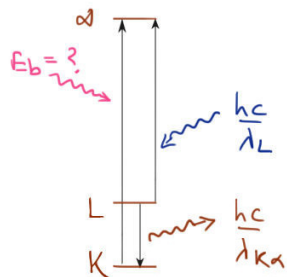
$$\Rightarrow K\Delta\nu = Kc R(z-1)^2 \frac{3}{4}$$

$$\Rightarrow z = \sqrt{\frac{4\Delta\nu}{3Rc}} + 1 = 22 //$$



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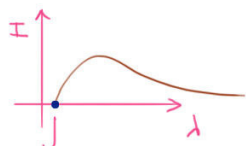
6.140. Calculate the binding energy of a K electron in vanadium $\rightarrow 23$ whose L absorption edge has the wavelength $\lambda_L = 2.4$ nm.



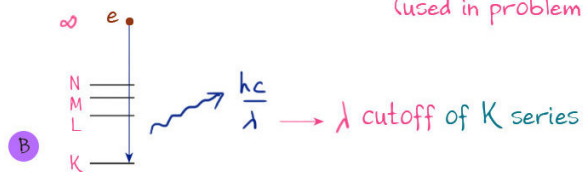
$$\begin{aligned}
 E_b &= \frac{hc}{\lambda_L} + \frac{hc}{\lambda_{K\alpha}} \\
 &= \frac{12400}{2.4 \text{ nm}} + 13.6 (Z-1)^2 \frac{3}{4} \\
 &= 5.4 \text{ keV} //
 \end{aligned}$$

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6.141. Find the binding energy of an L electron in titanium if the wavelength difference between the first line of the K series and its short-wave cut-off is $\Delta\lambda = 26$ pm.



A λ cutoff of continuous spectrum $= \frac{hc}{eV} = \frac{12400 \text{ eV}}{V}$
(used in problem 6.136)



Sol: $E_L = E_K - \frac{hc}{\lambda_{K\alpha}} = \frac{hc}{\lambda_K} - \frac{hc}{\lambda_{K\alpha}}$

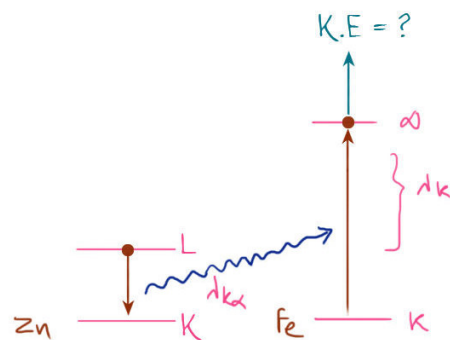
Also its given, $\lambda_{K\alpha} - \lambda_K = \Delta\lambda$

$\therefore E_L = \frac{hc}{\lambda_{K\alpha} - \Delta\lambda} - \frac{hc}{\lambda_{K\alpha}}$

(where, $\frac{1}{\lambda_{K\alpha}} = R(Z-1)^2 \frac{3}{4}$)

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6.142. Find the kinetic energy and the velocity of the photoelectrons liberated by K_α radiation of zinc from the K shell of iron whose K band absorption edge wavelength is $\lambda_K = 174 \text{ pm}$.



$$\frac{hc}{\lambda_{K\alpha}} = \frac{hc}{\lambda_K} + KE$$

(Zn) (Fe) (Fe's e⁻)

$$\Rightarrow KE = \frac{hc}{\lambda_{K\alpha}} - \frac{hc}{\lambda_K}$$

$$KE = 13.6 \frac{(Z-1)^2}{4} - \frac{12400}{1.74 \text{ \AA}} \text{ eV}$$

30

$$= 1.45 \text{ keV} //$$

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(1.45) \cdot 10^3 \cdot 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} = 2.26 \times 10^7 \text{ m/s} //$$

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6.214. Knowing the decay constant λ of a nucleus, find:

- (a) the probability of decay of the nucleus during the time from 0 to t ;
 (b) the mean lifetime τ of the nucleus.

A Probability of being decayed in $t = \frac{\text{Particles decayed in } t}{\text{Total particles}} = \frac{N_0 - N}{N_0} = \frac{N_0 - N_0 e^{-\lambda t}}{N_0} = 1 - e^{-\lambda t} //$

B $N = N_0 e^{-\lambda t}$

$$\Rightarrow \frac{dN}{dt} = -N_0 \lambda e^{-\lambda t}$$

$$\Rightarrow dN = -N_0 \lambda e^{-\lambda t} dt$$

Particles decayed in dt time at time $t = -dN$ (These particles have lifetime t .)

$$\therefore t_{av} = \frac{\int_{t=0}^{\infty} -dN t}{N_0} = \frac{\int_0^{\infty} \lambda e^{-\lambda t} t dt}{1} = \frac{1}{\lambda} //$$

total lifetime (for all particles)

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Solving integral $t_{av} = \int_0^{\infty} \lambda e^{-\lambda t} t dt$

$\lambda t \rightarrow x$

$$= \frac{1}{\lambda} \int_0^{\infty} x e^{-x} dx$$

Integration by parts

$$u dv = d(uv) - v du$$

$$\begin{aligned} u &\rightarrow x \\ dv &\rightarrow e^{-x} dx \Rightarrow \begin{aligned} du &= dx \\ v &= -e^{-x} \end{aligned} \end{aligned}$$

$$\begin{aligned} \therefore t_{av} &= \frac{1}{\lambda} \left(\int_0^{\infty} d[-x e^{-x}] - \int_0^{\infty} -e^{-x} dx \right) \\ &= \frac{1}{\lambda} \left(-x e^{-x} \Big|_0^{\infty} - e^{-x} \Big|_0^{\infty} \right) \\ &= \frac{1}{\lambda} \left(0 - (-1) \right) \\ &= \frac{1}{\lambda} // \end{aligned}$$

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6.215. What fraction of the radioactive cobalt nuclei whose half-life is 71.3 days decays during a month?

$$N = \frac{N_0}{2^n} \left\{ \text{where, } n = \frac{t}{t_{1/2}} \right\} \xrightarrow{\text{derivation}}$$

Sol: Fraction survived = $\frac{N}{N_0}$

$$\therefore \text{Fraction decayed} = 1 - \frac{N}{N_0} = 1 - \frac{1}{2^{30/71.3}} = \frac{1}{4} //$$

$$N = N_0 e^{-\lambda t}$$

Let, $t = n t_{1/2}$

$$= N_0 e^{-\lambda n t_{1/2}}$$

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

$$= N_0 e^{-\cancel{\lambda} n \cdot \frac{\ln 2}{\cancel{\lambda}}}$$

$$\Rightarrow N = \frac{N_0}{2^n}$$

where, $n = \frac{t}{t_{1/2}}$

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6.216. How many beta-particles are emitted during one hour by $1.0 \mu\text{g}$ of Na^{24} radionuclide whose half-life is 15 hours?



Sol: Number of beta emitted = Particles decayed

$$= N_0 - N = N_0 \left(1 - \frac{N}{N_0}\right) = N_0 \left(1 - \frac{1}{2^n}\right) \left\{ \text{where, } n = \frac{t}{t_{1/2}} \right\}$$

$$= N_0 \left(1 - \frac{1}{2^{1/15}}\right) = 1.2 \times 10^{15} //$$

$$\left(\frac{1 \times 10^{-6}}{24} \times 6.023 \times 10^{23} \right)$$

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6.217. To investigate the beta-decay of Mg^{23} radionuclide, a counter was activated at the moment $t = 0$. It registered N_1 beta-particles by a moment $t_1 = 2.0 \text{ s}$, and by a moment $t_2 = 3t_1$ the number of registered beta-particles was 2.66 times greater. Find the mean lifetime of the given nuclei.

Particles decayed: $N_1 = N_0 (1 - e^{-\lambda t_1})$

$$N_2 = N_0 (1 - e^{-\lambda t_2})$$

Its given, $\frac{N_2}{N_1} = 2.66$

$$\Rightarrow \frac{1 - e^{-\lambda t_2}}{1 - e^{-\lambda t_1}} = 2.66$$

$$\Rightarrow t_{av} = \frac{1}{\lambda} = 16 \text{ sec} //$$

Calculation:

$$1 - e^{-\lambda (3t_1)} = 2.66 (1 - e^{-\lambda t_1})$$

Let, $e^{-\lambda t_1} \rightarrow x$

$$1 - x^3 = 2.66 - 2.66x$$

$$x^3 - 2.66x + 1.66 = 0$$

$$x(x^2 - 1) - 1.66(x - 1) = 0$$

$$x^2 + x - 1.66 = 0$$

$$\Rightarrow x = 0.88 \text{ (As } x \text{ is positive)}$$

$$\Rightarrow e^{-2\lambda} = 0.88 \Rightarrow \frac{1}{\lambda} = \frac{-2}{\ln(0.88)} = 16 \text{ sec} //$$

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6.218. The activity of a certain preparation decreases 2.5 times after 7.0 days. Find its half-life.

Activity: $R = R_0 e^{-\lambda t} \Rightarrow R = \frac{R_0}{2^n} \rightarrow \frac{t}{t_{1/2}}$

$$\Rightarrow \left(\frac{R_0}{2.5} \right) = \frac{R_0}{2^{7/t_{1/2}}}$$

$$\Rightarrow 2^{7/t_{1/2}} = 2.5$$

$$\Rightarrow t_{1/2} = 5.3 \text{ days} //$$

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6.219. At the initial moment the activity of a certain radionuclide totalled 650 particles per minute. What will be the activity of the preparation after half its half-life period?

$$t = \frac{t_{1/2}}{2}$$

$$R = \frac{R_0}{2^n} \rightarrow \frac{t}{t_{1/2}}$$

$$\Rightarrow R = \frac{650}{2^{1/2}} = 460 \text{ particles/min} //$$

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6.220. Find the decay constant and the mean lifetime of Co^{55} radionuclide if its activity is known to decrease 4.0% per hour. The decay product is nonradioactive.

□□□

$$R = R_0 e^{-\lambda t}$$

After 1 hour, activity becomes: $0.96 R_0$

$$\Rightarrow 0.96 R_0 = R_0 e^{-\lambda}$$

$$\Rightarrow \lambda = 0.0408 \text{ hr}^{-1}$$

$$\Rightarrow t_{av} = \frac{1}{\lambda} = 24.5 \text{ hr}$$

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6.221. A U^{238} preparation of mass 1.0 g emits $1.24 \cdot 10^4$ alpha-particles per second. Find the half-life of this nuclide and the activity of the preparation.

□□□



Sol: One U atom emits one alpha particle. $\therefore$ Initial activity is given as: $R_0 = 1.24 \times 10^4$ per second

Initially: $R_0 = \lambda N_0$

$$\Rightarrow \lambda = \frac{R_0}{N_0} = \frac{1.24 \times 10^4}{\left(\frac{1}{238} \times 6.02 \times 10^{23}\right)} = 4.9 \times 10^{-18} \text{ per second}$$

$$\Rightarrow t_{1/2} = \frac{\ln 2}{\lambda} = 1.4 \times 10^{17} \text{ sec} = 4.5 \times 10^9 \text{ years}$$

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6.222. Determine the age of ancient wooden items if it is known that the specific activity of C^{14} nuclide in them amounts to $3/5$ of that in lately felled trees. The half-life of C^{14} nuclei is 5570 years.

Activity per unit mass

$$R = \frac{R_0}{2^n}$$

$$\Rightarrow \frac{3R_0}{5} = \frac{R_0}{2^{t/5570}}$$

$$\Rightarrow t \approx 4105 \text{ years} //$$

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6.223. In a uranium ore the ratio of U^{238} nuclei to Pb^{206} nuclei is $\eta = 2.8$. Evaluate the age of the ore, assuming all the lead Pb^{206} to be a final decay product of the uranium series. The half-life of U^{238} nuclei is $4.5 \cdot 10^9$ years.



ηx

x

$$N = \frac{N_0}{2^n} \quad \text{Total nuclei}$$

$$\eta x = \frac{\eta x + x}{2^{t/t_{1/2}}}$$

$$\Rightarrow \frac{\eta+1}{\eta} = 2^{t/t_{1/2}}$$

$$\Rightarrow \frac{3.8}{2.8} = 2^{\frac{t}{4.5 \times 10^9}} \Rightarrow t = 1.98 \times 10^9 \text{ years} //$$

Note: When $\eta = 1$, then $t = t_{1/2}$

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6.224. Calculate the specific activities of Na^{24} and U^{235} nuclides whose half-lives are 15 hours and $7.1 \cdot 10^8$ years respectively.

$$R = \lambda N = \frac{0.693}{t_{1/2}} N$$

Specific activities:

$$R_{\text{Na}} = \frac{0.693}{15 \times 3600} \times \frac{6.02 \times 10^{23}}{24} = 3.22 \times 10^{17} \text{ disintegrations/gm-sec}$$

$$R_{\text{U}^{235}} = \frac{0.693}{7.1 \times 10^8 \times 365 \times 24 \times 3600} \times \frac{6.02 \times 10^{23}}{235} = 8 \times 10^4 \text{ disintegrations/gm-sec}$$

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6.225. A small amount of solution containing Na^{24} radionuclide with activity $A = 2.0 \cdot 10^3$ disintegrations per second was injected in the bloodstream of a man. The activity of 1 cm^3 of blood sample taken $t = 5.0$ hours later turned out to be $A' = 16$ disintegrations per minute per cm^3 . The half-life of the radionuclide is $T = 15$ hours. Find the volume of the man's blood.

Let the volume be $V \text{ cm}^3$.

$$R = \frac{R_0}{2^n}$$

$\therefore$ for full body:

$$\Rightarrow V \times 16 \frac{1}{\text{min cm}^3} = \frac{2 \times 10^3}{2^{5/15}} \frac{1}{\text{sec}}$$

$$\Rightarrow V = \frac{60}{16} \frac{2 \times 10^3}{2^{5/15}} = 5.9 \times 10^3 \text{ cm}^3 //$$

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6.226. The specific activity of a preparation consisting of radioactive Co^{58} and nonradioactive Co^{59} is equal to $2.2 \cdot 10^{12} \text{ dis}/(\text{s} \cdot \text{g})$. The half-life of Co^{58} is 71.3 days. Find the ratio of the mass of radioactive cobalt in that preparation to the total mass of the preparation (in per cent).

For one gram of mixture:



Let, η $1-\eta$ $\eta = ?$

$$R = \lambda N$$

$\rightarrow \frac{\ln 2}{T_{1/2}}$

$$2.2 \times 10^{12} = \frac{0.693}{71.3 \times 24 \times 3600} \cdot \frac{\eta \times 6.02 \times 10^{23}}{58}$$

$$\Rightarrow \eta = 1.9 \times 10^{-3} = 0.19 \% //$$

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6.227. A certain preparation includes two beta-active components with different half-lives. The measurements resulted in the following dependence of the natural logarithm of preparation activity on time t expressed in hours:

t	0	1	2	3	5	7	10	14	20
$\ln A$	4.10	3.60	3.10	2.60	2.06	1.82	1.60	1.32	0.90

Find the half-lives of both components and the ratio of radioactive nuclei of these components at the moment $t = 0$.

Let them be N_1 & N_2 at $t = 0$

$$R = \lambda N$$

$$t=0 \quad A_1 = \lambda_1 N_1 + \lambda_2 N_2 \quad (1)$$

$$t_1=1 \quad A_2 = \lambda_1 N_1 e^{-\lambda_1 t_1} + \lambda_2 N_2 e^{-\lambda_2 t_1} \quad (2)$$

$$t_2=2 \quad A_3 = \lambda_1 N_1 e^{-\lambda_1 t_2} + \lambda_2 N_2 e^{-\lambda_2 t_2} \quad (3)$$

$$t_3=3 \quad A_4 = \lambda_1 N_1 e^{-\lambda_1 t_3} + \lambda_2 N_2 e^{-\lambda_2 t_3} \quad (4)$$

4 equations, 4 variables

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6.228. A P^{32} radionuclide with half-life $T = 14.3$ days is produced in a reactor at a constant rate $q = 2.7 \cdot 10^9$ nuclei per second. How soon after the beginning of production of that radionuclide will its activity be equal to $A = 1.0 \cdot 10^9$ dis/s?



$$t=0 \quad 0$$

$$t=t \quad N$$

$$\frac{dN}{dt} = q - \lambda N$$

$$\int_0^N \frac{dN}{q - \lambda N} = \int_0^t dt$$

$$\Rightarrow \frac{-1}{\lambda} \ln \frac{q - \lambda N}{q} = t \Rightarrow N = \frac{q}{\lambda} (1 - e^{-\lambda t})$$

$$R = \lambda N$$

$$= \lambda \frac{q}{\lambda} (1 - e^{-\lambda t})$$

$$\Rightarrow R = q (1 - e^{-\lambda t})$$

$$\Rightarrow 10^9 = 2.7 \times 10^9 (1 - \frac{1}{2} t_{14.3})$$

$$\Rightarrow t = 9.5 \text{ days} //$$

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6.229. A radionuclide A_1 with decay constant λ_1 transforms into a radionuclide A_2 with decay constant λ_2 . Assuming that at the initial moment the preparation contained only the radionuclide A_1 , find:

(a) the equation describing accumulation of the radionuclide A_2 with time;

(b) the time interval after which the activity of radionuclide A_2 reaches the maximum value.



$$t=0 \quad N_0 \quad 0$$

$$t=t \quad N_1 \quad N_2$$

$$A_1: N_1 = N_0 e^{-\lambda_1 t} \quad (1)$$

$$A_2: \frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \quad (2)$$

From 1,2:

$$\left(\frac{dN_2}{dt} + \lambda_2 N_2 = \lambda_1 N_0 e^{-\lambda_1 t} \right) \times e^{\lambda_2 t}$$

$$\Rightarrow \frac{d}{dt} (N_2 e^{\lambda_2 t}) = \lambda_1 N_0 e^{(\lambda_2 - \lambda_1)t}$$

$$\Rightarrow \int_0^{N_2 e^{\lambda_2 t}} d(N_2 e^{\lambda_2 t}) = \int_0^t \lambda_1 N_0 e^{(\lambda_2 - \lambda_1)t} dt$$

$$\Rightarrow N_2 = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t}) //$$

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$$B \text{ At max } N_2: \frac{dN_2}{dt} = 0$$

$$\Rightarrow -\lambda_1 e^{-\lambda_1 t} + \lambda_2 e^{-\lambda_2 t} = 0$$

$$\Rightarrow t = \frac{\ln \lambda_2 / \lambda_1}{\lambda_2 - \lambda_1} //$$

6.230. Solve the foregoing problem if $\lambda_1 = \lambda_2 = \lambda$.

A From previous problem:

$$N_2 = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t})$$

Putting, $\lambda_1 = \lambda$ (constant)

$$\Rightarrow N_2 = \frac{\lambda N_0}{\lambda_2 - \lambda} (e^{-\lambda t} - e^{-\lambda_2 t})$$

We need to find:

$$\lim_{\lambda_2 \rightarrow \lambda} N_2 = \lambda N_0 \lim_{\lambda_2 \rightarrow \lambda} \frac{(e^{-\lambda t} - e^{-\lambda_2 t})}{\lambda_2 - \lambda}$$

L'Hopital

$$= \lambda N_0 \lim_{\lambda_2 \rightarrow \lambda} \frac{\frac{d}{d\lambda_2} ()}{\frac{d}{d\lambda_2} ()} = \lambda N_0 \frac{t e^{-\lambda_2 t}}{1}$$

$\lambda_2 \rightarrow \lambda$

$$\therefore N_2 = \lambda N_0 t e^{-\lambda t}$$

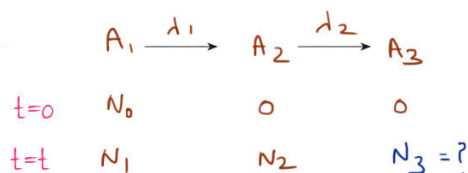
B From previous problem: At max N_2 : $t = \frac{\ln \lambda_2 / \lambda_1}{\lambda_2 - \lambda_1} //$

$$\lim_{\lambda_2 \rightarrow \lambda} t = \lim_{\lambda_2 \rightarrow \lambda} \frac{\ln (\lambda_2 / \lambda)}{\lambda_2 - \lambda}$$

$$\text{L'Hopital} \downarrow = \frac{\frac{d}{d\lambda_2} (\ln \lambda_2 - \ln \lambda)}{1} = \frac{1}{\lambda_2} \xrightarrow{\lambda_2 \rightarrow \lambda} \therefore t = \frac{1}{\lambda} //$$

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6.231. A radionuclide A_1 goes through the transformation chain $A_1 \rightarrow A_2 \rightarrow A_3$ (stable) with respective decay constants λ_1 and λ_2 . Assuming that at the initial moment the preparation contained only the radionuclide A_1 equal in quantity to N_{10} nuclei, find the equation describing accumulation of the stable isotope A_3 .



$$N_1 + N_2 + N_3 = N_0$$

$$\Rightarrow N_3 = N_0 - N_1 - N_2$$

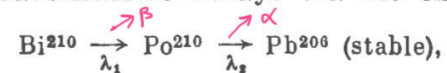
$$= N_0 - N_0 e^{-\lambda_1 t} - \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t})$$

$$= N_0 \left(1 + \frac{\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t}}{\lambda_2 - \lambda_1} \right) //$$

From 6.230

Youtube / Arihant Senior

6.232. A Bi^{210} radionuclide decays via the chain



where the decay constants are $\lambda_1 = 1.60 \cdot 10^{-6} \text{ s}^{-1}$, $\lambda_2 = 5.80 \cdot 10^{-8} \text{ s}^{-1}$. Calculate alpha- and beta-activities of the Bi^{210} preparation of mass 1.00 mg a month after its manufacture.

$$\begin{aligned} R_{\beta} &= \lambda_1 N_1 \\ &= \lambda_1 N_0 e^{-\lambda_1 t} \end{aligned}$$

$$\begin{aligned} R_{\alpha} &= \lambda_2 N_2 \\ &= \lambda_2 \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t}) \end{aligned}$$

From 6.230

Youtube / Arihant Senior

6.233. (a) What isotope is produced from the alpha-radioactive Ra^{226} as a result of five alpha-disintegrations and four β^- -disintegrations?

(b) How many alpha- and β^- -decays does U^{238} experience before turning finally into the stable Pb^{206} isotope?

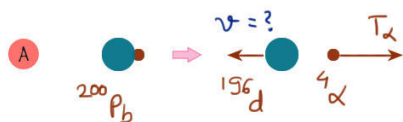
A
$$\begin{array}{c} 226 \\ 88 \end{array} \text{Ra} \xrightarrow[-4(-1e)]{-5(4_2\text{He})} \begin{array}{c} x \\ y \end{array} \text{Z} \Rightarrow \begin{aligned} x &= 226 - 20 = 206 \\ y &= 88 - 10 + 4 = 82 \end{aligned} \Rightarrow \begin{array}{c} 206 \\ 82 \end{array} \text{Pb}$$

B
$$\begin{array}{c} 238 \\ 92 \end{array} \text{U} \xrightarrow[-y(-1e)]{-x(4_2\text{He})} \begin{array}{c} 206 \\ 82 \end{array} \text{Pb}$$

$$\begin{aligned} 238 - 4x &= 206 \\ 92 - 2x + y &= 82 \end{aligned} \Rightarrow \begin{aligned} x &= \text{No of } \alpha = 8 \\ y &= \text{No of } \beta = 6 \end{aligned}$$

Youtube / Arihant Senior

6.234. A stationary Pb^{200} nucleus emits an alpha-particle with kinetic energy $T_\alpha = 5.77 \text{ MeV}$. Find the recoil velocity of a daughter nucleus. What fraction of the total energy liberated in this decay is accounted for by the recoil energy of the daughter nucleus?



(Daughter nucleus)

Momentum conservation

$$p_d = p_\alpha$$

$$\Rightarrow m_d v = \sqrt{2 m_\alpha T_\alpha}$$

$$\Rightarrow v = \frac{\sqrt{2 m_\alpha T_\alpha}}{m_d} //$$

B

$$E_d = \frac{p_d^2}{2m_d} = \frac{p_\alpha^2}{2m_d} = \frac{2m_\alpha T_\alpha}{2m_d}$$

$$\frac{E_d}{\text{Total Energy}} = \frac{E_d}{E_d + T_\alpha} = \frac{m_\alpha}{m_\alpha + m_d} //$$

$$= \frac{4}{4 + 196} = 0.02 //$$

Quite small

Youtube / Arihant Senior

6.235. Find the amount of heat generated by 1.00 mg of a Po^{210} preparation during the mean lifetime period of these nuclei if the emitted alpha-particles are known to possess the kinetic energy 5.3 MeV and practically all daughter nuclei are formed directly in the ground state.

Nuclei decayed: $\Delta N = N_0 (1 - e^{-\lambda t}) \rightarrow t = \frac{1}{\lambda}$

$$= \frac{1 \times 10^{-3}}{210} \times 6.02 \times 10^{23} \left(1 - \frac{1}{e}\right)$$

$\downarrow N_0$

$$= 2.87 \times 10^{18} \times 0.63$$

Heat generated: $Q \approx \Delta N \cdot E_\alpha = \Delta N \times 5.3 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$

$$= 1.54 \times 10^6 \text{ J} //$$

∴ Kinetic energy of alpha-particles gets converted to heat.

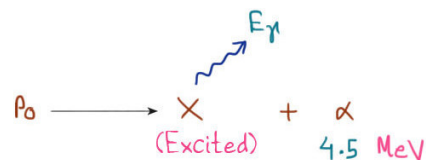
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6.236. The alpha-decay of Po^{210} nuclei (in the ground state) is accompanied by emission of two groups of alpha-particles with kinetic energies 5.30 and 4.50 MeV. Following the emission of these particles the daughter nuclei are found in the ground and excited states. Find the energy of gamma-quanta emitted by the excited nuclei.



$$\therefore 5.3 = E_\gamma + 4.5$$

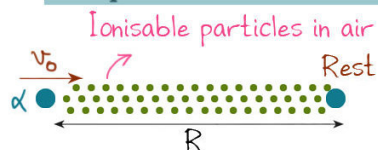
$$\Rightarrow E_\gamma = 0.8 \text{ MeV} //$$



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6.237. The mean path length of alpha-particles in air under standard conditions is defined by the formula $R = 0.98 \cdot 10^{-27} v_0^3 \text{ cm}$, where $v_0 \text{ (cm/s)}$ is the initial velocity of an alpha-particle. Using this formula, find for an alpha-particle with initial kinetic energy 7.0 MeV:

- its mean path length;
- the average number of ion pairs formed by the given alpha-particle over the whole path R as well as over its first half, assuming the ion pair formation energy to be equal to 34 eV.



Everytime alpha particle strikes $\bullet$, an ion pair is formed and alpha particle loses 34 eV of energy, until it eventually stops

$$\text{A } R = 0.98 \cdot 10^{-27} v_0^3 \quad v_0^3 = \left[\frac{2E}{m} \right]^{3/2}$$

$$\Rightarrow R = 0.98 \cdot 10^{-27} \left[\frac{2 \times 7 \times 10^6 \times 1.6 \times 10^{-19}}{4 \times 1.672 \times 10^{-27}} \cdot 10^4 \right]^{3/2} = 6.02 \text{ cm} //$$

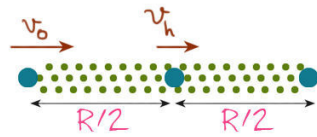
$[v^2]^{3/2} \Rightarrow (\text{m/s})^2 \rightarrow (\text{cm/s})^2$

B Average ion pairs =

$$\frac{E_0}{34 \text{ eV}} = \frac{7 \times 10^6}{34} = 2.06 \times 10^5 //$$

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C Let the half way velocity be v_h



$$R \propto v^3$$

$$\Rightarrow \frac{R_0}{v_0^3} = \frac{R/2}{(v_h)^3} \Rightarrow v_h = \frac{v_0}{2^{1/3}}$$

$$\Rightarrow v_h^2 = \frac{v_0^2}{2^{2/3}}$$

$$\Rightarrow E_h = \frac{E_0}{2^{2/3}} \quad (As E \propto v^2)$$

Ion pairs generated in First half = Total ion pairs - second half ionpairs

$$= \frac{E_0}{34} - \frac{E_h}{34}$$

$$= \frac{E_0}{34} \left(1 - \frac{1}{2^{2/3}}\right)$$

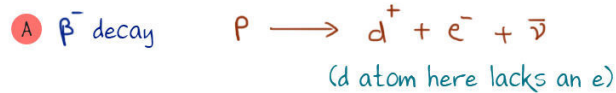
$$= 2.06 \times 10^5 (1 - 0.63)$$

$$= 7.6 \times 10^4 //$$

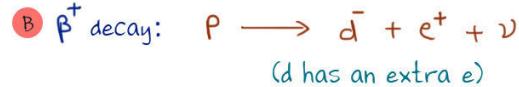
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6.238. Find the energy Q liberated in β^- and β^+ -decays and in K -capture if the masses of the parent atom M_p , the daughter atom M_d and an electron m are known.

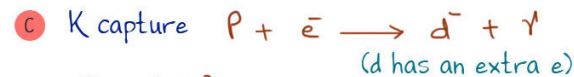
When an e^- is captured by the nucleus



$$\begin{aligned} Q &= \Delta mc^2 \\ &= (m_p - m_{d^+} - m_{e^-}) c^2 \\ &= [m_p - (m_{d^+} + m_{e^-})] c^2 \\ &= (m_p - m_d) c^2 // \end{aligned}$$



$$\begin{aligned} Q &= \Delta mc^2 \\ &= [m_p - m_{d^-} - m_{e^+}] c^2 \\ &= [m_p - (m_{d^-} - m_{e^-}) - m_{e^-} - m_{e^+}] c^2 \\ &= (m_p - m_d - 2m) c^2 // \end{aligned}$$



$$\begin{aligned} Q &= \Delta mc^2 \\ &= (m_p + m_{e^-} - m_{d^-}) c^2 \\ &= [m_p + m_{e^-} - (m_{d^-} - m_{e^-} + m_{e^-})] c^2 \\ &= (m_p - m_d) c^2 // \end{aligned}$$

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6.239. Taking the values of atomic masses from the tables, find the maximum kinetic energy of beta-particles emitted by Be^{10} nuclei and the corresponding kinetic energy of recoiling daughter nuclei formed directly in the ground state.



$$m_{\text{Be}} - m_{\text{B}} = 0.0006 \text{ amu} \\ = 6 \times 10^{-4} \text{ amu}$$

For maximum KE of electron, energy of $\bar{\nu}$ may be assumed 0.

$$Q = (m_{\text{Be}} - m_{\text{B}}) c^2 = KE_f - KE_i$$

(Be is at rest initially)

As mass of B is very high compared to e, so e will contain most KE

$$\Rightarrow 6 \times 10^{-4} \times (931.5) \text{ MeV} = 0.56 \text{ MeV} = KE_{\text{B}} + KE_e$$

$$KE_{\text{B}} = \frac{p^2}{2m_{\text{B}}} = \frac{(2m_e T)^2}{2m_{\text{B}}} = \frac{m_e T}{m_{\text{B}}} = \frac{9.1 \times 10^{-31}}{10 \times 1.66 \times 10^{-27}} \times 0.56 \times 10^6 \text{ eV} = 30.7 \text{ eV}$$

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Finding velocities, when one particle is very large compared to other



Problem: There is explosion and energy is released as $KE = (10^6 + 10^3) \text{ J}$
 $v_1 = ?$

For quick approximate answers

Step 1 Give whole energy to lighter particle (2kg).

$$v_2 = \sqrt{\frac{2T}{m_2}} = \sqrt{\frac{2(10^6 + 10^3)}{2}} \\ = 1000.5 \text{ m/s}$$

Step 2 p conservation.

$$m_1 v_1 = m_2 v_2 \\ \Rightarrow v_1 = \frac{2(1000.5)}{2000} = 1.0005 \text{ m/s}$$

Accurate answer:

p conservation:

$$2000 v_1 = 2 v_2$$

Energy conservation:

$$\frac{1}{2} (2000) v_1^2 + \frac{1}{2} (2) v_2^2 = 10^6 + 10^3$$

From 1,2:

$$1000 v_1^2 + (1000 v_1)^2 = 10^6 + 10^3$$

$$v_1 = 1 \text{ m/s}$$

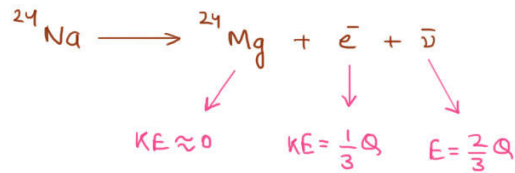
$$v_2 = 1000 \text{ m/s}$$

Note: When initial momentum is non-zero, accurate method is even more lengthy. Use approximate method to get velocities quickly

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6.240. Evaluate the amount of heat produced during a day by a β^- -active Na^{24} preparation of mass $m = 1.0$ mg. The beta-particles are assumed to possess an average kinetic energy equal to $1/3$ of the highest possible energy of the given decay. The half-life of Na^{24} is $T = 15$ hours.

$$\begin{aligned} {}^{24}\text{Na} &= 24 - 0.00903 \text{ amu} \\ {}^{24}\text{Mg} &= 24 - 0.01496 \text{ amu} \\ m_{\text{Na}} - m_{\text{Mg}} &= 5.93 \times 10^{-3} \text{ amu} \end{aligned}$$



(Turns to Heat) (Neutrinos escape)

Nuclei decayed in a day: $N_0 \left(1 - \frac{1}{2^{n/15}}\right)$

$$\begin{aligned} &= \frac{1 \times 10^{-3}}{24} \times 6.02 \times 10^{23} \left(1 - \frac{1}{2^{24/15}}\right) \\ &= 1.7 \times 10^{19} \quad (2) \end{aligned}$$

∴ For one nucleus

$$\begin{aligned} \text{Heat} &= \frac{Q}{3} = \frac{1}{3} (m_{\text{Na}} - m_{\text{Mg}}) c^2 \quad 6.238 \text{ (A)} \\ &= \frac{1}{3} (5.93 \times 10^{-3}) (931.5) \text{ MeV} \\ &= 1.8 \text{ MeV} \quad (1) \end{aligned}$$

From 1,2:

Total heat: $(1.8 \times 10^6 \times 1.6 \times 10^{-19}) \times 1.7 \times 10^{19} = 4.9 \times 10^6 \text{ J}$

Youtube / Arihant Senior

6.241. Taking the values of atomic masses from the tables, calculate the kinetic energies of a positron and a neutrino emitted by C^{11} nucleus for the case when the daughter nucleus does not recoil.

$$\begin{aligned} 1 \text{ amu} &= 931.5 \text{ MeV} \\ 1e &= 0.51 \text{ MeV} \\ (\because m_e c^2 &= 0.51) \end{aligned}$$

Method 1 Using non-relativistic mechanics

$$Q = (m_{\text{C}} - m_{\text{B}} - 2m_e) c^2 \quad 6.238 \text{ (B)}$$



$$\begin{aligned} &= (11.01143 - 11.0093) 931 - 2(0.51) \\ &= 0.963 \text{ MeV} \end{aligned}$$

$$\therefore E_\nu + E_e = 0.963 \text{ MeV}$$

$$E_\nu = \frac{hc}{\lambda} = pc$$

$$pc + \frac{p^2}{2m_e} = 0.963 \text{ MeV}$$

$$\Rightarrow pc + \frac{p^2 c^2}{2m_e c^2} = 0.963 \Rightarrow pc = E_\nu = 0.6 \text{ MeV}$$

$$\Rightarrow E_e = 0.963 - 0.6 = 0.36 \text{ MeV}$$

$$\begin{aligned} \frac{\frac{1}{2} m_e v_e^2}{m_e c^2} &= \frac{0.36}{0.51} \\ \Rightarrow v_e &= 1.18 c \end{aligned}$$

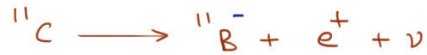
Not possible

$$0.51 \text{ MeV} \leftarrow$$

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Method 2 Using relativistic mechanics (accurate, $\because$ velocity of positron is high)

Energy available to neutrino and positron (including its rest energy) is given by:



$$\begin{aligned} Q &= (m_C - m_{B^-}) c^2 \\ &= [m_C - (m_B + m_e)] c^2 \\ &= (11.01143 - 11.0093) 931 - 1(0.51) \\ &= 1.47 \text{ MeV} \quad (1) \end{aligned}$$

$$\begin{aligned} \text{Also, } Q &= E_e + E_\nu \\ &= \sqrt{p^2 c^2 + m_e^2 c^4} + pc \quad (2) \end{aligned}$$

(Equation 1.8h in book)

From 1,2:

$$p^2 c^2 + (0.51)^2 = (1.47 - pc)^2$$

$$\Rightarrow pc = E_\nu = 0.646 \text{ MeV}, E_e = 1.47 - 0.646 = 0.824$$

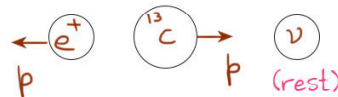
$$\text{KE of positron} = E_e - \text{rest energy} = E_e - m_e c^2 = 0.824 - 0.51 = 0.314 \text{ MeV}$$

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6.242. Find the kinetic energy of the recoil nucleus in the positron-ic decay of a N^{13} nucleus for the case when the energy of positrons is maximum.

Method 1 Using non-relativistic mechanics

In 6.241 daughter nucleus was at rest, here, Neutrino will be at rest (to maximise energy of positron)



$$\begin{aligned} Q &= (m_N - m_C - 2m_e) c^2 \quad 6.238 \text{ (B)} \\ &= (13.00574 - 13.00335) 931 - 2(0.51) \\ &= 2.22 - 2(0.51) \\ &= 1.21 \text{ MeV} \end{aligned}$$

Using approximation, e will have most KE:

$$\therefore E_e = \frac{p^2}{2m_e} \approx Q = 1.21 \text{ MeV}$$

$$E_C = \frac{p^2}{2m_C} = E_e \left(\frac{m_e}{m_C} \right)$$

$$\begin{aligned} &= \frac{1.21}{13 \times 1823} = 5 \times 10^{-5} \text{ MeV} \\ &= 0.05 \text{ keV} \end{aligned}$$

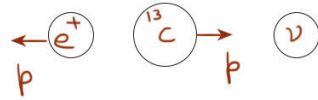
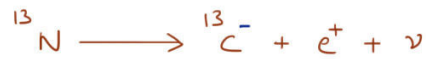
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$$\begin{aligned} \frac{\frac{1}{2} m_e v_e^2}{m_e c^2} &= \frac{1.21}{0.51} \\ \Rightarrow v_e &= 2.1c \end{aligned}$$

Not possible

Method 2 Using relativistic mechanics (accurate, ∴ velocity of positron is high)

□□□



Energy available to neutrino and positron (including its rest energy) is given by:

$$\begin{aligned} Q &= (m_N - m_C) c^2 \\ &= [m_N - (m_C + m_e)] c^2 \\ &= (13.00574 - 13.00335) 931 - 1(0.51) \\ &= 1.71 \text{ MeV} \end{aligned}$$

Also, $p_e c = \sqrt{E_e^2 - m_e^2 c^4}$ (equation 1.8h in book)

$$\Rightarrow p c \approx \sqrt{(1.71)^2 - (0.51)^2}$$

$$\Rightarrow p = \frac{1.63}{c} \text{ MeV}$$

$$\therefore E_C = \frac{p^2}{2m} = \frac{(1.63)^2}{2 m_C c^2} = \frac{(1.63)^2}{2 \cdot 13 (931)} = 0.11 \text{ keV}$$

Youtube / Arihant Senior

6.243. From the tables of atomic masses determine the velocity of a nucleus appearing as a result of K-capture in a Be^7 atom provided the daughter nucleus turns out to be in the ground state.

□□□



$$Q = \underbrace{(m_{\text{Be}} - m_{\text{Li}})}_{6.238 \text{ (C)}} c^2$$

$$\begin{aligned} &= (7.01693 - 7.01601) 931 \text{ MeV} \\ &= 0.86 \text{ MeV} \end{aligned}$$

∴ Neutrino will take up most of energy

$$E_\nu \approx Q$$

$$\Rightarrow p = \frac{Q}{c}$$

$$p = p$$

$$\begin{aligned} \Rightarrow v_{\text{Li}} &= \frac{p}{m} = \frac{Q}{m_C} = \frac{0.86 \times 10^6 \times 1.6 \times 10^{-19}}{7 \times 1.67 \times 10^{-27} \times 3 \times 10^8} \\ &= 3.9 \times 10^4 \text{ m/s} // \end{aligned}$$

Youtube / Arihant Senior

6.244. Passing down to the ground state, excited Ag^{109} nuclei emit either gamma quanta with energy 87 keV or K conversion electrons whose binding energy is 26 keV. Find the velocity of these electrons. Using relativistic mechanics

□ □ □

Energy released during de-excitation: $E = 87 \text{ keV}$

∴ For knocked out

K conversion electron: $E = E_b + KE$

$$\Rightarrow KE = \frac{1}{2} m_e v^2 = (87 - 26) = 61 \text{ keV}$$

Using, $T = (m - m_0) c^2$ & $m = \frac{m_0}{\sqrt{1 - (\frac{v}{c})^2}}$ (equations 1.8e and 1.8g in book)

$$61 \text{ keV} = \underbrace{m_e c^2}_{0.511 \text{ MeV}} \left(\frac{1}{\sqrt{1 - (\frac{v}{c})^2}} - 1 \right) \Rightarrow v = 0.45 c //$$

K-shell electrons that are knocked out during de-excitation of nuclei.

It's a non-radioactive decay process.

Youtube / Arihant Senior

6.245. A free stationary Ir^{191} nucleus with excitation energy $E = 129 \text{ keV}$ passes to the ground state, emitting a gamma quantum. Calculate the fractional change of gamma quanta energy due to recoil of the nucleus.

□ □ □

$$E \approx E_\gamma = pc = 129 \text{ keV} = (0.129) \text{ MeV}$$

$$\therefore E_{\text{Ir}} = \frac{p^2}{2m}$$

$$\begin{aligned} \therefore \text{Fraction of energy loss} &= \frac{E - E_\gamma}{E} = \frac{E_{\text{Ir}}}{E} = \frac{p^2}{2mE} = \frac{E^2}{2mEc^2} = \frac{E}{2mc^2} \\ &= \frac{(0.129)}{2(191)(931)} = 3.6 \times 10^{-7} \end{aligned}$$

Youtube / Arihant Senior

6.246. What must be the relative velocity of a source and an absorber consisting of free Ir^{191} nuclei to observe the maximum absorption of gamma quanta with energy $\epsilon = 129 \text{ keV}$?

For maximum absorption, final KE of system must be zero.
So momentum of incoming gamma photon must be canceled by momentum of nucleus.

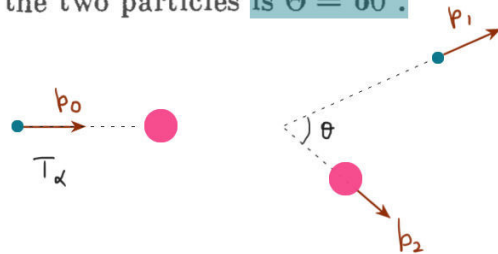


$$v_{\text{Ir}} = \frac{p}{m} = \frac{E/c}{m} = \frac{129 \times 10^{-3} \text{ MeV}}{m c} \times \frac{c}{c}$$

$$= \frac{.129 c}{191 \times 931} = 217 \text{ m/s} //$$

Youtube / Arihant Senior

6.249. An alpha-particle with kinetic energy $T_\alpha = 7.0 \text{ MeV}$ is scattered elastically by an initially stationary Li^6 nucleus. Find the kinetic energy of the recoil nucleus if the angle of divergence of the two particles is $\theta = 60^\circ$.



E conservation: $T_\alpha = T_1 + T_2$ 1

p conservation: $\vec{p}_0 = \vec{p}_1 + \vec{p}_2$

$$\Rightarrow p_0^2 = p_1^2 + p_2^2 + 2p_1 p_2 \cos \theta$$

$$\Rightarrow 2m_1 T_\alpha = 2m_1 T_1 + 2m_2 T_2 + 2\sqrt{2m_1 T_1} \sqrt{2m_2 T_2} \cos \theta$$
 2

From 1,2:

$$T_2 = \frac{T_\alpha}{1 + \frac{(m_1 - m_2)^2}{4m_1 m_2 \cos^2 \theta}} //$$

Youtube / Arihant Senior

6.250. A neutron collides elastically with an initially stationary deuteron. Find the fraction of the kinetic energy lost by the neutron

(a) in a head-on collision;

(b) in scattering at right angles.

deflection (ex, rutherfords gold foil scattering)



(Opposite directions as $m_n < m_d$)

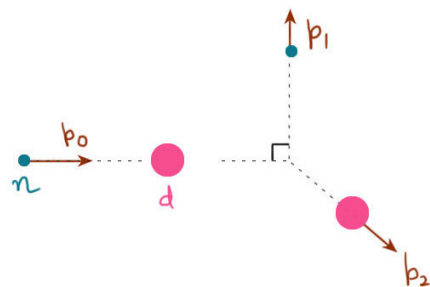
A E conservation: $T_0 = T_1 + T_2$

p conservation: $p_0 = p_2 - p_1$

$$\Rightarrow \sqrt{2m_1 T_0} = \sqrt{2m_2 T_2} - \sqrt{2m_1 T_1} \quad \text{--- } T_0 - T_2$$

$$\Rightarrow \left(\frac{\sqrt{m_1 T_0} - \sqrt{m_2 T_2}}{T_0} \right)^2 = \frac{m_1 (T_0 - T_2)}{T_0}$$

$$\Rightarrow \text{Fraction of E lost} = \frac{T_0 - T_1}{T_0} = \frac{T_2}{T_0} = \frac{4m_1 m_2}{(m_1 + m_2)^2} = \frac{4(1)(2)}{(1+2)^2} = \frac{8}{9} //$$



B $\vec{p}_0 = \vec{p}_1 + \vec{p}_2$

$$\Rightarrow \vec{p}_0 - \vec{p}_1 = \vec{p}_2$$

$$\Rightarrow p_0^2 + p_1^2 = p_2^2$$

($\because \vec{p}_0 \perp \vec{p}_1$)

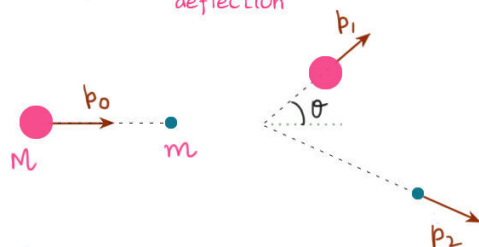
$$\Rightarrow \cancel{2m_1 T_0} + \cancel{2m_1 T_1} = 2m_2 T_2$$

$$\Rightarrow m_1 T_0 + m_1 (T_0 - T_2) = m_2 T_2$$

$$\Rightarrow \frac{T_2}{T_0} = \frac{2m_1}{m_1 + m_2} = \frac{2(1)}{1+2} = \frac{2}{3} //$$

6.251. Find the greatest possible angle through which a deuteron is scattered as a result of elastic collision with an initially stationary proton.

deflection



$$4M^2 \cos^2 \theta - 4(M^2 - m^2) \geq 0$$

$$\Rightarrow \cos^2 \theta \geq 1 - \frac{m^2}{M^2}$$

$$\Rightarrow \sin^2 \theta \leq \frac{m^2}{M^2}$$

$\because \theta$ cant be -ve $\Rightarrow \sin \theta \leq \frac{m}{M}$

$$\Rightarrow \sin \theta \leq \frac{1}{2} \Rightarrow \theta \leq 30^\circ //$$

E conservation: $T_0 = T_1 + T_2$

p conservation: $\vec{p}_0 = \vec{p}_1 + \vec{p}_2$

$$\Rightarrow \vec{p}_0 - \vec{p}_1 = \vec{p}_2$$

$$\Rightarrow p_0^2 + p_1^2 - 2p_0 p_1 \cos \theta = p_2^2$$

$$\Rightarrow 2MT_0 + 2MT_1 - 2\sqrt{2MT_0} \sqrt{2MT_1} \cos \theta = 2mT_2 \quad \text{--- } T_0 - T_1$$

$$\Rightarrow \frac{MT_0 + MT_1 - 2M\sqrt{T_0 T_1} \cos \theta}{T_1} = \frac{mT_0 - mT_1}{T_1}$$

$$\Rightarrow (M - m) \frac{T_0}{T_1} - 2M \cos \theta \sqrt{\frac{T_0}{T_1}} + (M + m) = 0$$

$$\Rightarrow D \geq 0, \text{ for real roots}$$

6.252. Assuming the radius of a nucleus to be equal to $R = 1.3 \sqrt[3]{A} \text{ fm}$, where A is its mass number, evaluate the density of nuclei and the number of nucleons per unit volume of the nucleus.

misprint fixed

$$1 \text{ fm} = 10^{-15} \text{ m}$$

$$\begin{aligned} \text{Nucleons per unit volume: } \frac{\text{Nucleons}}{\text{Volume of those nucleons}} &= \frac{A}{\frac{4}{3}\pi R^3} = \frac{3A}{4\pi(1.3)^3} \times 10^{-45} \\ &= 1.08 \times 10^{44} \text{ nucleons/m}^3 \\ &= 1.08 \times 10^{38} \text{ nucleons/cc} // \end{aligned}$$

Density: Mass per unit volume

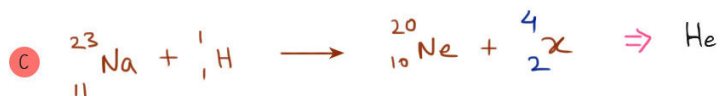
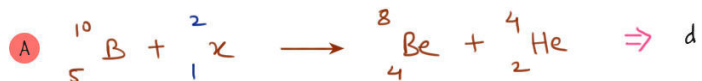
$$\begin{aligned} &= \text{mass per nucleon} \times \text{Nucleons per unit volume} \\ &= 1.67 \times 10^{-27} \times 1.08 \times 10^{38} \text{ kg/cc} \\ &= 1.8 \times 10^{11} \text{ kg/cc} // \end{aligned}$$

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6.253. Write missing symbols, denoted by x , in the following nuclear reactions:

- (a) $\text{B}^{10}(x, \alpha)\text{Be}^8$;
- (b) $\text{O}^{17}(d, n)x$;
- (c) $\text{Na}^{23}(p, x)\text{Ne}^{20}$;
- (d) $x(p, n)\text{Ar}^{37}$.

Conserving mass (mass number) and charge



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6.254. Demonstrate that the binding energy of a nucleus with mass number A and charge Z can be found from Eq. (6.6b).

For a given atom, binding energy of nucleus (in amu) is given as:

$$E_b = Z \Delta_H + (A-Z) \Delta_n - \Delta$$

$\downarrow$ $\downarrow$ $\downarrow$
 Mass surplus of: proton neutron atom

Relative to 1 amu, which in turn is defined as 1/12th the mass of C_{12} .
 Mass surplus is expressed in amu.

1

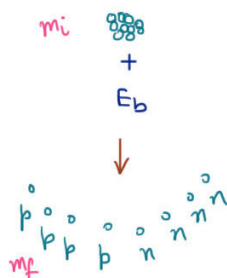
i.e.

$$m_p = 1 \text{ amu} + \Delta_H$$

$$m_n = 1 \text{ amu} + \Delta_n$$

$$m_A = A \text{ amu} + \Delta$$

Sol:



$$E_b = m_f - m_i \text{ (in amu terms, not Joules)}$$

$$= Z m_H + (A-Z) m_n - M$$

mass of separated nucleons mass of nucleus

From 1

$$= Z (1 + \Delta_H) + (A-Z) (1 + \Delta_n) - (A + \Delta)$$

$$= \cancel{Z} + Z \Delta_H + \cancel{A-Z} + (A-Z) \Delta_n - \cancel{A} - \Delta$$

Hence proved

Note: Actual binding energy will be $E_b \text{ (in amu)} \times 931$

6.255. Find the binding energy of a nucleus consisting of equal numbers of protons and neutrons and having the radius one and a half times smaller than that of Al^{27} nucleus.

$$\text{Total number of nucleons} = \left(\frac{V}{V_{Al}} \right) n_{Al} = \left(\frac{r^3}{(1.5r)^3} \right) \cdot 27 = 8 \Rightarrow {}^8\text{Be}$$

$$\begin{aligned} \therefore E_b &= (4 \Delta m_H + 4 \Delta m_n - \Delta m_{Be}) 931 \text{ MeV} \\ &= (4 \times 0.00783 + 4 \times 0.00867 - 0.00531) 931.5 \\ &= 56.5 \text{ MeV} // \end{aligned}$$

- 6.256. Making use of the tables of atomic masses, find:
 (a) the mean binding energy per one nucleon in O^{16} nucleus;
 (b) the binding energy of a neutron and an alpha-particle in a B^{11} nucleus;
 (c) the energy required for separation of an O^{16} nucleus into four identical particles.

$$Q = (m_i - m_f) c^2 = \kappa E_f - \kappa E_i = E_{bf} - E_{bi}$$

$$\begin{aligned} \text{A } E_b &= (8 \Delta m_p + 8 \Delta m_n - M_O) 931 \text{ MeV} \\ &= [8 \times 0.00783 + 8 \times 0.00867 - (-0.00509)] 931 \\ &= 127.1 \text{ MeV} // \quad E_{b/\text{nucleon}} = \frac{127.1}{16} \approx 8 \text{ MeV} // \end{aligned}$$

$$\begin{aligned} \text{B } {}^{11}\text{B} + E_b &\longrightarrow {}^{10}\text{B} + {}^1_0\text{n} \\ {}^{11}\text{B} &\xrightarrow{|||} {}^{10}\text{B} + {}^1_0\text{n} + Q \\ \therefore E_b = -Q &= -(M_{B_{11}} - M_{B_{10}} - m_n) 931.5 \\ &= (0.01231) 931 = 11.46 \text{ MeV} // \end{aligned}$$

$$\begin{aligned} \text{C } {}^{16}\text{O} + E_b &\longrightarrow 4 {}^4\text{He} \\ {}^{16}\text{O} &\xrightarrow{|||} 4 {}^4\text{He} + Q \\ \therefore E_b = -Q &= -(M_O - 4 M_{He}) 931 \text{ MeV} \\ &= (0.01549) 931 = 14.42 \text{ MeV} // \end{aligned}$$

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- 6.257. Find the difference in binding energies of a neutron and a proton in a B^{11} nucleus. Explain why there is the difference.

$${}^{11}\text{B} \longrightarrow {}^{10}\text{B} + {}^1_0\text{n} \Rightarrow E_{bn} = -Q = -(M_{B_{11}} - M_{B_{10}} - m_n)$$

$${}^{11}\text{B} \longrightarrow {}^{10}\text{Be} + {}^1_1\text{p} \Rightarrow E_{bp} = -Q = -(M_{B_{11}} - M_{Be_{10}} - m_p)$$

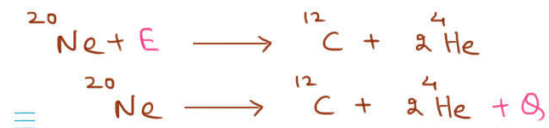
$$\begin{aligned} \Rightarrow E_{bn} - E_{bp} &= (M_{B_{10}} + m_n - M_{Be_{10}} - m_p) \\ &= (0.01294 + 0.00867 - 0.01354 - 0.00783) 931 \\ &= 0.223 \text{ MeV} // \end{aligned}$$

$$\Rightarrow E_{bn} > E_{bp}$$

Expected, as proton is easier to remove from nucleus due to repulsion. Hence its Binding energy is less.

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6.258. Find the energy required for separation of a Ne^{20} nucleus into two alpha-particles and a C^{12} nucleus if it is known that the binding energies per one nucleon in Ne^{20} , He^4 , and C^{12} nuclei are equal to 8.03, 7.07, and 7.68 MeV respectively.



$$\therefore E = -Q = -(E_{bf} - E_{bi})$$

$$\Rightarrow E = -(E_{bc} + 2E_{b\text{He}} - E_{b\text{Ne}})$$

$$\begin{aligned} &= E_{b\text{Ne}} - E_{bc} - 2E_{b\text{He}} \\ &= 20 \times 8.03 - 12 \times 7.68 - 2 \times (4 \times 7.07) \\ &= 11.9 \text{ MeV} // \end{aligned}$$

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6.259. Calculate in atomic mass units the mass of

(a) a Li^8 atom whose nucleus has the binding energy 41.3 MeV;

(b) a C^{10} nucleus whose binding energy per nucleon is equal to 6.04 MeV.

A



$$E_b = -Q = -(M_{\text{Li}} - 3m_p - 5m_n) \cdot 931$$

$$\rightarrow 41.3 = -(\Delta M_{\text{Li}} - 3\Delta H - 5\Delta n) \cdot 931$$

In amu's

$$\Rightarrow \frac{41.3}{931} = -(\Delta M_{\text{Li}} - 3 \times 0.00783 - 5 \times 0.00867)$$

$$\Rightarrow \Delta M_{\text{Li}} = 0.02248 \text{ amu}$$

$$\begin{aligned} \therefore M_{\text{Li}} &= 8 + \Delta M_{\text{Li}} \\ &= 8.02248 \text{ amu} // \end{aligned}$$

B



$$10 \times E_b = -(\Delta M_{\text{C}_{10}} - 6\Delta H - 4\Delta n) \cdot 931$$

6.04 0.00783 0.00867

$$\Rightarrow \Delta M_{\text{C}_{10}} = 0.01678 \text{ amu}$$

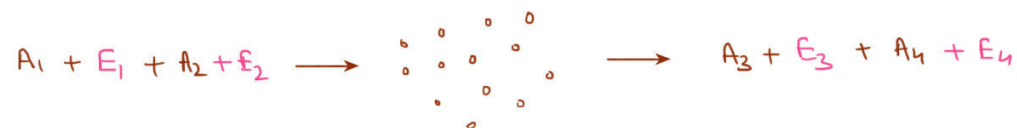
$$\begin{aligned} \therefore M_{\text{C}_{10}} &= 10 + \Delta M_{\text{C}_{10}} \\ &= 10.01678 \text{ amu} // \end{aligned}$$

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6.260. The nuclei involved in the nuclear reaction $A_1 + A_2 \rightarrow A_3 + A_4$ have the binding energies E_1, E_2, E_3 , and E_4 . Find the energy of this reaction.



|||



$$\therefore \text{Energy of reaction} = Q = E_3 + E_4 - E_1 - E_2 //$$

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6.261. Assuming that the splitting of a U^{235} nucleus liberates the energy of 200 MeV, find:

(a) the energy liberated in the fission of one kilogram of U^{235} isotope, and the mass of coal with calorific value of 30 kJ/g which is equivalent to that for one kg of U^{235} ;

(b) the mass of U^{235} isotope split during the explosion of the atomic bomb with 30 kt trotyl equivalent if the calorific value of trotyl is 4.1 kJ/g.

→ TNT (also, dynamite's calorific value is : 7.5 kJ/g)

A $E \text{ from } 1 \text{ kg } U^{235} = \left(\frac{1 \times 1000}{235} \times 6.02 \times 10^{23} \right) \times \left(200 \times 10^6 \times 1.6 \times 10^{-19} \right) = 8.21 \times 10^{10} \text{ kJ} //$
(weight of a big rat)

$\therefore \text{Equivalent coal} = \frac{8.21 \times 10^{10} \text{ kJ}}{30 \times 10^3 \text{ kJ/kg}} = 2.74 \times 10^6 \text{ kg} // \approx \text{(weight of 500 adult elephants)}$

B $\text{Equivalent } U^{235} = \left(30 \times 10^3 \text{ g} \times 4.1 \frac{\text{kJ}}{\text{g}} \right) \frac{1}{8.21 \times 10^{10} \text{ kJ/kg}} = 1.5 \text{ kg} //$

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6.262. What amount of heat is liberated during the formation of one gram of He^4 from deuterium H^2 ? What mass of coal with calorific value of 30 kJ/g is thermally equivalent to the magnitude obtained?



In formation of one molecule of He:

$$Q = (2 \times 0.0141 - 0.0026) \times 931 \text{ MeV} \\ = 23.8 \text{ MeV}$$

In forming one gram of He:

$$Q = (23.8) \frac{N_A}{4} = 3.5 \times 10^{24} \text{ MeV} = 5.6 \times 10^{11} \text{ J}$$

$$\therefore \text{Equivalent coal: } \frac{5.6 \times 10^{11} \text{ J}}{30 \times 10^3 \text{ J/g}} = 18.6 \text{ tonnes}$$

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6.263. Taking the values of atomic masses from the tables, calculate the energy per nucleon which is liberated in the nuclear reaction $\text{Li}^6 + \text{H}^2 \rightarrow 2\text{He}^4$. Compare the obtained magnitude with the energy per nucleon liberated in the fission of U^{235} nucleus.

Using 200 MeV/fission from 6.261

A $Q = [0.01513 + 0.0141 - 2 \times 0.0026] \times 931 \text{ MeV} \\ = 22.37 \text{ MeV}$

Number of nucleons on LHS of reaction = 8

$$\therefore Q \text{ per nucleon in given fusion reaction} = \frac{22.37}{8} = 2.8 \text{ MeV/nucleon}$$

B $Q \text{ per nucleon in fission of } \text{U}^{235} = \frac{200}{235} = 0.85 \text{ MeV/nucleon}$

So we observe that energy released per nucleon is higher for fusion than fission!

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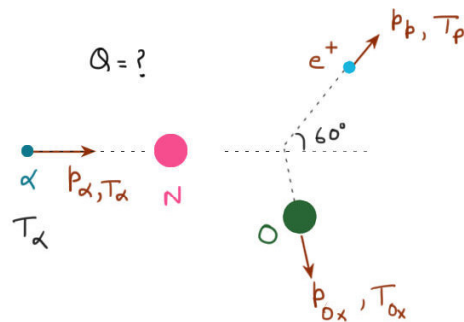
6.264. Find the energy of the reaction $\text{Li}^7 + p \rightarrow 2\text{He}^4$ if the binding energies per nucleon in Li^7 and He^4 nuclei are known to be equal to 5.60 and 7.06 MeV respectively.



$$\begin{aligned} Q &= E_{b2} - E_{b1} \\ &= 8(7.06) - 7(5.6) \\ &= 17.28 \text{ MeV} // \end{aligned}$$

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6.265. Find the energy of the reaction $\text{N}^{14}(\alpha, p)\text{O}^{17}$ if the kinetic energy of the incoming alpha-particle is $T_\alpha = 4.0 \text{ MeV}$ and the proton outgoing at an angle $\theta = 60^\circ$ to the motion direction of the alpha-particle has a kinetic energy $T_p = 2.09 \text{ MeV}$.



p conservation: $\vec{p}_\alpha = \vec{p}_p + \vec{p}_{Ox}$

$$\Rightarrow \vec{p}_\alpha - \vec{p}_p = \vec{p}_{Ox}$$

Diagram shows the vector subtraction: $\vec{p}_\alpha$ and $-\vec{p}_p$ are added to find $\vec{p}_{Ox}$ using the triangle rule with an angle of 120° between $\vec{p}_\alpha$ and $-\vec{p}_p$.

$$\Rightarrow p_\alpha^2 + p_p^2 + 2p_\alpha p_p \cos 120^\circ = p_{Ox}^2$$

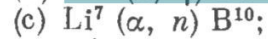
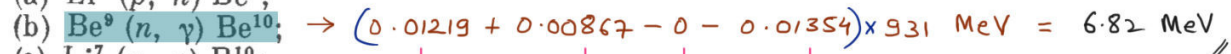
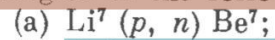
$$\Rightarrow 2m_\alpha T_\alpha + 2m_p T_p - 2\sqrt{2m_\alpha T_\alpha} \sqrt{2m_p T_p} = 2m_{Ox} T_{Ox}$$

E conservation: $Q = T_p + T_{Ox} - T_\alpha$

$$\Rightarrow Q = -12 \text{ MeV} //$$

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6.266. Making use of the tables of atomic masses, determine the energies of the following reactions:



A $Q = -1.65 \text{ MeV}$ -ve Q implies energy is needed to start reaction, ie its an Endoergic reaction

C $Q = -2.79 \text{ MeV}$

D $Q = 3.11 \text{ MeV}$

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6.267. Making use of the tables of atomic masses, find the velocity with which the products of the reaction $\text{B}^{10} (n, \alpha) \text{Li}^7$ come apart; the reaction proceeds via interaction of very slow neutrons with stationary boron nuclei.

$$= (\Delta_{\text{B}} + \Delta_n - \Delta_{\alpha} - \Delta_{\text{Li}}) 931 \text{ MeV} = 2.79 \text{ MeV}$$

E conservation: $Q = T_{\alpha} + T_{\text{Li}}$ 1

p conservation: $p_{\alpha} = p_{\text{Li}}$

$$\Rightarrow \sqrt{2m_{\alpha}T_{\alpha}} = \sqrt{2m_{\text{Li}}T_{\text{Li}}}$$

$$\Rightarrow m_{\alpha}T_{\alpha} = m_{\text{Li}}T_{\text{Li}}$$
 2

From 1,2: $T_{\alpha} = \frac{Q}{(1 + \frac{m_{\alpha}}{m_{\text{Li}}})} = \frac{1}{2}m_{\alpha}v_{\alpha}^2 \Rightarrow v_{\alpha} = \sqrt{\frac{2Q}{m_{\alpha}(1 + \frac{m_{\alpha}}{m_{\text{Li}}})}}$ //

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6.268. Protons striking a stationary lithium target activate a reaction $\text{Li}^7(p, n)\text{Be}^7$. At what value of the proton's kinetic energy can the resulting neutron be stationary?



$$Q = (\Delta_{\text{Li}} + \Delta_p - \Delta_n - \Delta_{\text{Be}}) 931 \text{ MeV} = -1.64 \text{ MeV}$$

E conservation: $Q = T_{\text{Be}} - T_p$ 1

p conservation: $p_p = p_{\text{Be}}$

$$\Rightarrow \sqrt{2m_p T_p} = \sqrt{2m_{\text{Be}} T_{\text{Be}}}$$

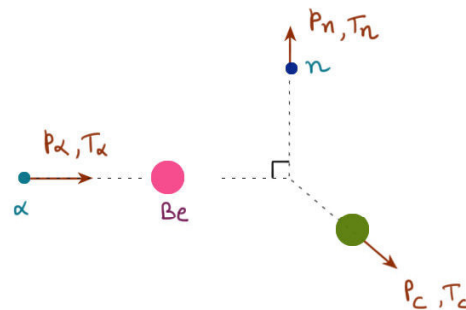
$$\Rightarrow m_p T_p = m_{\text{Be}} T_{\text{Be}} \quad 2$$

From 1,2:

$$T_p = \frac{Q}{\frac{m_p}{m_{\text{Be}}} - 1} = 1.91 \text{ MeV} //$$

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6.269. An alpha particle with kinetic energy $T_\alpha = 5.3 \text{ MeV}$ initiates a nuclear reaction $\text{Be}^9(\alpha, n)\text{C}^{12}$ with energy yield $Q = +5.7 \text{ MeV}$. Find the kinetic energy of the neutron outgoing at right angles to the motion direction of the alpha-particle.



p conservation: $\vec{p}_\alpha = \vec{p}_n + \vec{p}_C$

$$\Rightarrow \vec{p}_\alpha - \vec{p}_n = \vec{p}_C$$

$$\Rightarrow p_\alpha^2 + p_n^2 = p_C^2$$

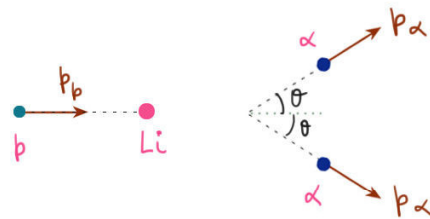
$$\Rightarrow 2m_\alpha T_\alpha + 2m_n p_n^2 = 2m_C T_C$$

E conservation: $Q = T_n + T_C - T_\alpha$

$$\Rightarrow T_n = 8.5 \text{ MeV} //$$

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6.270. Protons with kinetic energy $T_p = 1.0 \text{ MeV}$ striking a lithium target induce a nuclear reaction $p + \text{Li}^7 \rightarrow 2\text{He}^4$. Find the kinetic energy of each alpha-particle and the angle of their divergence provided their motion directions are symmetrical with respect to that of incoming protons.



E conservation: $Q = (\Delta p + \Delta \text{Li} - 2 \Delta \text{He})^{931} = 17.35 \text{ MeV}$
 $Q = 2T_\alpha - T_p \quad (1)$

p conservation: $p_p = 2p_\alpha \cos \theta$

$$\Rightarrow \sqrt{2m_p T_p} = 2\sqrt{2m_\alpha T_\alpha} \cos \theta$$

$$\Rightarrow m_p T_p = 4m_\alpha T_\alpha \cos^2 \theta \quad (2)$$

From 1,2: $T_\alpha = 3.1 \text{ MeV} //$

$$\theta = 85.25^\circ$$

$\therefore$ Angle of divergence $= 2\theta = 2 \times 85.25^\circ = 170.5^\circ //$

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6.271. A particle of mass m strikes a stationary nucleus of mass M and activates an endoergic reaction. Demonstrate that the threshold (minimal) kinetic energy required to initiate this reaction is defined by Eq. (6.6d).

$$T_{th} = \frac{m+M}{M} |Q|$$

Endothermic (i.e., Q is -ve)

Sol: If energy required is minimum:

- A 'collision' must be perfectly inelastic.
- B i.e., products must have same velocity
- C i.e., products must be at rest in com frame.



p conservation: $p_m = p$

$$\Rightarrow \sqrt{2mT_m} = \sqrt{2(m+M)T}$$

$$\Rightarrow mT_m = (m+M)T \quad (1)$$

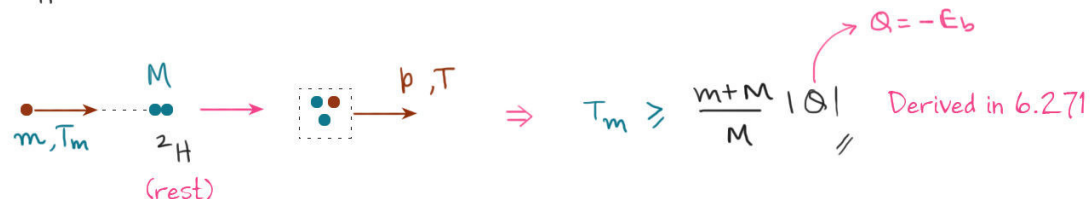
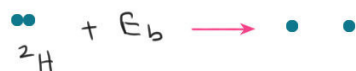
E conservation: $Q = T - T_m \quad (2)$

From 1,2, minimum KE of m to initiate reaction is:

$$T_m = -\frac{m+M}{M} Q = \frac{m+M}{M} |Q| //$$

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6.272. What kinetic energy must a proton possess to split a deuteron H^2 whose binding energy is $E_b = 2.2 \text{ MeV}$?



Sol: $T_p \geq \left(\frac{m_p + m_d}{m_d} \right) E_b$

$\Rightarrow T_p \geq \left(\frac{1+2}{2} \right) 2.2$

$\Rightarrow T_p \geq 3.3 \text{ MeV}$

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6.273. The irradiation of lithium and beryllium targets by a monoenergetic stream of protons reveals that the reaction $Li^7(p, n)Be^7$ — 1.65 MeV is initiated whereas the reaction $Be^9(p, n)B^9$ — 1.85 MeV does not take place. Find the possible values of kinetic energy of the protons.

Equal energy protons



$Li^7(p, n)Be^7 \checkmark \Rightarrow T_p \geq \frac{m_p + m_{Li}}{m_{Li}} (1.65) \Rightarrow T_p \geq 1.89$
 1.65 MeV

$Be^9(p, n)B^9 \times \Rightarrow T_p \leq \frac{m_p + m_B}{m_B} (1.85) \Rightarrow T_p \leq 2.05$
 1.85 MeV

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6.274. To activate the reaction (n, α) with stationary B^{11} nuclei, neutrons must have the threshold kinetic energy $T_{th} = 4.0$ MeV. Find the energy of this reaction.



$$T_n = \frac{m_n + m_B}{m_B} |Q|$$

$$\Rightarrow |Q| = \frac{1}{12} \cdot 4 = 3.7 \text{ MeV}$$

$$\Rightarrow Q = -3.74 \text{ MeV} //$$

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6.275. Calculate the threshold kinetic energies of protons required to activate the reactions (p, n) and (p, d) with Li^7 nuclei.



$$T_p = \left(\frac{m_p + m_{Li}}{m_{Li}} \right) |Q| = \left(\frac{1+7}{7} \right) |Q| = (\Delta p + \Delta Li - \Delta n - \Delta Be) \text{ g31} = -1.64 \text{ MeV}$$

$$= \frac{8}{7} (1.64) = 1.87 \text{ MeV} //$$



$$Q = (\Delta p + \Delta Li - \Delta d - \Delta Li^6) \text{ g31} = -5.02 \text{ MeV}$$

$$T_p = \frac{8}{7} (5.02) = 5.7 \text{ MeV} //$$

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6.276. Using the tabular values of atomic masses, find the threshold kinetic energy of an alpha particle required to activate the nuclear reaction $\text{Li}^7(\alpha, n)\text{B}^{10}$. What is the velocity of the B^{10} nucleus in this case?

$$T_\alpha = \frac{m_\alpha + M_{\text{Li}}}{M_{\text{Li}}} Q = \frac{4+7}{7} (\Delta_{\text{Li}} + \Delta_\alpha - \Delta_n - \Delta_{\text{B}}) 931 = -2.8 \text{ MeV}$$

$$= \frac{11}{7} \cdot (2.8) = 4.4 \text{ MeV} //$$

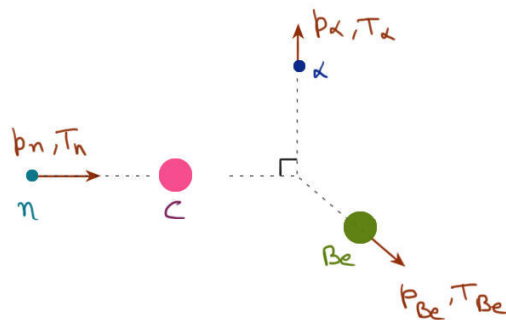
p conservation: $p_\alpha = p_{(n+B)}$

$$\Rightarrow \sqrt{2m_\alpha T_\alpha} = (m_n + m_B) v$$

$$\Rightarrow v = \frac{\sqrt{2m_\alpha T_\alpha}}{m_n + m_B} = 5.3 \times 10^6 \text{ m/s} //$$

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6.277. A neutron with kinetic energy $T_n = 10 \text{ MeV}$ activates a nuclear reaction $\text{C}^{12}(n, \alpha)\text{Be}^9$ whose threshold is $T_{th} = 6.17 \text{ MeV}$. Find the kinetic energy of the alpha-particles outgoing at right angles to the incoming neutrons' direction.



$$E \text{ conservation: } Q = T_\alpha + T_{\text{Be}} - T_n \quad (1)$$

$$p \text{ conservation: } \vec{p}_n = \vec{p}_\alpha + \vec{p}_{\text{Be}}$$

$$\Rightarrow \vec{p}_n - \vec{p}_\alpha = \vec{p}_{\text{Be}}$$

$$\Rightarrow p_n^2 + p_\alpha^2 = p_{\text{Be}}^2$$

$$\Rightarrow 2m_n T_n + 2m_\alpha T_\alpha = 2m_{\text{Be}} T_{\text{Be}} \quad (2)$$

$$T_{th} = \frac{m_n + m_C}{m_C} |Q|$$

$$\Rightarrow Q = -\frac{12}{13} T_{th} = -\frac{12}{13} (6.17) = -5.7 \text{ MeV}$$

$$\text{From 1,2: } T_\alpha = \frac{Q + T_n \left(1 - \frac{m_n}{m_{\text{Be}}}\right)}{1 + \frac{m_\alpha}{m_{\text{Be}}}} = 2.2 \text{ MeV} //$$

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6.278. How much, in per cent, does the threshold energy of gamma quantum exceed the binding energy of a deuteron ($E_b = 2.2 \text{ MeV}$) in the reaction $\gamma + {}^2\text{H} \rightarrow n + p$?

Cant use: $T_{th} = \left(\frac{m+M}{m}\right) Q$ as gamma photon is relativistic.

Sol: For minimum energy required, products will move together (i.e they will be at rest wrt center of mass)



∴ p conservation: $p_\gamma = p_{(n+p)}$

$$\Rightarrow \frac{E_\gamma}{c} = (m_n + m_p) v \quad (1)$$

E conservation: $E_b = E_\gamma - \frac{1}{2}(m_n + m_p) v^2 \quad (2)$

From 1,2: $E_\gamma = \frac{E_\gamma^2}{2(m_n + m_p)c^2} + E_b$

% required = $\frac{E_\gamma - E_b}{E_b} \times 100$

$$= \frac{E_\gamma^2}{E_b \cdot 2(m_n + m_p)c^2}$$

$$\approx \frac{E_b}{2(m_n + m_p)c^2} = \frac{2.2}{2(2)931} = 5.9 \times 10^{-4} \text{ MeV}$$

∴ KE of large products is negligible wrt E of relativistic gamma, from 2:

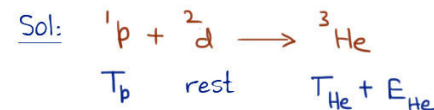
$$E_b = E_\gamma - \frac{1}{2}(m_n + m_p) v^2 \approx 0$$

$$\Rightarrow E_b \approx E_\gamma$$

6.279. A proton with kinetic energy $T_p = 1.5 \text{ MeV}$ is captured by a deuteron H^2 . Find the excitation energy of the formed nucleus.

Excitation energy: its the energy available within the unstable nucleus.

- It may be released in form of gamma rays, or knocking of an electron after which nucleus becomes stable.
- It is NOT kinetic energy.



E conservation:

$$Q = (\Delta_p + \Delta_d - \Delta_{\text{He}}) 931 = 5.49 \text{ MeV}$$

$$Q = T_{\text{He}} + E_{\text{He}} - T_p \quad (1)$$

p conservation:

$$p_p = p_{\text{He}}$$

$$\Rightarrow \sqrt{2m_p T_p} = \sqrt{2m_{\text{He}} T_{\text{He}}} \Rightarrow T_{\text{He}} = \frac{T_p}{3} \quad (2)$$

From 1,2: $E_{\text{He}} = Q + \frac{2T_p}{3}$

$$= 5.49 + \frac{2(1.5)}{3}$$

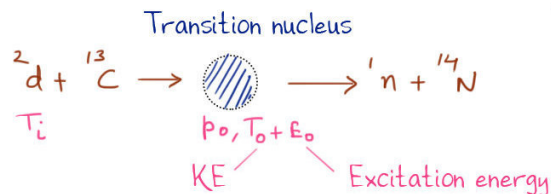
$$= 6.49 \text{ MeV}$$

6.280. The yield of the nuclear reaction $C^{13}(d, n)N^{14}$ has maximum magnitudes at the following values of kinetic energy T_i of bombarding deuterons: 0.60, 0.90, 1.55, and 1.80 MeV. Making use of the table of atomic masses, find the corresponding energy levels of the transitional nucleus through which this reaction proceeds.

→ The maximum 'contained' energy, which occurs at transition phase.

In other words, excitation energy of nucleus

E conservation:



$$= (\Delta_d + \Delta_c - \Delta_n - \Delta_N) 931 = 16.14 \text{ MeV}$$

$$Q = T_0 + E_0 - T_i \quad (1)$$

p conservation: $p_d = p_0$

$$\sqrt{2m_d T_i} = \sqrt{2(m_d + m_c) T_0}$$

$$\Rightarrow T_0 = \frac{m_d}{m_d + m_c} T_i = \frac{2}{15} T_i \quad (2)$$

From 1,2:

$$E_0 = Q + \frac{13}{15} T_i$$

∴ E_0 for, A $16.14 + \frac{13}{15}(0.6) = 16.66 \text{ MeV}$ B 16.92 MeV C 17.49 MeV D 17.7 MeV

Similarly

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6.289. Find the number of neutrons generated per unit time in a uranium reactor whose thermal power is $P = 100 \text{ MW}$ if the average number of neutrons liberated in each nuclear splitting is $\nu = 2.5$. Each splitting is assumed to release an energy $E = 200 \text{ MeV}$.

Unitary method

$$200 \text{ MeV} \longrightarrow 2.5 \text{ neutrons}$$

$$1 \text{ MeV} \longrightarrow \frac{2.5}{200} \text{ neutrons}$$

$$\text{In 1 sec: } \frac{100}{1.6 \times 10^{-19}} \text{ MeV} \longrightarrow \frac{2.5}{200} \times \frac{100}{1.6 \times 10^{-19}} \text{ neutrons} = 7.8 \times 10^{18} \text{ neutrons/sec}$$

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